1 Let C, P, and S denote the price of a cow, pig, and sheep respectively.

$2C-13P+5S=1000$

$3C+3P-9S=0$

$-5C+8P+6S=-600$

Solving, $C=1200, P=300, S=500$

$-3C+4P+3S=-900$ i.e. there is a deficit of 900 coins.

Alternative

$-2C+13P-5S=1000$

$-3C-3P+9S=0$

$5C-8P-6S=-600$

Solving, $C=-1200, P=-300, S=-500$

$3C-4P-3S=-900$ i.e. there is a deficit of 900 coins.

2 $x=\dfrac{\cos^2\theta}{k} \Rightarrow \dfrac{dx}{d\theta}=-\dfrac{2}{k}\cos\theta\sin\theta$

When $x=0$, $\dfrac{\cos^2\theta}{k}=0 \Rightarrow \theta=\dfrac{\pi}{2}$

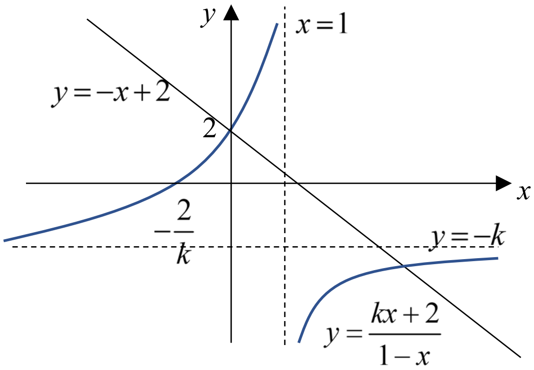
When $x=\dfrac{1}{2k}$, $\dfrac{\cos^2\theta}{k}=\dfrac{1}{2k} \Rightarrow \theta=\dfrac{\pi}{4}$

$$\int_0^{\frac{1}{2k}}\sqrt{\frac{kx}{1-kx}}\,dx=\int_{\frac{\pi}{2}}^{\frac{\pi}{4}}\sqrt{\frac{\cos^2\theta}{1-\cos^2\theta}}\left(-\frac{2}{k}\cos\theta\sin\theta\right)d\theta=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos\theta}{\sin\theta}\left(\frac{2}{k}\cos\theta\sin\theta\right)d\theta$$

$$=\frac{1}{k}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}2\cos^2\theta\,d\theta=\frac{1}{k}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos 2\theta+1\,d\theta=\frac{1}{k}\left[\frac{\sin 2\theta}{2}+\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$=\frac{1}{k}\left[\frac{\sin\pi}{2}+\frac{\pi}{2}-\left(\frac{\sin\frac{\pi}{2}}{2}+\frac{\pi}{4}\right)\right]=\frac{1}{k}\left(\frac{\pi}{4}-\frac{1}{2}\right)$$

3 **(i)**



(ii) To find the points of intersection,

$$\frac{kx+2}{1-x}=-x+2$$

$$kx+2=(-x+2)(1-x)=x^2-3x+2.$$

$$x[x-(3+k)]=0$$
$$x=0 \text{ or } x=3+k.$$

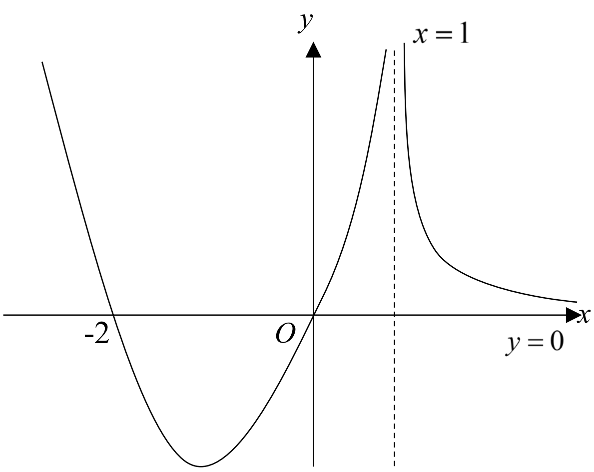
So, by interpreting the graph, $x<0$ or $1<x<3+k$

(iii) Replacing x with e^x:

$e^x<0$ (no soln $\because e^x>0$ for all x) or $1<e^x<3+k$

$\therefore 0<x<\ln(3+k)$

4 **(a)**



(bi) If $y=\frac{1}{g(x)}$ has $x=2$ as a vertical asymptote, then $y=g(x)$ has an $\underline{x}$-intercept at $x=2$. So $g(2)=0 \Rightarrow p=-2$.

