EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2021

General Certificate of Education Advanced Level

Higher 2

Section A: Pure Mathematics [40 marks]

1 Since all the coefficients are real, $z=\frac{1}{2}-\mathrm{i}$ is also a root.

Quadratic factor of $\mathrm{f}(z)$

$$\left[z-\left(\frac{1}{2}+\mathrm{i}\right)\right]\left[z-\left(\frac{1}{2}-\mathrm{i}\right)\right]=\left[\left(z-\frac{1}{2}\right)-\mathrm{i}\right]\left[\left(z-\frac{1}{2}\right)+\mathrm{i}\right]$$
$$=\left(z-\frac{1}{2}\right)^2-\mathrm{i}^2$$
$$=z^2-z+\frac{1}{4}-(-1)$$
$$=z^2-z+\frac{5}{4}$$

$$4z^3-12z^2+13z-10=\left(z^2-z+\frac{5}{4}\right)(Az+B)$$

Compare coefficients of z^3, $A=4$

Compare constant term, $B=-8$

So the 3rd factor is $4z-8$.

Thus the roots are $z=\frac{1}{2}+\mathrm{i},\frac{1}{2}-\mathrm{i},$ and 2.

2 **(i)** $$\mathrm{e}^{\mathrm{i}\left(\theta-\frac{\pi}{2}\right)}=\cos\left(\theta-\frac{\pi}{2}\right)+\mathrm{i}\sin\left(\theta-\frac{\pi}{2}\right)$$

$$=\cos\theta\cos\frac{\pi}{2}+\sin\theta\sin\frac{\pi}{2}+\mathrm{i}\left(\sin\theta\cos\frac{\pi}{2}-\cos\theta\sin\frac{\pi}{2}\right)$$
$$=0+\sin\theta+\mathrm{i}(0-\cos\theta)$$
$$=\sin\theta-\mathrm{i}\cos\theta$$

<u>Alternative 1</u>

$$\mathrm{e}^{\mathrm{i}\left(\theta-\frac{\pi}{2}\right)}=\mathrm{e}^{\mathrm{i}\theta}\times\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{2}\right)}=\mathrm{e}^{\mathrm{i}\theta}(-\mathrm{i})=(-\mathrm{i})(\cos\theta+\mathrm{i}\sin\theta)=\sin\theta-\mathrm{i}\cos\theta$$

Alternative 2

$$e^{i\left(\theta-\frac{\pi}{2}\right)}=\frac{e^{i\theta}}{e^{i\left(\frac{\pi}{2}\right)}}=\frac{e^{i\theta}}{i}=\frac{e^{i\theta}}{i}\times\frac{-i}{-i}=-i\left(\cos\theta+i\sin\theta\right)=\sin\theta-i\cos\theta$$

(ii) $1-z^2=1-\left(\cos\theta+i\sin\theta\right)^2$

$$\begin{aligned}&=1-\left(\cos^2\theta+2i\cos\theta\sin\theta-\sin^2\theta\right)\\&=1-\cos^2\theta+\sin^2\theta-2i\cos\theta\sin\theta\\&=2\sin^2\theta-2i\cos\theta\sin\theta\\&=2\sin\theta\left(\sin\theta-i\cos\theta\right)\\&=2\sin\theta e^{i\left(-\frac{\pi}{2}+\theta\right)}\text{[from (i)]}\end{aligned}$$

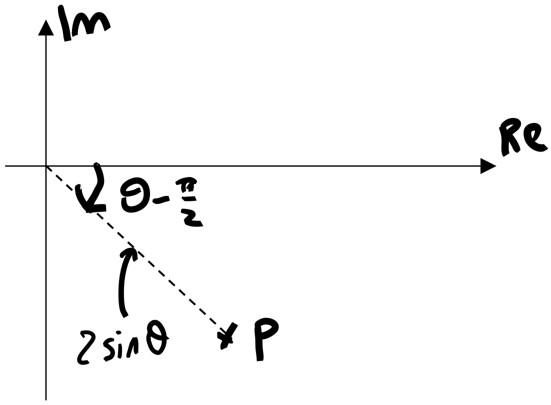
Thus $\arg\left(1-z^2\right)=\theta-\frac{\pi}{2}$ and $\left|1-z^2\right|=2\sin\theta$.

