Fairfield Methodist School (Secondary)
2021 Secondary 4 Express Preliminary Examination
Additional Mathematics Paper 1
Marking Scheme

Qn		Description	Allocation of marks
1		$x+y=16$ $y=16-x$ --- (1) Apply Cosine Rule: $x^2+y^2-2xy\cos 120°=14^2$ $x^2+y^2+xy=196$ --- (2) Sub (1) into (2): $x^2+(16-x)^2+x(16-x)=196$ $x^2+x^2-32x+256+16x-x^2=196$ $x^2-16x+60=0$ $(x-6)(x-10)=0$ $x=6$ or $x=10$ $y=10$ or $y=6$ In either case, area of triangle is $=\frac{1}{2}(6)(10)\sin 60°$ $=30\times\frac{\sqrt{3}}{2}$ $=15\sqrt{3}$ cm^2	B1: (1) B1: (2) M1: Substitution A1: **at least one *x*, *y* pair correct** A1
2	**(i)**	$\frac{360°}{b}=180°$ $b=2$ (shown)	 AG1
2	**(ii)**	Minimum value = 250 – 60 = 190 Watts	 B1
2	**(iii)**	$209=250+60\cos(2x)$ $60\cos(2x)=-41$ $\cos(2x)=-\frac{41}{60}$ $2x=\cos^{-1}\left(-\frac{41}{60}\right)$ $2x=133.10°$ $x=66.6°$ (to 1 dp) Using the symmetry of the graph, the range of values is $-90°\le x<-66.6°$ or $66.6°<x\le 90°$	 B1: Make cos(2*x*) subject B1: 66.6 degrees B1: Both ranges

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3	**(i)**	$T_{r+1}=\binom{7}{r}\left(x^5\right)^{7-r}\left(\frac{1}{x}\right)^r$ $=\binom{7}{r}x^{35-6r}$ However, when r is a whole number, <u>35 is odd and 6r is even</u>, so $35-6r$ is odd **OR** a <u>common factor of 2 cannot be extracted</u> from $35-6r$. Hence, there are no even powers of x.	B1 (substitution into general term) B1 B1 (explanation)
3	**(ii)**	From part **(i)**, we can ignore the first binomial term completely as it has no even powers of x. Expand the second binomial term: $(kx)^7+7(kx)^6(3)+...=1344x^6+...$ Comparing coefficients: $21k^6=1344$ $k^6=64$ $k=2$ (as k is positive) The coefficient of x^4 is $\binom{7}{3}(2)^4(3)^3=15120$.	M1: Compare coefficients A1 B1
4		$\frac{dy}{dx}=\frac{(x-1)2-(2x-5)}{(x-1)^2}-12$ $=\frac{2x-2-2x+5}{(x-1)^2}-12$ $=\frac{3}{(x-1)^2}-12$ For increasing function, $\frac{dy}{dx}>0$. $\frac{3}{(x-1)^2}-12>0$ $\frac{3-12(x-1)^2}{(x-1)^2}>0$ $\frac{-12x^2+24x-9}{(x-1)^2}>0$ As $(x-1)^2\geq 0$ for all x, $-12x^2+24x-9>0$ $-3(2x-1)(2x-3)>0$ $\frac{1}{2}<x<\frac{3}{2},\ x\neq 1$ **or** $\frac{1}{2}<x<1$ and $1<x<\frac{3}{2}$.	M1 (quotient rule or product rule applied correctly) A1 M1: dy/dx > 0 M1: Factorise A1