(bii) If $y=\dfrac{1}{\mathrm{g}(x)}$ has a minimum point at $y=\dfrac{1}{5}$, then $y=\mathrm{g}(x)$ has a maximum point at $y=5$.

$\mathrm{g}'(x)=3x^2-3=3(x+1)(x-1)$ so turning points of $y=\mathrm{g}(x)$ are at $x=\pm 1$.

$\mathrm{g}''(x)=6x$ so, by second derivative test, $x=1$ is a minimum point while $x=-1$ is a maximum point. So turning point is at $(-1,5)$.

$(-1)^3-3(-1)+p=5 \Rightarrow p=3$.

Alternatively,

If $y=\dfrac{1}{\mathrm{g}(x)}$ has a minimum point at $y=\dfrac{1}{5}$, then $y=\mathrm{g}(x)$ has a maximum point at $y=5$.

From GC, observe that $y=x^3-3x$ has a maximum point at $y=2$. Since we need to move the maximum point to $y=5$, $p=3$

5 **(a)(i)** $w=x^2+y^2$

Differentiating implicitly w.r.t. *x*,

$$\frac{\mathrm{d}w}{\mathrm{d}x}=2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}=2\left(x+y\frac{\mathrm{d}y}{\mathrm{d}x}\right)$$

Thus, we have $x+y\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{2}\dfrac{\mathrm{d}w}{\mathrm{d}x}$

Substitute into the given DE: $\dfrac{1}{2}\dfrac{\mathrm{d}w}{\mathrm{d}x}=\sqrt{w} \Rightarrow \dfrac{\mathrm{d}w}{\mathrm{d}x}=2\sqrt{w}$

(a)(ii) Separating variables: $\dfrac{1}{\sqrt{w}}\dfrac{\mathrm{d}w}{\mathrm{d}x}=2$

Integrating w.r.t *x*, $\displaystyle\int \frac{1}{\sqrt{w}}\frac{\mathrm{d}w}{\mathrm{d}x}\mathrm{d}x=\int 2\ \mathrm{d}x$

$$\Rightarrow \int w^{-\frac{1}{2}}\mathrm{d}w=\int 2\ \mathrm{d}x \Rightarrow 2w^{\frac{1}{2}}=2x+C$$

Substituting back, $2\sqrt{x^2+y^2}=2x+C$

When $x=3$, $y=4$: $2\sqrt{3^2+4^2}=2(3)+C \Rightarrow C=4$

The particular solution is $\sqrt{x^2+y^2}=x+2$ (or $x^2+y^2=(x+2)^2$)

(b) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{1}{x}$

Integrate w.r.t. x, $\frac{dy}{dx} = \ln x + C$ (note: no need $|x|$ because $x > 0$)

Integrate w.r.t. x, $y = x\ln x - \int x \cdot \frac{1}{x}\,dx + Cx$

$= x\ln x - x + Cx + D$

$= x\ln x + C_1 x + D$ (where $C_1 = C - 1$)

6	**(i)**	Differentiating implicitly w.r.t. x, $\frac{1}{y+1}\frac{dy}{dx} = -\sec^2 x$. So $\frac{dy}{dx} = -(y+1)\sec^2 x$ --(1).
	(ii)	Differentiating again w.r.t. x, $\frac{d^2y}{dx^2} = -\frac{dy}{dx}\sec^2 x - 2(y+1)\sec^2 x\tan x$ --(2). When $x = 0$, $\ln(y+1) = 1 - \tan 0$ $y = e - 1$ Substituting $x = 0, y = e - 1$ into (1): $\frac{dy}{dx} = \frac{-(e-1+1)}{\cos^2 0} = -e$. Substituting into (2): $\frac{d^2y}{dx^2} = \frac{-(-e)}{\cos^2 0} - \frac{2(e)\tan 0}{\cos^2 0} = e$. So $y = (e-1) - ex + \frac{e}{2}x^2 + \ldots$
	(iii)	Rearranging, $y = e^{1-\tan x} - 1$. When x is small, $\tan x \approx x$. $y \approx e(e^{-x}) - 1$ $\approx e(1 - x + \frac{x^2}{2}) - 1$ $= (e-1) - ex + \frac{e}{2}x^2$
	(iv)	$e^{1-\tan x} \approx e - ex + \frac{e}{2}x^2$ so $\int_{-0.1}^{0} e^{1-\tan x}\,dx \approx \int_{-0.1}^{0} e - ex + \frac{e}{2}x^2\,dx = 0.28587$ (to 5 dp)

