Marking Scheme

Sec 4 Express Preliminary Examination 2021 Additional Mathematics Paper 2

Qn. No.	**Solution**	**Marks**	**Partial Marks**	**Guidance**
1(a)	$xy = 20------(1)$ $2x-11=2y---(2)$ $x=\frac{2y+11}{2}----(3)$ Sub. (3) into (1): $\left(\frac{2y+11}{2}\right)y=20$ $(2y+11)y=40$ $2y^2+11y-40=0$ $(2y-5)(y+8)=0$ $y=\frac{5}{2}$ or -8 $x=8$ or $-\frac{5}{2}$ Hence A and B are $\left(8,\frac{5}{2}\right)$ and $\left(-\frac{5}{2},-8\right)$.	4m	M1 M1 A1, A1	Substitution Factorise or use general formula
1(b)	$\frac{6x^3-14}{2x^3+x}=3+\frac{-3x-14}{x(2x^2+1)}$ $=3+\frac{A}{x}+\frac{Bx+C}{2x^2+1}$ $=3+\frac{A(2x^2+1)+x(Bx+C)}{x(2x^2+1)}$ By comparing coefficients: $A=-14$ $C=-3$ $2A+B=0$ $-28+B=0$ $B=28$ $\therefore \frac{6x^3-14}{2x^3+x}=3-\frac{14}{x}+\frac{28x-3}{2x^2+1}$	6m	B1 B1 M1 A2 A1	Decompose to proper fraction Partial fraction Alternate method: use substitution Any 2 answers Simplified form

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2(i)	$y=4-e^{\frac{1}{2}x}$ At A, $x=0$, $y=4-e^{\frac{1}{2}(0)}=3$ $\therefore A(0,3)$ $\frac{dy}{dx}=-\frac{1}{2}e^{\frac{1}{2}x}$ At A, $\frac{dy}{dx}=-\frac{1}{2}e^{\frac{1}{2}(0)}=-\frac{1}{2}$ Equation of line AC (tangent at A) is $y=-\frac{1}{2}x+3$	4m	B1 B1 M1 A1	Find x and y coordinates of A M1ft: Attempts to find gradient at A using x-coord. found
2(ii)	At B, $y=0$, $4-e^{\frac{1}{2}x}=0$ $\frac{1}{2}x=\ln 4$ $x=2\ln 4$ or $\ln 16$ $\therefore B(\ln 16,0)$ At C, $y=0$, $-\frac{1}{2}x+3=0$ $x=6$ $\therefore C(6,0)$ Area of shaded region $=\frac{1}{2}(6)(3)-\int_0^{\ln 16}(4-e^{\frac{1}{2}x})dx$ $=9-\left[4x-2e^{\frac{1}{2}x}\right]_0^{\ln 16}$ $=9-\left[4\ln 16-2e^{\frac{1}{2}\ln 16}-\left(0-2e^{\frac{1}{2}(0)}\right)\right]$ $=9-5.09035$ $=3.90965$ $=3.91$ units^2 (to 3 s.f.)	5m	B1 B1 M1 A1 A1	Non-exact value of x is unacceptable B1ft: from eqn. of line AC M1ft: <u>Correct order</u> of subtraction Note: Can also use area under line AC $\left[\int_0^6(-\frac{1}{2}x+3)dx\right]$ A1: Integration of area under the curve

Qn. No.	Solution	Marks	Partial Marks	Guidance
3(i)	$\frac{3x}{3x-2}=1+\frac{2}{3x-2}$ $\int\frac{3x}{3x-2}dx=\int\left(1+\frac{2}{3x-2}\right)dx$ $=x+\frac{2}{3}\ln(3x-2)+c$	3m	B1 B2	Partial fractions B1 for $x + c$, B1 for $\frac{2}{3}\ln(3x-2)$
3(ii)	$\frac{d}{dx}x\ln(3x-2)$ $=x\left(\frac{3}{3x-2}\right)+\ln(3x-2)(1)$ $=\frac{3x}{3x-2}+\ln(3x-2)$	2m	B2	If <u>answer is incorrect</u>, give M1 for use of product law with differentiation of $\ln(3x-2)$ done correctly. B2: 1m for each term
3(iii)	$\int\ln(3x-2)dx$ $=x\ln(3x-2)-\int\frac{3x}{3x-2}dx$ $=x\ln(3x-2)-\int1+\frac{2}{3x-2}dx$ $=x\ln(3x-2)-\left[x+\frac{2}{3}\ln(3x-2)\right]+c$ $=x\ln(3x-2)-x-\frac{2}{3}\ln(3x-2)+c$	3m	M1 M1 A1	Able to link to (i) & (ii) Integrate correctly Answer includes constant
4(a)	Sub. $y=-5x-2$ into $y=mx^2+3$ $-5x-2=mx^2+3$ $mx^2+5x+5=0$ b^2-4ac $=5^2-4(m)(5)$ $=25-20m$	4m	M1 M1	