Alternative

$$1-z^2=z\left(\frac{1}{z}-z\right)=e^{i\theta}\left(-2i\sin\theta\right)=2\sin\theta\times e^{i\left(\theta-\frac{\pi}{2}\right)}\quad\left(\because -i=e^{i\left(-\frac{\pi}{2}\right)}\right)$$

Thus $\arg\left(1-z^2\right)=\theta-\frac{\pi}{2}$ and $\left|1-z^2\right|=2\sin\theta$.

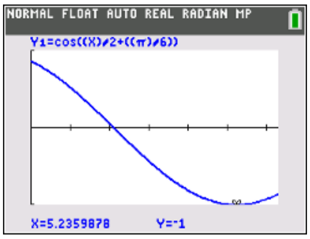
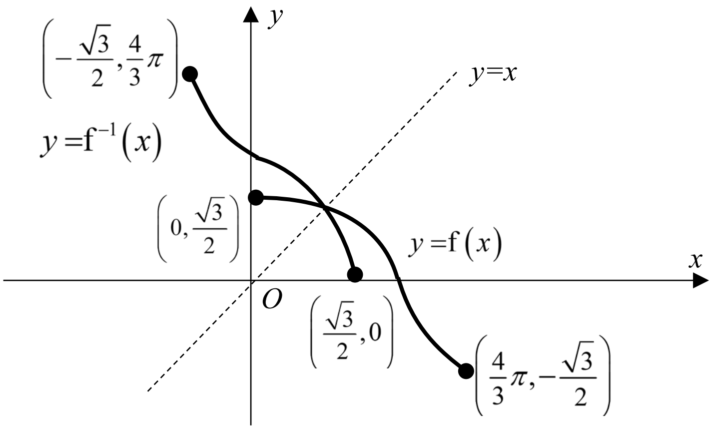
(iii) Let point P represent the complex number $1-z^2$.



(iv)

$$\begin{aligned}\arg\left(\frac{z^*}{z^3\left(1-z^2\right)}\right)&=\arg\left(z^*\right)-\arg\left(z^3\right)-\arg\left(1-z^2\right)\\&=-\arg\left(z\right)-3\arg\left(z\right)-\arg\left(1-z^2\right)\\&=-\theta-3\theta-\left(\theta-\frac{\pi}{2}\right)\\&=\frac{\pi}{2}-5\theta\end{aligned}$$

	Since $\left(\dfrac{z^*}{z^3\left(1-z^2\right)}\right)$ is real, $\arg\left(\dfrac{z^*}{z^3\left(1-z^2\right)}\right)=k\pi, k\in\mathbb{Z}$ $\dfrac{\pi}{2}-5\theta=k\pi$ $\theta=\left(\dfrac{1}{10}-\dfrac{k}{5}\right)\pi, k\in\mathbb{Z}$ Since $0<\theta<\dfrac{\pi}{2}$, $\theta=\dfrac{\pi}{10},\dfrac{3}{10}\pi$

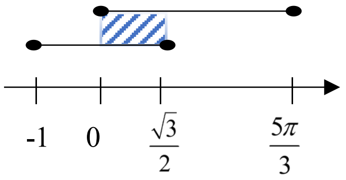
3	**(a)(i)** From graph on GC, largest k is when $y=-1$  $\cos\left(\dfrac{x}{2}+\dfrac{\pi}{6}\right)=-1$ i.e. $\Rightarrow \dfrac{x}{2}+\dfrac{\pi}{6}=\pi$ $\Rightarrow x=2\left(\pi-\dfrac{\pi}{6}\right)=\dfrac{5\pi}{3}$
	(a)(ii) 
	(a)(iii) $\mathrm{ff}^{-1}(x)=\mathrm{f}^{-1}\mathrm{f}(x)=x$ The two functions are equal when the domains intersect. From graph in **(a)(ii)**, $0\le x\le\dfrac{\sqrt{3}}{2}$.