7 **(i)** Let $\dfrac{4r-1}{3^{r+2}}=\dfrac{Ar}{3^r}-\dfrac{B(r+2)}{3^{r+2}}=\dfrac{(9A-B)r-2B}{3^{r+2}}$

Comparing constants: $B=0.5$

Comparing coefficients of r: $4=9A-0.5\Rightarrow A=0.5$

$$\therefore \frac{4r-1}{3^{r+2}}=\frac{1}{2}\left[\frac{r}{3^r}-\frac{r+2}{3^{r+2}}\right]$$

(ii) $\displaystyle\sum_{r=1}^{n}\frac{4r-1}{3^{r+2}}=\frac{1}{2}\sum_{r=1}^{n}\left[\frac{r}{3^r}-\frac{r+2}{3^{r+2}}\right].$

$$\begin{aligned}
=\frac{1}{2}\Bigg[&\frac{1}{3}-\frac{3}{3^3}\\
&+\frac{2}{3^2}-\frac{4}{3^4}\\
&+\frac{3}{3^3}-\frac{5}{3^5}\\
&\;.\\
&\;.\\
&\;.\\
&+\frac{n-2}{3^{n-2}}-\frac{n}{3^n}\\
&+\frac{n-1}{3^{n-1}}-\frac{n+1}{3^{n+1}}\\
&+\frac{n}{3^n}-\frac{n+2}{3^{n+2}}\Bigg]
\end{aligned}$$

$$=\frac{1}{2}\left(\frac{5}{9}-\frac{n+1}{3^{n+1}}-\frac{n+2}{3^{n+2}}\right)$$ (or equivalent)

(iii) Series converges as $\displaystyle\lim_{n\to\infty}\frac{n+1}{3^{n+1}}=\lim_{n\to\infty}\frac{n+2}{3^{n+2}}=0$. $\displaystyle\sum_{r=1}^{\infty}\frac{4r-1}{3^{r+2}}=\frac{5}{18}$.

(iv) $\displaystyle\sum_{r=1}^{\infty}\frac{1}{3^{r+2}}=\frac{1}{3^3}+\frac{1}{3^4}+\ldots=\frac{\frac{1}{27}}{1-\frac{1}{3}}=\frac{1}{18}$.

Hence,

$$\sum_{r=1}^{\infty}\frac{4r-1}{3^{r+2}}=\sum_{r=1}^{\infty}\left(\frac{4r}{3^{r+2}}-\frac{1}{3^{r+2}}\right)$$

$$\sum_{r=1}^{\infty}\frac{4r-1}{3^{r+2}}=\sum_{r=1}^{\infty}\frac{4r}{3^{r+2}}-\sum_{r=1}^{\infty}\frac{1}{3^{r+2}}$$

From part **(iii)** and the sum just found,

$$\frac{5}{18}=\sum_{r=1}^{\infty}\frac{4r}{3^{r+2}}-\frac{1}{18}$$

$$\sum_{r=1}^{\infty}\frac{4r}{3^{r+2}}=\frac{5}{18}+\frac{1}{18}=\frac{1}{3}$$

$$\sum_{r=1}^{\infty}\frac{4r}{3^{2}\cdot 3^{r}}=\frac{1}{3}$$

$$\frac{4}{9}\sum_{r=1}^{\infty}\frac{r}{3^{r}}=\frac{1}{3}$$

$$\therefore \sum_{r=1}^{\infty}\frac{r}{3^{r}}=\frac{1}{3}\times\frac{9}{4}=\frac{3}{4}.$$