Qn		Description	Allocation of marks
		Alternative Solution $y=\frac{2x-5-12x(x-1)}{x-1}$ $=\frac{2x-5-12x^2+12x}{x-1}$ $=\frac{-12x^2+14x-5}{x-1}$ $\frac{dy}{dx}=\frac{(x-1)(-24x+14)-(-12x^2+14x-5)}{(x-1)^2}$ $=\frac{-24x^2+14x+24x-14+12x^2-14x+5}{(x-1)^2}$ $=\frac{-12x^2+24x-9}{(x-1)^2}$ For increasing function, $\frac{dy}{dx}>0$. As $(x-1)^2\geq 0$ for all x, $-12x^2+24x-9>0$ $-3(2x-1)(2x-3)>0$ $\frac{1}{2}<x<\frac{3}{2},\ x\neq 1$ **or** $\frac{1}{2}<x<1$ and $1<x<\frac{3}{2}$.	 [M1] [A1] [M1] [M1] [A1]
5	**(i)**	When $t=0, P=0: 0=\log_{1.02}(c)\Rightarrow c=1$ (shown) When $t=3, P=15: 15=\log_{1.02}(9m+1)$ $9m+1=1.02^{15}$ $m=\frac{1.02^{15}-1}{9}$ $m=0.0384$	B1 (show) M1: Convert from log to exp form A1
5	**(ii)**	When $t=24$: $P=\log_{1.02}\left(0.0384(24)^2+1\right)$ $=\frac{\ln\left(0.0384(24)^2+1\right)}{\ln 1.02}$ $=158.6\%$ The model is not appropriate after two years as **percentages cannot be larger than 100%.**	 B1 B1 (explanation)

Qn		Description	Allocation of marks
6	**(i)**	$\frac{\mathrm{d}y}{\mathrm{d}x}=\int 6\sin 2x\, dx$ $=6\left(\frac{-\cos 2x}{2}\right)+c$ $-3\cos 2x+c$ Gradient of tangent = 5 When $x=\frac{\pi}{2},\frac{dy}{dx}=5:$ $-3\cos 2\left(\frac{\pi}{2}\right)+c=5$ $-3(-1)+c=5$ $c=2$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-3\cos 2x+2$	M1 M1 (correct gradient of tangent) A1 (withhold A1 if no *c*)
6	**(ii)**	$y=\int -3\cos 2x+2\, dx$ $=-3\left(\frac{\sin 2x}{2}\right)+2x+c'$ $=-\frac{3}{2}\sin 2x+2x+c'$ When $x=\frac{\pi}{12},y=-\frac{3}{4}:$ $-\frac{3}{4}=-\frac{3}{2}\sin 2\left(\frac{\pi}{12}\right)+2\left(\frac{\pi}{12}\right)+c'$ $-\frac{3}{4}=-\frac{3}{4}+\frac{\pi}{6}+c'$ $c'=-\frac{\pi}{6}$ $y=-\frac{3}{2}\sin 2x+2x-\frac{\pi}{6}$	M1 M1 A1 (withhold A1 if no *c*)

Qn		Description	Allocation of marks
7	**(i)**	$2x^2-4x+6=2(x^2-2x)+6$ $=2(x^2-2x+\left(\frac{-2}{2}\right)^2-\left(\frac{-2}{2}\right)^2)+6$ $=2\left[(x-1)^2-1)\right]+6$ $=2(x-1)^2-2+6$ $=2(x-1)^2+4$ $-x^2-4x-1=-(x^2+4x)-1$ $=-(x^2+4x+\left(\frac{4}{2}\right)^2-\left(\frac{4}{2}\right)^2)-1$ $=-\left[(x+2)^2-4)\right]-1$ $=-(x+2)^2+4-1$ $=-(x+2)^2+3$	M1 A1 M1 A1
7	**(ii)**	Maximum point of $-(x+2)^2+3$ is $(-2,\ 3)$ Minimum point of $2(x-1)^2+4$ is $(1,\ 4)$ Therefore the max value of $-(x+2)^2+3$ which is 3 is less than the min value of $2(x-1)^2+4$ which is 4, the two curves <u>will not intersect</u>.	B1 (or on diagram) B1 (or on diagram) B1 (explanation or diagram shown)
8a	**(i)**	$-90° \le x \le 90°$	B1
8a	**(ii)**	$-90° < x < 90°$	B1
8b	**(i)**	$\sin 75° = \sin(30°+45°)$ $=\sin 30° \cos 45° + \cos 30° \sin 45°$ $=\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$ $=\frac{1}{2\sqrt{2}}+\frac{\sqrt{3}}{2\sqrt{2}}$ $=\frac{1+\sqrt{3}}{2\sqrt{2}}$ (shown)	 M1: Correct special trigo ratios AG1