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	Since $m<1$, $-20m>-20$ $25-20m>5$ Since the <u>discriminant > 0</u>, there are <u>real roots/ solutions</u>. So the line intersects the curve.		M1 A1 (AG)	 Correct argument and conclusion
4(b) (i)	Since the curve $y=ax^2+bx+a$ lies completely above the x-axis, $a>0$ and $b^2-4a^2<0$ (or $b^2<4a^2$)	2m	 B1 B1	
4(b) (ii)	Since b = 2, $2^2-4a^2<0$ $4-4a^2<0$ [or $1-a^2<0$] $(2+2a)(2-2a)<0$ [or $(1+a)(1-a)<0$] $a<-1$ or $a>1$ Since a > 0, hence a > 1.	3m	 M1 A1 A1	 M1 for substituting and simplifying the inequality
5(a) (i)	$y=\dfrac{\cos x}{3+\sin x}$ $\dfrac{dy}{dx}=\dfrac{(3+\sin x)(-\sin x)-\cos x(\cos x)}{(3+\sin x)^2}$ $=\dfrac{-3\sin x-\sin^2 x-\cos^2 x}{(3+\sin x)^2}$ $=\dfrac{-3\sin x-1}{(3+\sin x)^2}$ (Shown)	3m	 M2 AG1	 M1 for use of quotient or product rule. Dependent on working
5(a) (ii)	At the turning point, $\dfrac{dy}{dx}=0$. $\dfrac{-3\sin x-1}{(3+\sin x)^2}=0$ $-3\sin x-1=0$ $\sin x=-\dfrac{1}{3}$ Ref. $\angle=\sin^{-1}\dfrac{1}{3}$ $=0.33984$	5m	 M1	 M1: equate derivative to zero and simplify

<table>
<tr><th>Qn. No.</th><th>Solution</th><th>Marks</th><th>Partial Marks</th><th>Guidance</th></tr>
<tr><td></td><td>Since $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$,
$x = -0.340$ (to 3 s.f.)

Either use 2nd derivative test,
$\frac{d^2y}{dx^2} = \frac{(3+\sin x)^2(-3\cos x)-(-3\sin x-1)2(3+\sin x)(\cos x)}{(3+\sin x)^4}$
Sub. $x = -0.33984$,
$\frac{d^2y}{dx^2} = \frac{-20.11323758}{50.56768022}$
$= -0.397749 < 0$
The curve has a <u>maximum turning point</u> at $x = -0.340$ (to 3 s.f.).

Or use 1st derivative test,
<table><tr><td>x</td><td>$x < -0.33984$</td><td>$x = -0.33984$</td><td>$x > -0.33984$</td></tr><tr><td>$\frac{dy}{dx}$</td><td>+</td><td>0</td><td>–</td></tr><tr><td>Shape of tangent</td><td>/</td><td>—</td><td>\</td></tr></table>
The curve has a <u>maximum turning point</u> at $x = -0.340$ (to 3 s.f.).</td><td></td><td>A1

Either

M1
A1

A1

Or

B2

A1</td><td>M1 for use of quotient or product rule.
A1: Lose 1m for any error.

Correct inequality and conclusion about nature of turning point

Table
B1: sign of dy/dx
B1: shape of tangent

Make a conclusion about nature of turning point</td></tr>
<tr><td>5(b)</td><td>Given $\frac{dx}{dt} = 0.4$ units/s
$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$
$= \frac{-3\sin x - 1}{(3+\sin x)^2} \times 0.4$
Sub. $x = 0.5$,
$\frac{dy}{dt} = -0.20140 \times 0.4$
$= -0.0806$ units/s (to 3 s.f.)</td><td>3m</td><td>M1