$\therefore \mathrm{ff}^{-1}(x) = \mathrm{f}^{-1}\mathrm{f}(x) = x$ for $0 \le x \le \dfrac{\sqrt{3}}{2}$

<u>Alternative: (consider domains)</u>

$$D_{\mathrm{f}^{-1}\mathrm{f}} = D_{\mathrm{f}} = \left[0, \frac{5\pi}{3}\right]$$

$$D_{\mathrm{ff}^{-1}} = D_{\mathrm{f}^{-1}} = R_{\mathrm{f}} = \left[-1, \frac{\sqrt{3}}{2}\right]$$



$\therefore \mathrm{ff}^{-1}(x) = \mathrm{f}^{-1}\mathrm{f}(x) = x$ for $0 \le x \le \dfrac{\sqrt{3}}{2}$

(b) $\mathrm{h} : x \mapsto 2x^2 + 3, \qquad x \in \mathbb{R}, x \le 0,$

$\mathrm{hg} : x \mapsto 2x + 3 - 2a, \qquad x \in \mathbb{R}^+, x > a.$

$$\mathrm{h}[\mathrm{g}(x)] = 2x + 3 - 2a$$

$$2[\mathrm{g}(x)]^2 + 3 = 2x + 3 - 2a$$

$$[\mathrm{g}(x)]^2 = x - a$$

$$\mathrm{g}(x) = -\sqrt{x-a} \qquad \because R_{\mathrm{g}} \subseteq D_{\mathrm{h}} = (-\infty, 0]$$

$$D_{\mathrm{g}} = D_{\mathrm{hg}} = (a, \infty)$$

<u>Alternative: find h^{-1}</u>

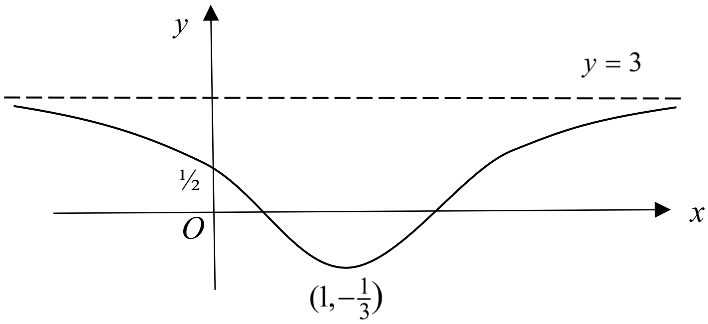
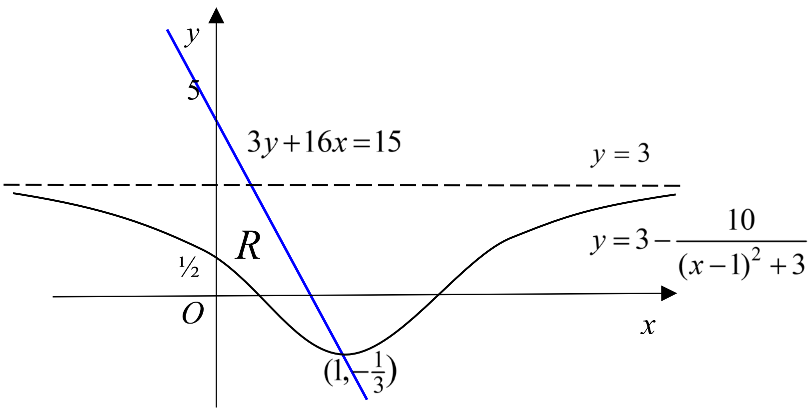
Let $y = 2x^2 + 3$. Then $x = -\sqrt{\dfrac{y-3}{2}}$ or $\sqrt{\dfrac{y-3}{2}}$ (reject $\because x \le 0$).

So $\mathrm{h}^{-1}(x) = -\sqrt{\dfrac{x-3}{2}}$.