8 (i) total area $=\frac{2}{n}\left\{\left[\left(\frac{2}{n}\right)^{2}+2\right]+\left[\left(\frac{4}{n}\right)^{2}+2\right]+...+\left[\left(\frac{2(n-1)}{n}\right)^{2}+2\right]+\left[\left(\frac{2n}{n}\right)^{2}+2\right]\right\}$

$$=\frac{2}{n}\sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^{2}+2\right]$$
$$=\frac{2}{n}\sum_{i=1}^{n}\left(\frac{4}{n^{2}}i^{2}+2\right)$$
$$=\frac{2}{n}\left[\left(\frac{4}{n^{2}}\right)\sum_{i=1}^{n}i^{2}+\sum_{i=1}^{n}2\right]$$
$$=\frac{2}{n}\left(\frac{4}{n^{2}}\frac{n(n+1)(2n+1)}{6}+2n\right)$$
$$=\frac{4(n+1)(2n+1)}{3n^{2}}+4 \quad \text{(shown)}$$

(ii) Area of $R=\int_{0}^{2}x^{2}+2\ \mathrm{d}x=\left[\frac{x^{3}}{3}+2x\right]_{0}^{2}=\frac{20}{3}$

Let A_n denote the total area when n rectangles are used.

So we need $A_n-R=\frac{4(n+1)(2n+1)}{3n^{2}}+4-\frac{20}{3}<0.1$

From GC,

$n=40$: $A_{40}-R=0.1008$
$n=41$: $A_{41}-R=0.0984$

So least $n=41$

(iii) [Note that area of a trapezium $=\frac{1}{2}(\text{sum of parallel sides})\times\text{height}$.]

For n trapeziums, width $=\frac{2}{n}$. So for $n=4$, width $=\frac{2}{4}=\frac{1}{2}$

Let B_n denote the total area when n trapeziums are used.

$$B_4 = \frac{1}{2}\left(f(0)+f\left(\frac{1}{2}\right)\right)\times\frac{1}{2}+\frac{1}{2}\left(f\left(\frac{1}{2}\right)+f\left(\frac{2}{2}\right)\right)\times\frac{1}{2}+\frac{1}{2}\left(f\left(\frac{2}{2}\right)+f\left(\frac{3}{2}\right)\right)\times\frac{1}{2}+\frac{1}{2}\left(f\left(\frac{3}{2}\right)+f(2)\right)\times\frac{1}{2}$$

$$=\frac{1}{2}\left[\left(f(0)+f\left(\frac{1}{2}\right)\right)+\left(f\left(\frac{1}{2}\right)+f\left(\frac{2}{2}\right)\right)+\left(f\left(\frac{2}{2}\right)+f\left(\frac{3}{2}\right)\right)+\left(f\left(\frac{3}{2}\right)+f(2)\right)\right]\times\frac{1}{2}$$

$$=\frac{1}{4}\left[f(0)+2f\left(\frac{1}{2}\right)+2f\left(\frac{2}{2}\right)+2f\left(\frac{3}{2}\right)+f(2)\right]$$

$$=6.75\text{ units}^2$$

$B_4 - R = 0.0833$

(iv) Note that $B_4 - R = 0.0833 < 0.1$.

In (ii), it takes 41 rectangles to be less than 0.1 units^2 of the actual area, whereas in (iii) it only takes 4 trapeziums to achieve that. So using trapeziums to estimate the area is more efficient.

[Possible way to organise information from the two parts to help identify the more significant aspect to comment on:

	Shape	n	Difference
(ii)	rectangles	41	< 0.1
(iii)	trapeziums	4	0.0833
Comparison		~ 10 times	~ equal

9 **(i)** $\frac{dx}{dt} = 1+e^t$, $\frac{dy}{dt} = 3-e^{-t}$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{3-e^{-t}}{1+e^t}$$

$$\frac{3-e^{-t}}{1+e^t} = 0$$

$3 - e^{-t} = 0$

$t = -\ln 3$

$x = -\ln 3 + e^{-\ln 3}$, $y = -3\ln 3 + e^{\ln 3}$

$= -\ln 3 + \frac{1}{3}$, $= -3\ln 3 + 3$

coordinates of the stationary point is $\left(-\ln 3+\frac{1}{3}, -3\ln 3+3\right)$