M1

A1</td><td>Form a chain rule

Sub. x and $\frac{dx}{dt}$ values</td></tr>
</table>

<table>
<tr><th>Qn. No.</th><th>Solution</th><th>Mark s</th><th>Partial Marks</th><th>Guidance</th></tr>
<tr><td>6(a) (i)</td><td>
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>y</td><td>14</td><td>50</td><td>66</td><td>104</td><td>150</td><td>205</td></tr>
<tr><td>$\frac{y}{x}$</td><td>14</td><td>25</td><td>22</td><td>26</td><td>30</td><td>34.2</td></tr>
</table>
Incorrect reading of $y = 50$.</td><td>2m</td><td>P1

B1</td><td>See last page</td></tr>
<tr><td>6(a) (ii)</td><td>Drawing of straight line.

The correct value of y occurs when
$\frac{y}{x} = 18$,
$y = 18 \times 2 = 36$.

Hence, the correct value of y is 36.</td><td>2m</td><td>L1

B1</td><td>Dependent on graph</td></tr>
<tr><td>6(a) (iii)</td><td>$y = px(x+k)$
$\frac{y}{x} = px + pk$

From the graph,
gradient of line $= \frac{28-12}{4.5-0.5}$
$p = 4$

$\frac{y}{x}$-intercept $= 10$
$pk = 10$
Sub. $p = 4$,
$k = 2.5$</td><td>3m</td><td>B1

B1

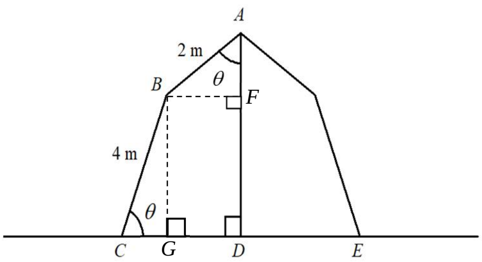
B1</td><td>Form eqn. of str. line

Estimated value 3.5 to 4.5

Estimated value</td></tr>
</table>

Qn. No.	Solution	Marks	Partial Marks	Guidance
6(b)	$\frac{x^2}{a}-\frac{3y^2}{b}=1$ $x^2b-3y^2a=ab$ $y^2=\frac{b}{3a}x^2-\frac{b}{3}$ Sub. gradient $\frac{b}{3a}=\frac{4}{27}$ and $\left(3,-\frac{8}{9}\right)$, $-\frac{8}{9}=\frac{4}{27}(3)-\frac{b}{3}$ $b=4$ Sub. $b=4$ into gradient $\frac{4}{3a}=\frac{4}{27}$ $3a=27$ $a=9$ $\therefore\ a=9,\ b=4$	4m	B1 M1 A1 A1	Form eqn. of str. line
7(i)	Gradient of ST $=\frac{2-(-6)}{5-1}$ $=2$ Gradient of perpendicular to ST is $-\frac{1}{2}$. Mid-point of ST $=\left(\frac{1+5}{2},\frac{-6+2}{2}\right)$ $=(3,-2)$ Eqn. of perpendicular bisector of ST is $y-(-2)=-\frac{1}{2}(x-3)$ $y+2=-\frac{1}{2}(x-3)$ or $y=-\frac{1}{2}x-\frac{1}{2}$	4m	M1 A1 B1 A1	Eqn. in any equivalent form (AEF)

Qn. No.	Solution	Marks	Partial Marks	Guidance
7(ii)	$y = x - 8$ ---- (1) $y = -\frac{1}{2}x - \frac{1}{2}$ ---- (2) Sub. (1) into (2): $x - 8 = -\frac{1}{2}x - \frac{1}{2}$ $\frac{3}{2}x = 7\frac{1}{2}$ $x = 5$ Sub. into (1): $y = 5 - 8$ $= -3$ $\therefore$ centre of circle O is $(5, -3)$. Length OS (or OT) $= \sqrt{(1-5)^2 + (-6-(-3))^2}$ $= \sqrt{25}$ $= 5$ units Eqn. of circle is $(x-5)^2 + (y+3)^2 = 25$ or $x^2 + y^2 - 10x + 6y + 9 = 0$	4m	M1 A1 M1 A1	Solve simultaneous eqns.
7(iii)	Mid-point of SR $= \left(\frac{1+9}{2}, \frac{-6+0}{2}\right)$ $= (5, -3)$ which is the centre O. $\therefore$ SR is a diameter of the circle.(Shown) <u>Or</u> length of SR = $\sqrt{(1-9)^2 + (-6-0)^2}$ = 10 Radius = 5 Since SR is twice the radius and both S and R lie on the circle, then SR is the diameter of the circle. <u>Or</u>	2m	M1 A1 (AG)	M1: Use mid-point formula. A1: Show that mid-point of SR is equal to the coord. of centre of circle