Then $\mathrm{h}^{-1}(\mathrm{hg}(x)) = -\sqrt{\dfrac{\mathrm{hg}(x) - 3}{2}}$

$\therefore \mathrm{g}(x) = -\sqrt{\dfrac{2x + 3 - 2a - 3}{2}} = -\sqrt{x-a}$

$D_{\mathrm{g}} = D_{\mathrm{hg}} = (a, \infty)$

4	**(i)**	$y=3-\dfrac{10}{x^2-2x+4} \Rightarrow \dfrac{dy}{dx}=\dfrac{10(2x-2)}{(x^2-2x+4)^2}$ At the stationary point, $\dfrac{dy}{dx}=0 \Rightarrow x=1,\ y=3-\dfrac{10}{3}=-\dfrac{1}{3}$ $\therefore$ stationary point $\left(1,-\frac{1}{3}\right)$ <u>Alternatively</u>, from equation of C, the graph of $y=3-\dfrac{10}{(x-1)^2+3}$ is symmetrical about $x=1$. Therefore the minimum point is $\left(1,-\frac{1}{3}\right)$.
	(ii)	$y=3-\dfrac{10}{x^2-2x+4}=3-\dfrac{10}{(x-1)^2+3}$ 
	(iii)	

$$\text{Required area} = \int_0^1 \left(5-\frac{16}{3}x\right)-\left(3-\frac{10}{3+(x-1)^2}\right)\mathrm{d}x$$

$$= \int_0^1 \left(2-\frac{16}{3}x+\frac{10}{3+(x-1)^2}\right)\mathrm{d}x$$

$$= \left[2x-\frac{8}{3}x^2+\frac{10}{\sqrt{3}}\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right)\right]_0^1$$

$$= -\frac{2}{3}+\frac{10}{\sqrt{3}}\left(0-\left(-\frac{\pi}{6}\right)\right)$$

$$= \frac{5\pi}{3\sqrt{3}}-\frac{2}{3}$$

$$= \frac{1}{3}\left(\frac{5\pi}{\sqrt{3}}-2\right) \text{ unit}^2$$

(iv) $y = 3-\dfrac{10}{(x-1)^2+3}$

$$\Rightarrow (x-1)^2+3 = \frac{10}{3-y}$$

$$\Rightarrow x-1 = \pm\sqrt{\frac{10}{3-y}-3}$$

$$\Rightarrow x = 1-\sqrt{\frac{10}{3-y}-3} \quad (\because \text{ region is in the interval } x<1)$$

$$\text{volume} = \pi\int_{-\frac{1}{3}}^{5}\left(\frac{15-3y}{16}\right)^2 \mathrm{d}y - \pi\int_{-\frac{1}{3}}^{\frac{1}{2}}\left(1-\sqrt{\frac{10}{3-y}-3}\right)^2 \mathrm{d}y$$

$$\approx 5.0693$$

$$= 5.07 \text{ unit}^3 \quad (3 \text{ s.f.})$$

<u>Alternatively</u>, (using formula for volume of cone)

$$\text{volume} = \frac{1}{3}\pi(1)^2\left[5-\left(-\frac{1}{3}\right)\right]-\pi\int_{-\frac{1}{3}}^{\frac{1}{2}}\left(1-\sqrt{\frac{10}{3-y}-3}\right)^2 \mathrm{d}y$$

$$= \frac{16}{9}\pi - 0.16416\pi$$

$$\approx 5.0693$$

$$= 5.07 \text{ unit}^3 \quad (3 \text{ s.f.})$$

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5 **(i)** By ratio theorem, $\overrightarrow{OP} = \dfrac{m\mathbf{b}+n\mathbf{a}}{m+n}$.

$$\overrightarrow{OC} = 3\overrightarrow{OP} = 3\left(\frac{m\mathbf{b}+n\mathbf{a}}{m+n}\right)$$

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 3\left(\frac{m\mathbf{b}+n\mathbf{a}}{m+n}\right) - \mathbf{a} = \frac{3m}{m+n}\mathbf{b} + \frac{3n}{m+n}\mathbf{a} - \mathbf{a}$$

$$= \frac{3m}{m+n}\mathbf{b} + \left(\frac{3n}{m+n} - 1\right)\mathbf{a} = \frac{3m}{m+n}\mathbf{b} + \left(\frac{3n-n-m}{m+n}\right)\mathbf{a} = \frac{3m}{m+n}\mathbf{b} + \left(\frac{2n-m}{m+n}\right)\mathbf{a}$$

