NAME: _____ **( )** **CLASS:** _____

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2021
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS 4049/01

Paper 1

Date: 27 August 2021 **Duration: 2 hours 15 minutes**

Candidates answer on Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use

Table of Penalties		**Question Number**		
Presentation	□ 1 □ 2		Parent's/Guardian's Signature	**90**
Rounding Off	□ 1			

Setter: Mr Wilson Ho and Mr Nicholas Chee

This question paper consists of 19 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{r}a^{n-r}b^r + ... + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

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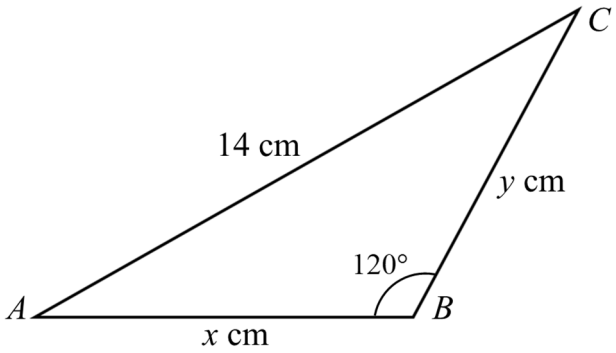
1



In triangle ABC, $AB = x$ cm, $BC = y$ cm, $AC = 14$ cm and angle $ABC = 120°$.

Given that the perimeter of triangle ABC is 30 cm, find the exact area of triangle ABC. [5]

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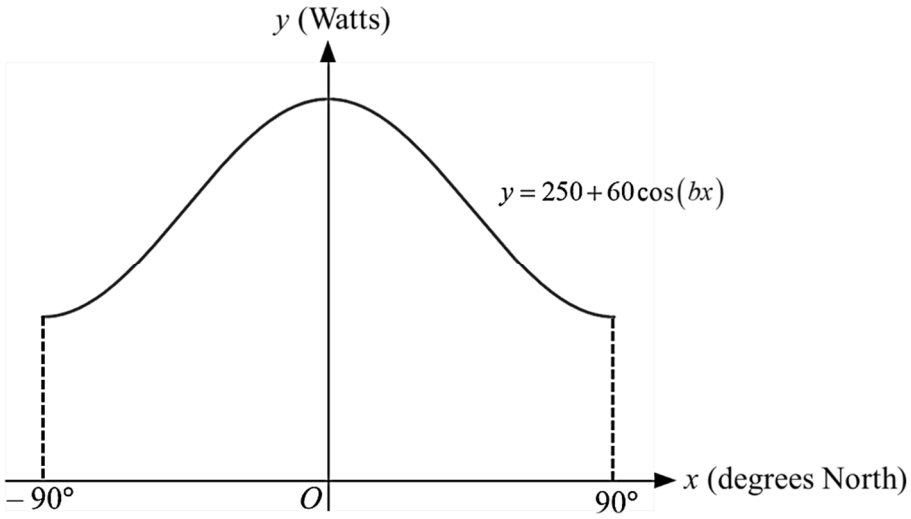
2 In climate studies, the relationship between daily solar radiation, y Watts, and the latitude on earth, x degrees North, is modelled by the equation

$$y = 250 + 60\cos(bx) \text{ for } -90° \le x \le 90°, \text{ where } b \text{ is a constant.}$$

The graph below shows the graph of y against x.



(i) Show that $b = 2$. [1]

(ii) Write down the minimum daily solar radiation based on the model. [1]

(iii) A place has a *polar* climate if it receives a daily solar radiation of less than 209 Watts. Find the range of values of x which represent latitudes with a *polar* climate. [3]

3 **(i)** By considering the general term in the expansion of $\left(x^5+\frac{1}{x}\right)^7$, explain why there are no even powers of x in its expansion. [3]

(ii) Given that the coefficient of x^6 in the expansion of $\left(x^5+\frac{1}{x}\right)^7+(kx+3)^7$ is 1344, where k is a positive constant, find the coefficient of x^4 in the expansion. [3]

4 A curve has equation $y = \frac{2x-5}{x-1} - 12x$, where $x \neq 1$.

Find the range of values of x for which y is an increasing function of x. [5]

5 The percentage of a fully vaccinated population, P %, in a given country, t months after the start of their vaccination programme is modelled by the equation:

$$P = \log_{1.02}\left(mt^2 + c\right), \text{ where } m \text{ and } c \text{ are constants.}$$

In that country, none of the population was fully vaccinated right before the start of the vaccination programme. 3 months later, 15% of the population was fully vaccinated.