Qn		Description	Allocation of marks
8b	**(ii)**	$\sec 150° = \dfrac{1}{\cos 150°}$ $= \dfrac{1}{\cos 2(75°)}$ $= \dfrac{1}{1-2\sin^2 75°}$ $= \dfrac{1}{1-2\left(\dfrac{1+\sqrt{3}}{2\sqrt{2}}\right)^2}$ $= \dfrac{1}{1-2\left(\dfrac{4+2\sqrt{3}}{8}\right)}$ $= \dfrac{1}{1-\left(1+\dfrac{\sqrt{3}}{2}\right)}$ $= -\dfrac{2}{\sqrt{3}}$ $= -\dfrac{2}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$ $= -\dfrac{2}{3}\sqrt{3}$	M1 (double angle) M1 (expand the square of (i) correctly) M1: simplify fraction to get this term A1
9	**(i)**	$LHS = \dfrac{\cos\theta}{\sin\theta} - 2\sin\theta\cos\theta$ $= \dfrac{\cos\theta - 2\sin^2\theta\cos\theta}{\sin\theta}$ $= \dfrac{\cos\theta\left(1-2\sin^2\theta\right)}{\sin\theta}$ $= \dfrac{\cos\theta\cos 2\theta}{\sin\theta}$ $= \cot\theta\cos 2\theta$ $= RHS$ (proved) <u>Alternative Method</u> $RHS = \dfrac{\cos\theta}{\sin\theta}\left(1-2\sin^2\theta\right)$ $= \dfrac{\cos\theta}{\sin\theta} - \dfrac{2\sin^2\theta\cos\theta}{\sin\theta}$ $= \cot\theta - \dfrac{\sin 2\theta\sin\theta}{\sin\theta}$ $= \cot\theta - \sin 2\theta$ $= LHS$ (proved)	M1: $\cot\theta$ and $\sin 2\theta$ M1: Combine fraction and take out common factor AG1 [M1]: cot and double angle formula [M1]: Expand and split fraction [AG1]

Qn		Description	Allocation of marks
9	**(ii)**	$3\cot\theta - 3\sin 2\theta = \cos 2\theta$ $\cot\theta - \sin 2\theta = \frac{1}{3}\cos 2\theta$ $\cot\theta\cos 2\theta = \frac{1}{3}\cos 2\theta$ $3\cot\theta\cos 2\theta - \cos 2\theta = 0$ $\cos 2\theta\left(3\cot\theta - 1\right) = 0$ $\cos 2\theta = 0$ or $\cot\theta = \frac{1}{3} \Rightarrow \tan\theta = 3$ $2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$, $\theta = 1.25$ $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$, $\theta = 1.25$ or $\theta = 0.785, 2.36$, $\theta = 1.25$ (to 3 sf)	M1: Use part (i) and factorise M1: Change cot to tan A1 for any pair of solns A1 for all correct
10	**(i)**	Arc length PQ = $r\theta$ $2r + r\theta = 16$ $r = \frac{16}{2+\theta}$ $A = \frac{1}{2}\left(\frac{16}{2+\theta}\right)^2\theta$ $= \frac{(256)\theta}{2(2+\theta)^2}$ $= \frac{128\theta}{(2+\theta)^2}$ (shown)	M1: Make r subject M1: Allow FT1 for their r if area formula correct AG1
10	**(ii)**	$\frac{\mathrm{d}A}{\mathrm{d}\theta} = \frac{(2+\theta)^2 128 - 128\theta(2)(2+\theta)}{(2+\theta)^4}$ $= \frac{(2+\theta)\left[128(2+\theta) - 256\theta\right]}{(2+\theta)^4}$ $= \frac{256 - 128\theta}{(2+\theta)^3}$ At stationary point, $\frac{\mathrm{d}A}{\mathrm{d}\theta} = 0$. $\frac{256 - 128\theta}{(2+\theta)^3} = 0$ $256 - 128\theta = 0$ $128\theta = 256$ $\theta = 2$ Max $A = \frac{128(2)}{16}$ $= 16 \text{ cm}^2$	M2 (1st mark for correct *form* of quotient rule, 2nd mark for correct answer) M1 A1

