NAME: ____________________________ (   )   CLASS: _______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2021
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS   4049/02

Paper 2

Date: 26 August 2021   **Duration: 2 hours 15 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Table of Penalties		**Question Number**		
Presentation	□ 2			**90**
Rounding off	□ 1		Parent's/Guardian's Signature	

Setter: Mdm Haliza

This question paper consists of **20** printed pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

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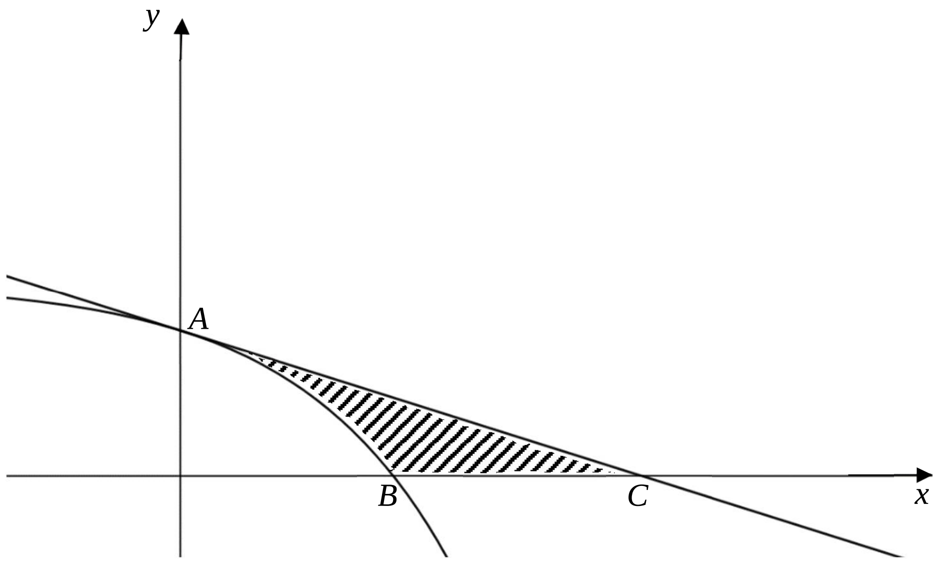
Answer **all** the questions.

1 **(a)** The straight line $2x-11=2y$ meets the curve $xy=20$ at the points A and B.

Find the coordinates of A and of B. [4]

1 **(b)** Express $\dfrac{6x^3 - 14}{2x^3 + x}$ in partial fractions. [6]

2 The diagram shows part of the curve $y = 4 - e^{\frac{1}{2}x}$ which cuts the axes at A and at B. The tangent to the curve at A meets the x-axis at C.



(i) Find the equation of the line AC. [4]

2 **(ii)** Find the area of the shaded region bounded by the tangent AC, the curve and the x-axis. [5]

3 **(i)** Express $\dfrac{3x}{3x-2}$ in the form $a+\dfrac{b}{3x-2}$.

Hence find $\displaystyle\int \frac{3x}{3x-2}\,dx$. [3]

(ii) Differentiate $x\ln(3x-2)$. [2]

(iii) Using the results from parts **(i)** and **(ii)**, find $\displaystyle\int \ln(3x-2)\,dx$. [3]

4 **(a)** Explain whether the line $y = -5x - 2$ intersects the curve $y = mx^2 + 3$ where $m < 1$. [4]

(b) The curve with equation $y = ax^2 + bx + a$, where a and b are constants, lies completely above the x-axis.

(i) Write down the conditions which must apply to a and b. [2]

4 **(b)** **(ii)** Given that $b = 2$, find the range of values of a. [3]

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5 **(a)** The equation of a curve is $y = \dfrac{\cos x}{3 + \sin x}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{-3\sin x - 1}{(3 + \sin x)^2}$. [3]

5 **(a)** **(ii)** Find the x-coordinate of the turning point of the curve for $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$ and determine the nature of this point. [5]

5 **(b)** Find the rate of change of y when $x = 0.5$, given that x changes at a rate of 0.4 units per second at this instant. [3]

6 **(a)** The table below shows some experimental values of x and y which are known to be related by the equation $y = px(x+k)$. One of the values of y was recorded wrongly.

x	1	2	3	4	5	6
y	14	50	66	104	150	205

(i) On the grid on the next page, plot $\dfrac{y}{x}$ against x and hence determine which value of y, in the table above, is the incorrect recording. [2]

(ii) Draw the straight line graph and use it to estimate a value of y to replace the incorrect recording of y found in part (i). [2]

(iii) Use your graph to estimate the value of p and of k . [3]

6 **(b)** Two variables x and y are related by the equation $\frac{x^2}{a}-\frac{3y^2}{b}=1$, where a and b are constants. When the graph of y^2 against x^2 is drawn, a straight line is obtained.