(i) Show that $c = 1$, and find m to 3 significant figures. [3]

(ii) Explain, with relevant calculations, why the given model is inappropriate two years after the start of the vaccination programme. [2]

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6 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6\sin 2x$. The normal to the curve has a gradient of $-\frac{1}{5}$ at the point where $x = \frac{\pi}{2}$. Also, the curve passes through the point $\left(\frac{\pi}{12}, -\frac{3}{4}\right)$.

(i) Find an expression for the gradient function of the curve. [3]

(ii) Find the equation of the curve. [3]

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7 **(i)** Express each of $2x^2 - 4x + 6$ and $-x^2 - 4x - 1$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [4]

(ii) Use your answers from part **(i)** to explain why the curves with equations $y = 2x^2 - 4x + 6$ and $y = -x^2 - 4x - 1$ will not intersect. [3]

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8 Do not use a calculator for this question.

(a) State the values, in degrees, between which each of the following must lie:

(i) the principal value of $\sin^{-1} x$, [1]

(ii) the principal value of $\tan^{-1} x$. [1]

(b) **(i)** Show that $\sin 75° = \dfrac{1+\sqrt{3}}{2\sqrt{2}}$. [2]

(ii) **Hence**, express $\sec 150°$ in the form $a\sqrt{3}$, where a is a rational number. [4]

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9 **(i)** Prove the identity $\cot\theta - \sin 2\theta = \cot\theta \cos 2\theta$. [3]

(ii) **Hence**, solve the equation $3\cot\theta - 3\sin 2\theta = \cos 2\theta$, for $0 \le \theta \le \pi$ radians. [4]

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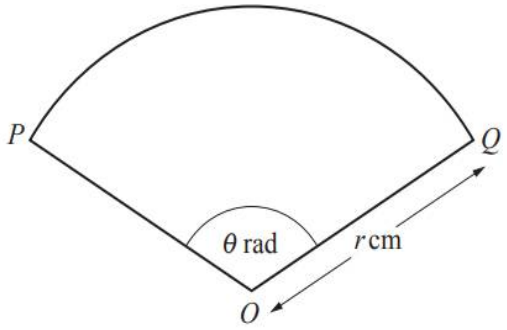
10



The diagram shows the sector OPQ of a circle, centre O, radius r cm, where angle $POQ = \theta$ radians. The perimeter of the sector is 16 cm.

$\left[\text{Arc length} = r\theta \text{ and Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}\right]$

(i) Show that area, A cm^2, of the sector is given by $A = \dfrac{128\,\theta}{(2+\theta)^2}$. [3]

10 It is given that θ can vary and A has a maximum value.

(ii) Find the maximum value of A. [4]

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11 The function $f(x) = 2x^3 + 5x^2 + kx + p$ is divisible by $(2x-3)$ and $f'(x)$ leaves a remainder of 42 when divided by $(x-2)$.

(i) Find the value of *k* and of *p*. [4]

(ii) Find the remainder when $f(x)$ is divided by $(x+1)(x-2)$. [2]

11 **(iii)** Using the values of k and p found in part **(i)**, show that $f(x) = (2x-3)(ax^2+bx+c)$, where a, b and c are integers to be found. [3]

(iv) **Hence**, show that $f(x) = 0$ has only one real solution and state this solution. [2]

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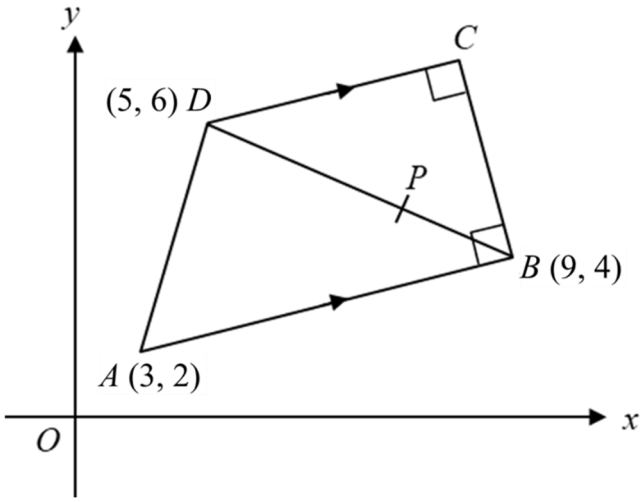
12 Solutions to this question by accurate drawing will not be accepted.