Qn. No.	Solution	Marks	Partial Marks	Guidance
	Gradient of $SR = \frac{0+6}{9-1} = \frac{3}{4}$ Eqn. of SR is $y-0=\frac{3}{4}(x-9)$ $y=\frac{3}{4}x-\frac{27}{4}$ Sub. $x = 5$, $y=\frac{3}{4}(5)-\frac{27}{4}$ $=-3$ Since centre of circle $(5,-3)$ lies on the line SR, then SR must be the diameter of the circle.			
8(i)	 $\sin\theta = \frac{BG}{4}$ $BG = 4\sin\theta = FD$ $\cos\theta = \frac{AF}{2}$ $AF = 2\cos\theta$ $AD = AF + FD$ $= 4\sin\theta + 2\cos\theta$ (shown)	2m	B1 B1 (AG)	
8(ii)	$AD = 4\sin\theta + 2\cos\theta$ $= R\cos(\theta-\alpha)$ $= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ By comparing coefficients of θ:	3m	B1 B1	Form 2 eqns.

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	$2 = R\cos\alpha$ — (1) $4 = R\sin\alpha$ — (2) $\frac{(2)}{(1)}: \tan\alpha = \frac{4}{2} = 2$ $\alpha = 63.435°$ $(1)^2 + (2)^2 : R^2\sin^2\alpha + R^2\cos^2\alpha = 4^2 + 2^2$ $R^2 = 20$ $R = \sqrt{20}$ or $2\sqrt{5}$ $\therefore AD = 2\sqrt{5}\cos(\theta - 63.4)°$ (to 1 d.p.) or $AD = \sqrt{20}\cos(\theta - 63.4)°$ (to 1 d.p.)		B1	Must form eqn for *AD* or subtract 1m
8(iii)	Max. $AD = 2\sqrt{5}$ or $\sqrt{20}$ or 4.47 m (to 3s.f.) When $\cos(\theta - 63.435)° = 1$ $\theta - 63.435 = 0$ $\theta = 63.4°$ (to 1 d.p.)	2m	B1 B1	
8(iv)	$2\sqrt{5}\cos(\theta - 63.435)° = 4.1$ $\cos(\theta - 63.435)° = 0.91679$ Ref. $\angle$ = 23.539° $\theta - 63.435 = -23.539, 23.539$ $\theta = 39.896°, 86.974°$ $\theta = 39.9°, 87.0°$ (to 1 d.p.) Khalid and Travis could have used different angles of θ to pitch their tents.	3m	M1 A1 A1	2 values of θ Valid explanation relating to the 2 angles of θ