Given that this line passes through the point $\left(3,-\frac{8}{9}\right)$ and has a gradient $\frac{4}{27}$, find the value of a and of b. [4]

7 The points $S(1,-6)$ and $T(5,2)$ both lie on a circle. The centre of the circle, O, lies on the line $y = x - 8$.

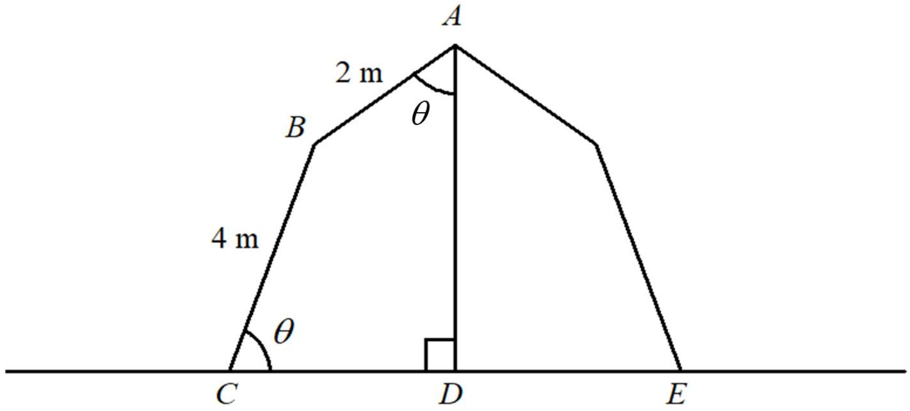
(i) Find the equation of the perpendicular bisector of *ST*. [4]

7 **(ii)** Find the equation of the circle. [4]

(iii) The point $R(9, 0)$ lies on the circle.

Show that *SR* is a diameter of the circle. [2]

8



The diagram above shows a vertical cross-section through a canvas tent in which AB = 2 m, BC = 4 m, $\angle BAD = \angle BCD = \theta\,°$ and CDE is the horizontal ground.

The diagram is symmetrical about the only vertical pole AD.

(i) Show that $AD = 4\sin\theta + 2\cos\theta$. [2]

(ii) Express AD in the form $R\cos(\theta - \alpha)$, where R is positive and α is acute. [3]

8 **(iii)** State the maximum length of *AD* and the corresponding value of θ. [2]

(iv) Khalid and Travis each pitched their own canvas tents as shown in the diagram on page 17, using identical poles and canvas sheets.
If *AD* = 4.1 m, explain why their tents do not look identical? Justify your answer with mathematical proofs. [3]

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9 **(a)** Find the values of x such that $3^{2x} - 3^{x} = 3^{x+2} - 9$. [4]

(b) Given that $\dfrac{8}{\log_x y} - \log_x y + \log_y x = 0$, express y in terms of x. [4]

9 **(c)** Show that the equation $\log_2(7-2x)-\log_2(x-5)=4$ has no real solutions. [4]

~ End of Paper ~

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FMS(S) Sec 4 Exp Prelim Examination 2021
Additional Mathematics Paper 2

Answer Key

1(a)	$\left(8, \frac{5}{2}\right)$ and $\left(-\frac{5}{2}, -8\right)$	**1(b)**	$3-\frac{14}{x}+\frac{28x-3}{2x^2+1}$
2(i)	$y=-\frac{1}{2}x+3$	**2(ii)**	3.91 units^2 (to 3 s.f.)
3(i)	$x+\frac{2}{3}\ln(3x-2)+c$	**3(ii)**	$\frac{3x}{3x-2}+\ln(3x-2)$
3(iii)	$x\ln(3x-2)-x-\frac{2}{3}\ln(3x-2)$	**4(a)**	Since $m<1$, $-20m>-20$ $25-20m>5$ Since the discriminant > 0, there are real roots/ solutions. So the line intersects the curve.
4(b)(i)	$a>0$ and $b^2-4a^2<0$ (or $b^2<4a^2$)	**4(b)(ii)**	$a > 1$
5(a)(ii)	The curve has a maximum turning point at $x=-0.340$ (to 3 s.f.).	**5(b)**	-0.0806 units/s (to 3 s.f.)
6(a)(i)	$y = 50$	**6(a)(ii)**	$y = 36$
6(a)(iii)	$p=4, k=2.5$	**6(b)**	$a=9,\ b=4$
7(i)	$y=-\frac{1}{2}x-\frac{1}{2}$	**7(ii)**	$(x-5)^2+(y+3)^2=25$ or $x^2+y^2-10x+6y+9=0$
7(iii)	Use mid-point of *SR* = centre of circle or length of *SR* = twice the radius or equation of *SR* passes through centre of circle.	**8(ii)**	$AD=2\sqrt{5}\ \cos(\theta-63.4)°$ (to 1 d.p.) or $AD=\sqrt{20}\ \cos(\theta-63.4)°$ (to 1 d.p.)
8(iii)	Max. $AD=2\sqrt{5}$ or $\sqrt{20}$ or 4.47 m (to 3s.f.) when $\theta=63.4°$ (to 1 d.p.)	**8(iv)**	$\theta=39.9°, 87.0°$ (to 1 d.p.) Khalid and Travis could have used different angles of θ to pitch their tents.
9(a)	$x=0$ or $x=2$	**9(b)**	$y=x^3$ or x^{-3} $\left(\text{or } \frac{1}{x^3}\right)$

9(c)	$x=\dfrac{29}{6}$ (or equivalent). $\log_2\left(-\dfrac{8}{3}\right)$ and $\log_2\left(-\dfrac{1}{6}\right)$ do not exist, therefore there are no real solutions.	**6(b)**