<table>
<tr><th colspan="2">Qn</th><th>Description</th><th>Allocation of marks</th></tr>
<tr><td>11</td><td>(i)</td><td>$f'(x)=6x^2+10x+k$
Sub x = 2 and equate to 42 by Remainder Theorem:
$24+20+k=42$
k = 42 – 44
k = – 2
By Factor Theorem,
$f(\frac{3}{2}) = 2\left(\frac{3}{2}\right)^3+5\left(\frac{3}{2}\right)^2-2\left(\frac{3}{2}\right)+p=0$
$2\left(\frac{27}{8}\right)+5\left(\frac{9}{4}\right)-2\left(\frac{3}{2}\right)+p=0$
$\frac{27+45-12}{4}+p=0$
$15+p=0$
$p=-15$</td><td>M1: Sub into correct expression and equate

A1

M1

A1</td></tr>
<tr><td>11</td><td>(ii)</td><td>$$\begin{array}{r|rrrr} & 2x & +7 & & \\ \hline x^2-x-2 & 2x^3 & +5x^2 & -2x & -15 \\ - & 2x^3 & -2x^2 & -4x & \\ \hline & & 7x^2 & +2x & -15 \\ - & & 7x^2 & -7x & -14 \\ \hline & & & 9x & -1 \end{array}$$
The remainder is $(9x-1)$.</td><td>M1: Leading term in quotient correct

A1</td></tr>
<tr><td>11</td><td>(iii)</td><td>$2x^3+5x^2-2x-15=(2x-3)(ax^2+bx+c)$
Comparing leading coefficient and constant term:
$2ax^3 = 2x^3$ and $-3c = -15$
So, a = 1 and c = 5.
Comparing coefficient of x:
$-3b + 2c = -2$
$-3b = -12$
b = 4
Therefore, $a=1, b=4, c=5$.
$f(x)=(2x-3)(x^2+4x+5)$ (shown)</td><td>B2 (any two correct, soi)

AG1 (shown with a, b, c)</td></tr>
<tr><td>11</td><td>(iv)</td><td>$2x^3+5x^2-2x-15=(2x-3)(x^2+4x+5)=0$
$2x-3=0 \Rightarrow x=\frac{3}{2}$ or 1.5
or $x^2+4x+5=0$
$b^2-4ac = 16-4(1)(5)$
$= -4 < 0$
As discriminant is less than 0, there are no other real roots
or $x=\frac{-4\pm\sqrt{-4}}{2}$ (NA, no real roots)</td><td>B1 (real root)