(ii) $\frac{dx}{dt} = 1+e^t$, $\frac{dy}{dt} = 3-e^{-t}$

$$\frac{dy}{dt} = \frac{dx}{dt}$$

$3-e^{-t}=1+e^{t}$

$e^{2t}-2e^{t}+1=0$

$\left(e^{t}-1\right)^{2}=0$

$e^{t}=1$

$t=0$

$x=0+e^{0}=1$; $y=3(0)+e^{0}=1$

coordinates of point P is $(1,1)$

gradient of normal at $P=-\dfrac{1+e^{0}}{3-e^{-0}}=-1$

Equation of normal at P: $y-1=-1(x-1)\Rightarrow y=-x+2$

(iii) Sub $x=t+e^{t}$, $y=3t+e^{-t}$ into $y=-x+2$, $3t+e^{-t}=-\left(t+e^{t}\right)+2$

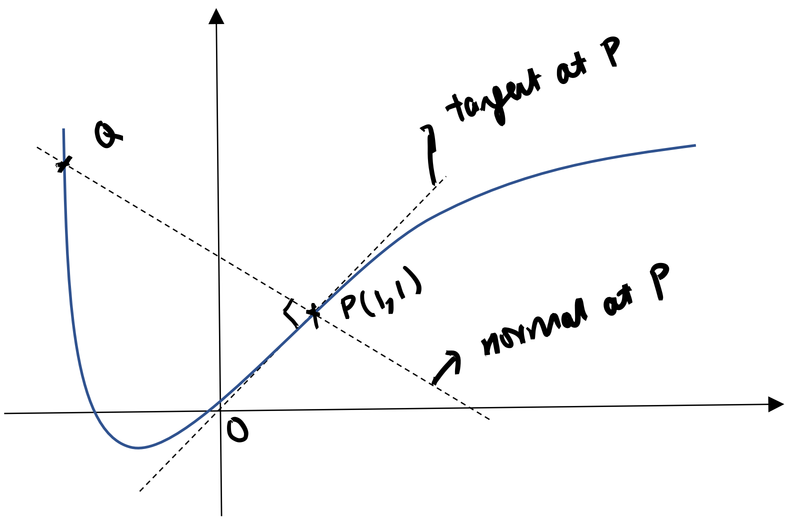
Using GC, $t=0$ or $t=-2.466497$

Point Q:

$\left(-2.466497+e^{-2.466497},3(-2.466497)+e^{2.466497}\right)$

$(-2.38,4.38)$

(iv) Gradient of normal at P is -1. Thus gradient of QP is -1. Gradient of line OP is 1 since P is point (1,1). Since the product of their gradients is -1, OP is perpendicular to QP and thus triangle OPQ is a right-angled triangle.



Alternative

$OQ^{2}=24.871,\ OP^{2}=2,\ PQ^{2}=22.871$

Since $OQ^{2}=OP^{2}+PQ^{2}$, by Pythagoras' Theorem, $OP\perp PQ$ and so triangle OPQ is a right-angled triangle.

$\overrightarrow{OP} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\overrightarrow{PQ} = \begin{pmatrix} -2.38-1 \\ 4.38-1 \end{pmatrix} = \begin{pmatrix} -3.38 \\ 3.38 \end{pmatrix}$

Since $\overrightarrow{OP} \cdot \overrightarrow{PQ} = 0$, then $OP \perp PQ$ and so triangle OPQ is a right-angled triangle.

10 **(i)** Amt immediately after 2nd dose $= 120 \times \left(\frac{1}{2}\right)^3 + 120 = 135\text{mg (shown)}$.

(ii) amt of drug $= 120 + 120 \times \left(\frac{1}{8}\right) + ... + 120 \times \left(\frac{1}{8}\right)^{n-1} = \dfrac{120\left(1 - \left(\frac{1}{8}\right)^n\right)}{1 - \frac{1}{8}} = \dfrac{960}{7}\left(1 - \left(\frac{1}{8}\right)^n\right)$

(iii) As $n \to \infty$, $\left(\frac{1}{8}\right)^n \to 0$ so long-term amt = 137mg (3s.f.) (or $\frac{960}{7}$ mg)

(iv) Amt in Nya's bloodstream after kth dose $= \dfrac{120\left(1 - 0.6^k\right)}{1 - 0.6} = 300\left(1 - 0.6^k\right)$

We want $300\left(1 - 0.6^k\right) > 280$

Method 1 (from GC)

k	$300\left(1 - 0.6^k\right)$
5	276.67
6	286

$\therefore$ least $k = 6$, i.e. Nya will experience heart palpitations after the 6th dose.