(ii) Area of triangle $ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$$= \frac{1}{2}\left|(\mathbf{b}-\mathbf{a}) \times \left(\left(\frac{3m}{m+n}\right)\mathbf{b} + \left(\frac{2n-m}{m+n}\right)\mathbf{a}\right)\right|$$

$$= \frac{1}{2}\left|\frac{3m}{m+n}(\mathbf{b}\times\mathbf{a}) - \frac{2n-m}{m+n}(\mathbf{a}\times\mathbf{b})\right| \text{ (since } \mathbf{a}\times\mathbf{a} = \mathbf{b}\times\mathbf{b} = 0)$$

$$= \frac{1}{2}\left|\frac{3m}{m+n}(\mathbf{b}\times\mathbf{a}) + \frac{2n-m}{m+n}(\mathbf{b}\times\mathbf{a})\right|$$

$$= \frac{1}{2}\left|\frac{2m+2n}{m+n}(\mathbf{b}\times\mathbf{a})\right| = |(\mathbf{b}\times\mathbf{a})| = |\mathbf{a}||\mathbf{b}|\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}|\mathbf{a}|^2$$

(iii) $\overrightarrow{AC} = \frac{3m}{m+n}\mathbf{b} + \left(\frac{2n-m}{m+n}\right)\mathbf{a}$

Since $\overrightarrow{AC}$ is parallel to $\overrightarrow{OB}$, $\frac{2n-m}{m+n} = 0$

Thus $2n - m = 0 \Rightarrow \frac{m}{n} = 2$

Thus, $AP : PB = 2 : 1$

Section B: Probability and Statistics [60 marks]

6 **(i)** $\mathrm{P}(A' | B) = \frac{13}{20} \Rightarrow \frac{\mathrm{P}(A' \cap B)}{\mathrm{P}(B)} = \frac{13}{20}$

$$\frac{\mathrm{P}(B) - \mathrm{P}(A\cap B)}{\mathrm{P}(B)} = \frac{13}{20}$$

$$\mathrm{P}(B)\left(1 - \frac{13}{20}\right) = \mathrm{P}(A \cap B)$$

$$\mathrm{P}(B) = \frac{7/60}{1 - 13/20} = \frac{1}{3} \text{ (shown)}$$

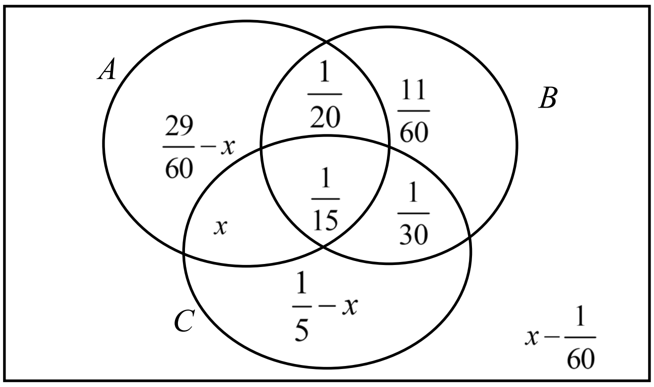
<u>Alternative</u>

$\mathrm{P}(A | B) = \frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)} \quad \Rightarrow 1 - \mathrm{P}(A' | B) = \frac{7/60}{\mathrm{P}(B)}$

$$\Rightarrow \mathrm{P}(B)=\frac{7/60}{1-13/20}=\frac{1}{3} \text{ (shown)}$$

(ii) Since $\mathrm{P}(B)\times \mathrm{P}(C)=\frac{1}{3}\times\frac{3}{10}=\frac{1}{10}=\mathrm{P}(B\cap C)$, B and C are independent.