The diagram shows a trapezium *ABCD* with *AB* parallel to *DC*, and *BC* perpendicular to *AB*. The coordinates of *A*, *B* and *D* are (3, 2), (9, 4) and (5, 6) respectively.

(i) Find the equation of line *DC*. [2]

(ii) Find the coordinates of *C*. [3]

Continuation of working space for question **12(ii)**.

(iii) Find the area of trapezium *ABCD*. [2]

(iv) Find the coordinates of the point *P* on *BD* such that $3BP = PD$. [1]

13 A cyclist, travelling along a straight path, passes a point P with an acceleration of 1.8 km/h^2. Five hours after passing P, the cyclist reaches the point Q. The velocity of the cyclist, v km/h, t hours after passing P is given by

$$v = 3 + 3kt - 2kt^2, \text{ for } 0 \le t \le 5$$

where k is a constant.

(i) Find the time at which the cyclist is at instantaneous rest. [4]

(ii) Explain why the total distance travelled by the cyclist five hours after passing P is not given by the distance PQ. [1]

13 (iii) **Hence**, find the average speed of the cyclist for the five hour journey after passing P. [5]

~ End of Paper ~

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1	$15\sqrt{3}$ cm^2	**9i**	$LHS = \dfrac{\cos\theta}{\sin\theta} - 2\sin\theta\cos\theta$ $= \dfrac{\cos\theta\left(1-2\sin^2\theta\right)}{\sin\theta}$ $= RHS$ (proved)
2i	Period $= 180° \Rightarrow b = 2$ (shown)		
2ii	190 Watts		
2iii	$-90° \le x < -66.6°$ or $66.6° < x \le 90°$		
3i	$T_{r+1} = \binom{7}{r} x^{35-6r}$. **35 is odd and 6*r* is even**.		
3ii	15120		
4	$\dfrac{1}{2} < x < 1$ and $1 < x < \dfrac{3}{2}$	**9ii**	$\theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}$, $\theta = 1.25$
5i	$0 = \log_{1.02}(c) \Rightarrow c = 1$ (shown); m = 0.0384	**10i**	$2r + r\theta = 16$, so $r = \dfrac{16}{2+\theta}$ $A = \dfrac{1}{2}\left(\dfrac{16}{2+\theta}\right)^2 \theta$ $= \dfrac{128\theta}{(2+\theta)^2}$ (shown)
5ii	P = 158.6% is **larger than 100%**		
6i	$\dfrac{dy}{dx} = -3\cos 2x + 2$		
6ii	$y = -\dfrac{3}{2}\sin 2x + 2x - \dfrac{\pi}{6}$		
7i	$2x^2 - 4x + 6 = 2(x-1)^2 + 4$ $-x^2 - 4x - 1 = -(x+2)^2 + 3$	**10ii**	Max A = 16 cm^2
7ii	Max of $-(x+2)^2 + 3$ which is 3 is less than the min of $2(x-1)^2 + 4$ which is 4.	**11i**	$k = -2$, $p = -15$ **11ii** $(9x-1)$
		11iii	$f(x) = (2x-3)(x^2+4x+5)$ with a = 1, b = 4 and c = 5.
		11iv	$2x - 3 = 0 \Rightarrow x = 1.5$ $b^2 - 4ac = 16 - 4(1)(5)$ $= -4 < 0$, no real roots
8ai	$-90° \le x \le 90°$ **8aii** $-90° < x < 90°$		
8bi	$\left(\dfrac{1}{2}\right)\left(\dfrac{1}{\sqrt{2}}\right) + \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{1+\sqrt{3}}{2\sqrt{2}}$ (shown)	**12i**	$y - 6 = \dfrac{1}{3}(x-5)$ or $y = \dfrac{1}{3}x + \dfrac{13}{3}$
8bii	$\sec 150° = \dfrac{1}{1-2\sin^2 75°}$ $= \dfrac{1}{1-2\left(\dfrac{4+2\sqrt{3}}{8}\right)}$ $= \dfrac{1}{1-\left(1+\dfrac{\sqrt{3}}{2}\right)}$ $= -\dfrac{2}{\sqrt{3}}$ $= -\dfrac{2}{3}\sqrt{3}$	**12ii**	$C(8, 7)$
		12iii	15 units^2 **12iv** $P(8, 4.5)$
		13i	k = 0.6, and t = 2.5 hours
		13ii	The cyclist **changed direction** at t = 2.5 hours, so the total distance is greater than PQ.
		13iii	5.25 km/h

END OF PAPER 1 ANSWER KEY