Method 2 (algebraic)

$$300\left(1 - 0.6^k\right) > 280 \Rightarrow 0.6^k < \frac{1}{15} \Rightarrow k > \frac{\ln\left(\frac{1}{15}\right)}{\ln 0.6} = 5.3013$$

$\therefore$ least $k = 6$, i.e. Nya will experience heart palpitations after the 6th dose.

(v) Let m be the number of complete days taken from the onset of palpitations till completely cleared.

We want $286 \times 0.6^m < 1$

m	286×0.6^m
11	1.0376
12	0.6226

$\therefore$ least $m = 12$

(vi) Let n be the number of days taken to reach 25mg. Then $3 + (n-1)(2) = 25 \Rightarrow n = 12$.

Total mg $= \dfrac{12}{2}\left[2(3) + 11(2)\right] + (28 - 12) \times 25 = 568\text{mg}$

11	**(i)**	$p_1 : \mathbf{r}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}=0 \quad p_2 : \mathbf{r}\cdot\begin{pmatrix}-3\\6\\-20\end{pmatrix}=90$ Let θ be the required acute angle. $\cos\theta=\dfrac{\left\lvert\begin{pmatrix}0\\0\\1\end{pmatrix}\cdot\begin{pmatrix}-3\\6\\-20\end{pmatrix}\right\rvert}{\sqrt{9+36+400}}=\dfrac{20}{\sqrt{445}}$ $\Rightarrow\theta=18.542^\circ=18.5^\circ \quad (1\text{ d.p.})$
	(ii)	The line AD is the line of intersection between p_1 and p_2. $p_1 : z=0$ $p_2 : -3x+6y-20z=90.$ **Method 1 (use GC PlySmlt2)** Solving $z=0$ and $-3x+6y-20z=90$ $\left.\begin{aligned}x&=-30+2y\\ y&=y\\ z&=0\end{aligned}\right\\} \ \mathbf{r}=\begin{pmatrix}-30\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\0\end{pmatrix}, \lambda\in\mathbb{R}$ **Method 2 (find direction of line AD)** A direction of line AD is $\mathbf{n}_1\times\mathbf{n}_2=\begin{pmatrix}0\\0\\1\end{pmatrix}\times\begin{pmatrix}-3\\6\\-20\end{pmatrix}=\begin{pmatrix}6\\3\\0\end{pmatrix}=3\begin{pmatrix}2\\1\\0\end{pmatrix}$ Also, at A, $x=0$ and $z=0$ $\Rightarrow -3(0)+6y-20(0)=90\Rightarrow 6y=90\Rightarrow y=15$ So $A(0,15,0)$. So eqn of $l_{AD} : \mathbf{r}=\begin{pmatrix}0\\15\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\0\end{pmatrix}, \lambda\in\mathbb{R}$ **Method 3 (solve 2 eqns manually)** Sub $z=0$ into $-3x+6y-20z=90$: $-3x+6y=90\Rightarrow y=15+\dfrac{1}{2}x \quad \text{OR} \quad x=-30+2y$

Let $x=\alpha,\ \alpha\in\mathbb{R}$ Then $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}\alpha\\15+\frac{1}{2}\alpha\\0\end{pmatrix}$ $l_{AD}:\mathbf{r}=\begin{pmatrix}0\\15\\0\end{pmatrix}+\alpha\begin{pmatrix}1\\\frac{1}{2}\\0\end{pmatrix},\ \alpha\in\mathbb{R}$ OR $\mathbf{r}=\begin{pmatrix}0\\15\\0\end{pmatrix}+\alpha\begin{pmatrix}2\\1\\0\end{pmatrix},\ \alpha\in\mathbb{R}$	Let $y=\beta,\quad \beta\in\mathrm{R}$ Then $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-30+2\beta\\\beta\\0\end{pmatrix}$ $l_{AD}:\mathbf{r}=\begin{pmatrix}-30\\0\\0\end{pmatrix}+\beta\begin{pmatrix}2\\1\\0\end{pmatrix},\beta\in\mathbb{R}$