(iii)



Let $x=\mathrm{P}(A\cap B'\cap C)$

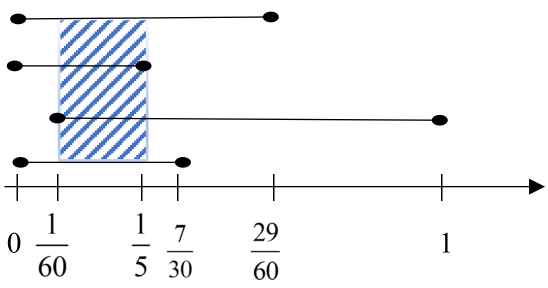
$$*\ \mathrm{P}(A\cap(B'\cap C'))=\mathrm{P}(A)-\mathrm{P}(A\cap B)-x=\frac{29}{60}-x \Rightarrow 0\le x\le\frac{29}{60}$$

$$\mathrm{P}(C\cap(A'\cap B'))=\mathrm{P}(C)-\mathrm{P}(B\cap C)-x=\frac{1}{5}-x \Rightarrow 0\le x\le\frac{1}{5}$$

$$\mathrm{P}(A'\cap B'\cap C')=1-\mathrm{P}(B)-\frac{29}{60}-\left(\frac{1}{5}-x\right)=x-\frac{1}{60}\Rightarrow\frac{1}{60}\le x\le 1$$

$$\mathrm{P}(A\cap C)=x+\frac{1}{15}\le\min\left[\mathrm{P}(A),\mathrm{P}(C)\right]=\frac{3}{10}\Rightarrow 0\le x\le\frac{7}{30}$$

Taking intersection of above restrictions on x:



$$\therefore\ \frac{1}{60}\le\mathrm{P}(A\cap B'\cap C)\le\frac{1}{5}$$

<u>Alternatively, from the venn diagram,</u>

x is max when $\left(\frac{1}{5}-x\right)$ is min, i.e. $\Rightarrow x=\frac{1}{5}$

Note: when $x=\frac{1}{5}$, $x-\frac{1}{60}=\frac{11}{60}$ and $\frac{29}{60}-x=\frac{17}{60}$. All probabilities are non-negative.

x is min when $\left(\frac{1}{5}-x\right)$ is max, i.e. $\left(x-\frac{1}{60}\right)$ is min $\Rightarrow x=\frac{1}{60}$

Note: when $x=\frac{1}{60}$, $\frac{1}{5}-x=\frac{11}{60}$ and $\frac{29}{60}-x=\frac{7}{15}$. All probabilities are non-negative.

$\therefore\ \frac{1}{60}\le \mathrm{P}\left(A\cap B'\cap C\right)\le\frac{1}{5}$

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7 **(i)** If $S=3$, his second sock must not match his first sock and his third sock must match either of his first two socks.

$\therefore \mathrm{P}(S=3)=\frac{6}{7}\times\frac{2}{6}=\frac{2}{7}$.

(ii) After drawing 5 socks, he will be certain of having drawn a matching pair because there are only 4 pairs of socks. So he will never need to draw a sixth (or seventh, or eighth) sock.

(iii)

S	2	3	4	5
$\mathrm{P}(S=s)$	$\frac{1}{7}$	$\frac{2}{7}$	$\mathrm{P}(S=4)=\frac{6}{7}\times\frac{4}{6}\times\frac{3}{5}$ $=\frac{12}{35}$	$\mathrm{P}(S=5)=\frac{6}{7}\times\frac{4}{6}\times\frac{2}{5}=\frac{8}{35}$ Or $\mathrm{P}(S=5)=1-\left(\frac{1}{7}+\frac{2}{7}+\frac{12}{35}\right)=\frac{8}{35}$

Note: $\mathrm{P}(S=1,6,7,8)=0$ (implied if not seen)

(iv) From GC, $\mathrm{E}(S)=\frac{128}{35}$ or 3.66 ($3sf$)

$$\begin{aligned}\mathrm{Var}(S)&=E\left(S^2\right)-\left[E(S)\right]^2\\&=\left[2^2\left(\frac{1}{7}\right)+2^2\left(\frac{2}{7}\right)+2^2\left(\frac{12}{35}\right)+2^2\left(\frac{8}{35}\right)\right]-\left[\frac{128}{35}\right]^2\\&=\frac{1186}{1225}\text{ or }0.968\ (3sf)\end{aligned}$$