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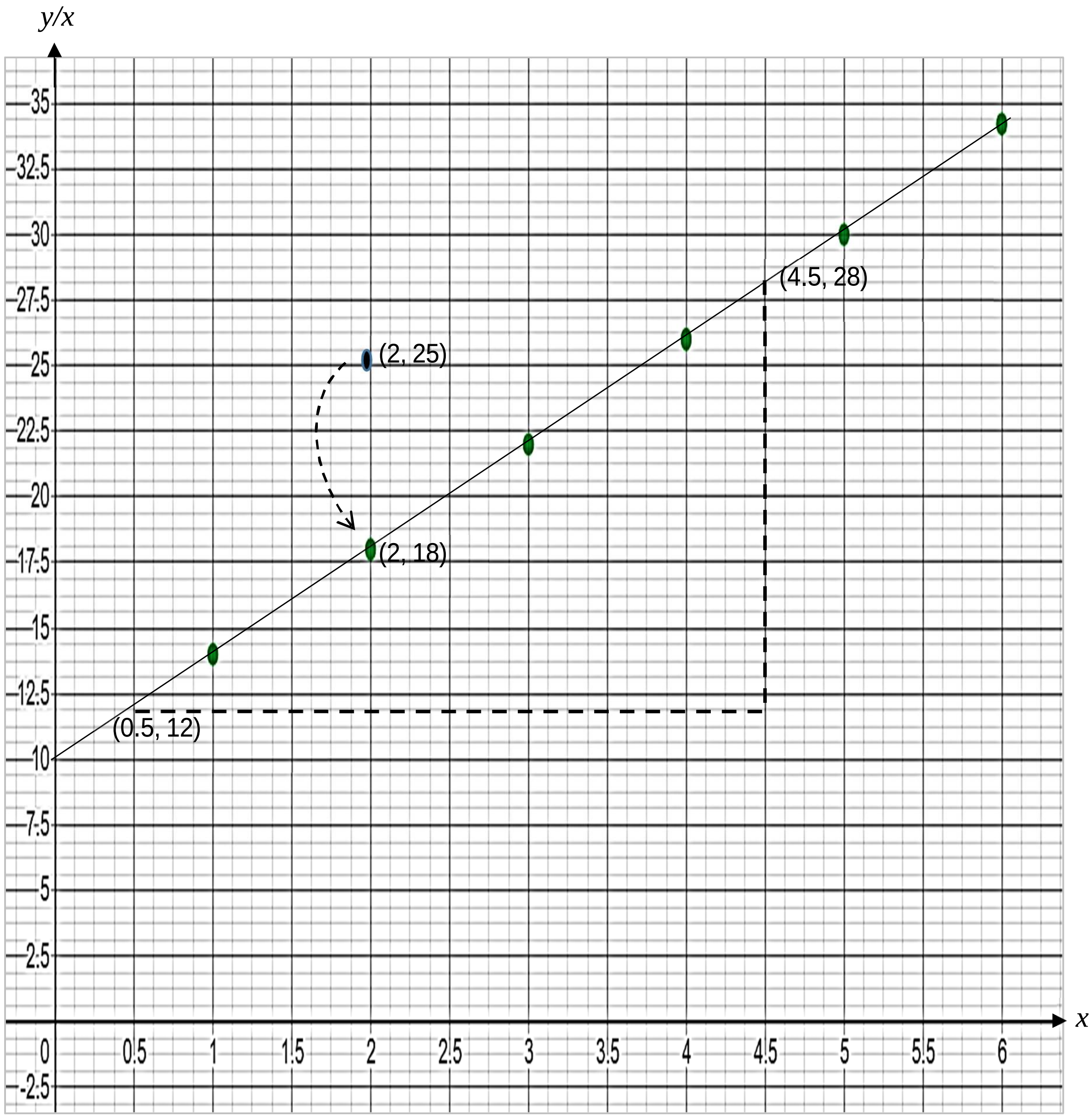
9(a)	$3^{2x}-3^x=3^{x+2}-9$ Sub. $u=3^x$, $u^2-u=9u-9$ $u^2-10u+9=0$ $(u-1)(u-9)=0$ $u=1$ or $u=9$ $3^x=1$ $x=\frac{\lg 1}{\lg 3}$ $x=0$ or $3^x=9$ $\quad=3^2$ $x=2$ $\therefore x=0$ or $x=2$	4m	 M1 M1 A1 A1	 Substitution method Factorise
9(b)	$\frac{8}{\log_x y}-\log_x y+\log_y x=0$ $\frac{8}{\log_x y}-\log_x y+\frac{\log_x x}{\log_x y}=0$ $8-(\log_x y)^2+1=0$ $(\log_x y)^2=9$ $\log_x y=\pm 3$ $y=x^3$ or x^{-3} $\left(\text{or } \frac{1}{x^3}\right)$	4m	 M1 M1 A2	 M1: Apply change of base law M1: Simplify into quadratic eqn. 1m for each answer

Qn. No.	Solution	Marks	Partial Marks	Guidance
9(c)	$\log_2(7-2x)-\log_2(x-5)=4$ $\log_2\dfrac{(7-2x)}{x-5}=4$ $\dfrac{(7-2x)}{x-5}=2^4$ $7-2x=16x-80$ $18x=87$ $x=\dfrac{29}{6}$ or equivalent However, $\log_2\left(7-2\left(\dfrac{29}{6}\right)\right)=\log_2\left(-\dfrac{8}{3}\right)$ $\left[\text{or } \log_2\left(\dfrac{29}{6}-5\right)=\log_2\left(-\dfrac{1}{6}\right)\right]$ This does not exist, therefore there are no real solutions.	4m	 M1 M1 A1 A1	 M1: Apply Quotient Law M1: Convert to index form Correct argument and conclusion

6(b)



~ End of marking scheme ~