B1 (with explanation, allow for FT)</td></tr>
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Qn		Description	Allocation of marks
12	**(i)**	Gradient of $DC = \frac{3-1}{8-2}$ $= \frac{1}{3}$ As line DC passes through $D(5, 6)$: $y-6=\frac{1}{3}(x-5)$ or $y=\frac{1}{3}x+\frac{13}{3}$	 B1 B1
12	**(ii)**	Gradient of $BC = -3$ As line BC passes through $B(9, 4)$, $y-4=-3(x-9)$ $y-4=-3x+27$ $y=-3x+31$ **--- (1)** From **(i):** $y=\frac{1}{3}x+\frac{13}{3}$ **--- (2)** Equating **(1)** and **(2)**: $-3x+31=\frac{1}{3}x+\frac{13}{3}$ $-9x+93=x+13$ $10x=80$ $x=8$ Sub to **(1):** $y=-3(8)+31=7$ Hence, $C(8,7)$.	B1: Gradient of BC M1: Use of simultaneous equation A1
12	**(iii)**	Area of trapezium $ABCD$ $=\frac{1}{2}\begin{vmatrix} 3 & 9 & 8 & 5 & 3 \\ 2 & 4 & 7 & 6 & 2 \end{vmatrix}$ $=\frac{1}{2}[(12+63+48+10)-(18+32+35+18)]$ $=\frac{1}{2}[133-103]$ $=15 \text{ units}^2$ *<u>Alternative Method</u>* $AB=\sqrt{(9-3)^2+(4-2)^2}=\sqrt{40}$ units $DC=\sqrt{(8-5)^2+(7-6)^2}=\sqrt{10}$ units $BC=\sqrt{(9-8)^2+(7-4)^2}=\sqrt{10}$ units Area of trapezium $=\frac{1}{2}\times\left(2\sqrt{10}+\sqrt{10}\right)\times\sqrt{10}$ $=\frac{1}{2}\times 3\sqrt{10}\times\sqrt{10}$ $= 15 \text{ units}^2$	 M1: FT A1 [M1]: Any correctly applied distance formula with FT [A1]
12	**(iv)**	From D to B, x increases by 4 and y decreases by 2. Using similar triangles with $DP : PB = 3 : 1$,	

Qn		Description	Allocation of marks
		From D to P, x increases by $4\times\frac{3}{4}$ = 3 units and y decreases by $2\times\frac{3}{4}=1.5$ units. Hence, $P(5+3,6-1.5)=P(8,4.5)$	B1
13	**(i)**	$a=3k-4kt$ When t = 0, a = 1.8: $1.8=3k$ $k=0.6$ $v=3+1.8t-1.2t^2$ When v = 0: $3+1.8t-1.2t^2=0$ $12t^2-18t-30=0$ $6(t+1)(2t-5)=0$ As t is positive, $t=2.5$ h .	B1 B1 M1: Equate expression of v to 0 A1
13	**(ii)**	The cyclist **<u>changed direction</u>** at t = 2.5 h, so the total distance travelled would be greater than PQ.	B1
13	**(iii)**	$s=\int 3+1.8t-1.2t^2 dt$ $s=3t+0.9t^2-0.4t^3+c$ When t = 0, s = 0: c = 0 $s=3t+0.9t^2-0.4t^3$ When t = 2.5: $s=3(2.5)+0.9(2.5)^2-0.4(2.5)^3$ $s=6.875$ km When t = 5: $s=3(5)+0.9(5)^2-0.4(5)^3$ $s=-12.5$ km Total distance = $(6.875\times 2)+12.5$ = 26.25 km Average speed = $\frac{26.25}{5}$ = **5.25 km/h** <u>Alternative Method</u> Tot Dist = $\left\|\int_0^{2.5}\left(3+1.8t-1.2t^2\right)dt\right\|+\left\|\int_{2.5}^{5}\left(3+1.8t-1.2t^2\right)dt\right\|$ $=\left[3t+0.9t^2-0.4t^3\right]_0^{2.5}+\left[-3t-0.9t^2+0.4t^3\right]_{2.5}^{5}$ $=6.875+(12.5+6.875)$ $=26.25$ km Average speed = $\frac{26.25}{5}$ = **5.25 km/h**	FT1: for integration M1: Sub for t at turning point M1: Sub for t at end point A1 B1 [FT1: Integrate correctly] [M1: Sub for t at turning point] [M1: Sub for t at end point, modulus correct] [A1] [B1]

END OF MARKING SCHEME