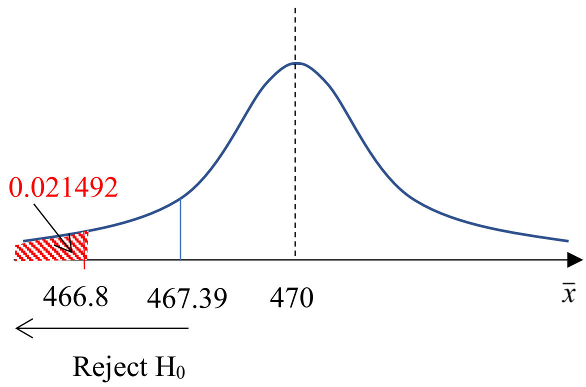
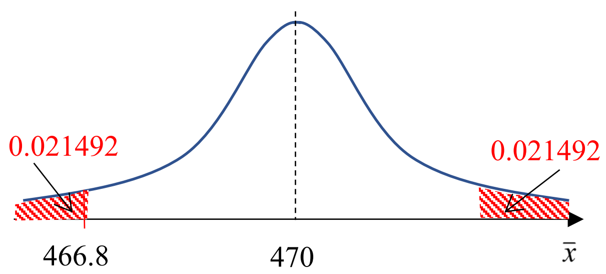
(v) Observe that $T=S_1+S_2+S_3$.

$\mathrm{P}(T=9)>\mathrm{P}(S_1=3,S_2=3,S_3=3)=\left(\frac{2}{7}\right)^3$ because the event in the RHS is a subset of the event in the LHS.

More concretely, in order for 9 socks to be drawn in 3 days, it is possible but not necessary that 3 socks be drawn everyday. It is also possible that 2 socks are drawn on the first day, 3 are drawn on the second day and 4 are drawn on the last day.

(vi) Since $T = S_1 + S_2 + S_3$, $\mathrm{Var}(T) = 3\mathrm{Var}(S) = \dfrac{3558}{1225}$ or 2.90 (3sf)

8	**(i)**	# of ways to arrange the 3 gemstones = 3! # of ways to arrange 11 units = (11-1)! Total # of ways $= 3! \times (11-1)! = 21772800$
	(ii)	# of ways $= {}^6C_3 \times {}^4C_3 \times {}^3C_3 \times (3-1)! \times 3! \times 3! = 5760$
	(iii)	# of ways with Ls all together and Es $= 7! \times 3!$ # of ways to slot and arrange Ss $= {}^8C_4 \times 4!$ Total # of ways $= 7! \times 3! \times {}^8C_4 \times 4! = 50803200$
	(iv)	#of ways = all ways – 0 E – 0 S – 0 L $= {}^{13}C_7 - {}^7C_7 - {}^{10}C_7 - {}^9C_7 = 1559$ <u>Alternative: direct listing</u> Possible cases for ESL: 511: 72 412: 180, 421: 270 331: 240, 313: 80, 322: 360, 232: 180, 223: 90, 241: 45, 142: 18, 133: 24

9	**(i)**	The term 'random sample' means that each light bulb had an equal probability of being selected for the sample, and the selection of light bulbs were independent of each other.
	(ii)	Let X be the brightness of a randomly chosen lightbulb with population mean μ. To test $H_0 : \mu = 470$ vs $H_1 : \mu < 470$ at 5% level of significance Under H_0, $\overline{X} \sim N\left(470, \frac{100}{40}\right)$ where [$\mu_0 = 470,\ \sigma = 10,\ \overline{x} = 466.8,\ n = 40$] Reject H_0 if p-value < 0.05 [alternative: s $z < -1.6449$ or $\overline{x} < 467.39$]  From GC, p-value $= 0.021492 = 0.0215$ (3 s.f) < 0.05 [alternatively, z-value = -2.0238] Hence, we reject H_0 and conclude that there is sufficient evidence at 5% level of significance that the mean brightness is less than 470 lumens. The p-value of 0.0215 represents the probability of observing a sample mean brightness of 466.8 lumens or less, assuming that the mean brightness is indeed 470 lumens.
	(iii)	For a 2-tail test, new p-value = $2(0.021492) = 0.042984$ which is still less than 0.05 and we will reject H_0.  Hence, the conclusion to reject H_0 in part (ii) is **not** affected. [Alternatively, new critical region is $z < -1.9599$ or $z > 1.9599$ $\overline{x} < 466.90$ or $\overline{x} > 473.10$

Since the z-value (or $\bar{x}$) is unchanged at -2.0238 (or 466.8), we will still reject H_0.