(iii) Given that $M(20,2,1.5)$ is the mid-point of *ST*,

$$\overrightarrow{OM}=\frac{\overrightarrow{OS}+\overrightarrow{OT}}{2}\Rightarrow\overrightarrow{OT}=2\begin{pmatrix}20\\2\\1.5\end{pmatrix}-\begin{pmatrix}18\\2.5\\2\end{pmatrix}=\begin{pmatrix}22\\1.5\\1\end{pmatrix}$$

$\therefore T(22,1.5,1)$

(iv) Since *X* is nearest to *S*, we can see that *X* is the foot of perpendicular from *S* to the line *AD*.

$$\overrightarrow{OX}=\begin{pmatrix}-30\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\0\end{pmatrix}\text{ for some }\lambda\in\mathbb{R}$$

$$\overrightarrow{SX}=\overrightarrow{OX}-\overrightarrow{OS}=\left[\begin{pmatrix}-30\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\0\end{pmatrix}\right]-\begin{pmatrix}18\\2.5\\2\end{pmatrix}=\begin{pmatrix}-48\\-2.5\\-2\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\0\end{pmatrix}$$

Since $\overrightarrow{SX}\perp l_{AD}$, $\left[\begin{pmatrix}-48\\-2.5\\-2\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\0\end{pmatrix}\right]\cdot\begin{pmatrix}2\\1\\0\end{pmatrix}=0$

$$-96-2.5+\lambda(4+1)=0$$

$$\Rightarrow\lambda=19.7$$

$$\overrightarrow{OX}=\begin{pmatrix}-30\\0\\0\end{pmatrix}+19.7\begin{pmatrix}2\\1\\0\end{pmatrix}=\begin{pmatrix}9.4\\19.7\\0\end{pmatrix}\ \therefore X(9.4,19.7,0)$$

[Remark: The steps if you used $\overrightarrow{OX}=\begin{pmatrix}0\\15\\0\end{pmatrix}+\alpha\begin{pmatrix}2\\1\\0\end{pmatrix}$ for some $\alpha\in\mathbb{R}$ are similar.]

(v) **Method 1** ($\perp$ dist from Q to $p_1=\perp$ dist from Q to p_2)

$\perp$ dist from Q to $p_1=k$ ($\because Q(2,15,k)$, and p_1 is horizontal)

$\perp$ dist from Q to p_2 :

A point on p_2 is $(-30, 0, 0)$. Call this point R.

So $\overrightarrow{QR} = \begin{pmatrix} -30 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 15 \\ k \end{pmatrix} = \begin{pmatrix} -32 \\ -15 \\ -k \end{pmatrix}$

Then $\perp$ dist from Q to $p_2 = \left|\overrightarrow{QR}.\hat{\mathbf{n}}_2\right| = \dfrac{\left|\begin{pmatrix} -32 \\ -15 \\ -k \end{pmatrix}.\begin{pmatrix} -3 \\ 6 \\ -20 \end{pmatrix}\right|}{\sqrt{445}} = \dfrac{6+20k}{\sqrt{445}}$

So $k = \dfrac{|6+20k|}{\sqrt{445}} \Rightarrow k = 5.4793 = 5.48$ (3s.f.)

[Remark: The steps if you used $(0,15,0)$ - which is point A – are similar.]

Method 2 (instead of using LoP* above)

$\perp$ dist from Q to $p_2 = \dfrac{\left|\begin{pmatrix} 2 \\ 15 \\ k \end{pmatrix}.\begin{pmatrix} -3 \\ 6 \\ -20 \end{pmatrix} - 90\right|}{\sqrt{445}} = \dfrac{|-6-20k|}{\sqrt{445}} = \dfrac{6+20k}{\sqrt{445}}$

So $k = \dfrac{6+20k}{\sqrt{445}} \Rightarrow k = 5.4793 = 5.48$ (3s.f.)