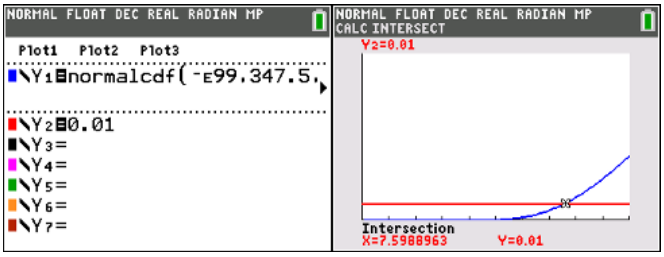
(iv) unbiased estimate of population mean, $\bar{x} = \dfrac{-125}{50} + 350 = 347.5$

unbiased estimate of population variance, $s^2 = \dfrac{4704}{49} = 96$

(v) Test $H_0 : \mu_y = 350$ vs $H_1 : \mu_y < 350$

Under H_0, since $n = 50$ is large, $\bar{Y} \sim N\left(350, \dfrac{\sigma^2}{50}\right)$ approximately by Central Limit Theorem

H_0 is not rejected, i.e. $P(\bar{Y} < 347.5) \geq 0.01$



From GC, $\sigma \geq 7.60$ (3 s.f)

Alternatively,

z-value $= \dfrac{347.5 - 350}{\sigma / \sqrt{50}} \geq -2.3263$

$-2.5\sqrt{50} \geq -2.3263\sigma$

$\sigma \geq 7.60$

10 **(i)** Let Y be the width (in mm) of a box. $Y \sim N(240.5, 0.02)$

Let $C = W_1 + W_2 + ...W_{24} - Y$

$C \sim N(-0.5, 0.092)$

$$\begin{aligned} P(\text{pens fit into box}) &= P(W_1 + W_2 + ...W_{24} < Y) \\ &= P(C < 0) \\ &= 0.95037 \\ &= 0.950 \text{ (to 3 sf)} \end{aligned}$$

(ii) Let W be the width (in mm) of a pen. $W \sim N(10, 0.003)$

$P(\text{pen is defective}) = 1 - P(9.9 < W < 10.1) = 1 - 0.93211 = 0.067889 = 0.0679$ (to 3sf)

(iii) Let X be the number of defective pens in a box of 12. $X \sim B(12, 0.067889)$

	$P(\text{batch passes inspection})$ $= P(X \le 1) + P(X = 2)P(X = 0)$ $= 0.806087 + (0.150598)(0.430142)$ $= 0.870865 = 0.871$ (to 3 sf)
(iv)	$P(\le 3 \text{ defective pens found} \mid \text{batch did not pass inspection})$ $= \dfrac{P(X=2)P(X=1)+P(X=3)}{P(\text{batch did not pass inspection})}$ $= \dfrac{(0.150598)(0.375945)+0.036562}{1-0.870865}$ $= \dfrac{0.0931785}{0.129135} = 0.722$ (to 3 sf)
(v)	Let S be the number of special pencils in a box of 12. $S \sim B(12, 0.07)$ Since $n = 40$ is large, by Central Limit Theorem, $\overline{S} \sim N\left(0.84, \dfrac{0.7812}{40}\right)$ approximately. Let T be the number of special crayons in a box of 12. $T \sim B(12, 0.06)$ Since $n = 40$ is large, by Central Limit Theorem, $\overline{T} \sim N\left(0.72, \dfrac{0.6768}{40}\right)$ approximately. We want to find $\mathrm{P}(\overline{S} > \overline{T})$, i.e. $\mathrm{P}(\overline{S} - \overline{T} > 0)$. $\overline{S} - \overline{T} \sim \mathrm{N}(0.12, 0.03645)$ $P(\overline{S} - \overline{T} > 0) = 0.73517 \approx 0.735$ (3 s.f.)