1 A particle is moving along the curve with equation $3x^2 + xy = 5$. Given that $\frac{dy}{dt} = 4$ when $x = 1$, find the rate of decrease of x at this instant. [4]

2 A triangle ABC is such that $AC = \sqrt{2}$, $BC = 4$ and angle $ACB = \frac{1}{4}\pi + \theta$. Given that θ is sufficiently small for θ^3 and higher powers of θ to be neglected, show that

$$AB \approx \sqrt{10}\left[1 + a\theta + b\theta^2\right],$$

where a and b are real constants. [5]

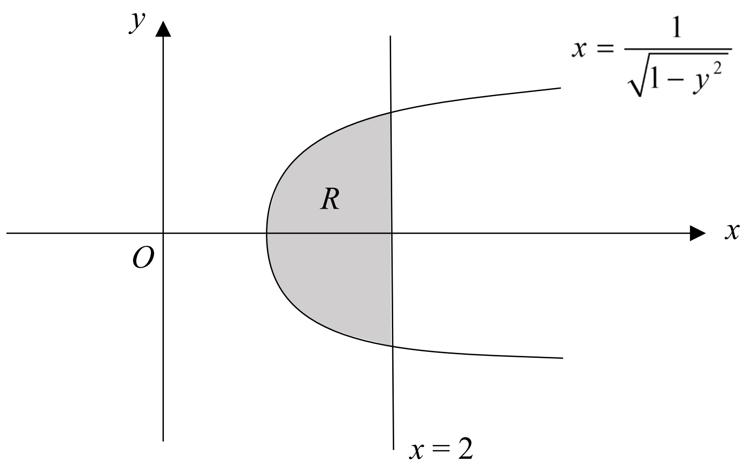
3 The curve C has equation $y = \frac{x^2 + px + 3}{x + q}$, where p and q are non-zero constants and $x \neq -q$. It is given that the asymptotes of C are $y = x + 3$ and $x = 2$.

(i) Find the values of p and q. Hence sketch C, stating the coordinates of any axial intercepts and turning points. [4]

Use the values of p and q that you have found in part **(i)** to answer part **(ii)**.

(ii) By drawing a suitable curve on the same diagram in part **(i)**, find the range of values of b, where b is a positive constant, such that there are 2 real roots to the equation $(x-5)^2 + b\left(\frac{x^2 + px + 3}{x + q}\right)^2 = 1.$ [2]

4 The diagram shows the shaded region R bounded by curve C with equation $x = \frac{1}{\sqrt{1 - y^2}}$ and the line $x = 2$.



(i) Find the exact volume of the solid generated when R is rotated through 2π radians about the y-axis. [5]

(ii) Write down the equation of the curve obtained when C is translated by 2 units in the negative x-direction. Hence, or otherwise, find the volume of the solid generated when R is rotated through 2π radians about the line $x = 2$. [3]

5 The complex numbers z_1 and z_2 are such that

$$z_1 = 1 + b\text{i},\ \ b > 1,\ \ \arg(z_1) = \alpha \ \text{ and}$$

$$z_2 = 1 - c\text{i},\ \ 0 < c < 1,\ \ \arg(z_2) = \beta.$$

Let Z_1 and Z_2 be points representing z_1 and z_2 respectively on the Argand diagram.

(i) Indicate Z_1 and Z_2 on an Argand diagram. [2]

Z_3 is the point representing z_3 on an Argand diagram such that $|z_3| = |z_1|$, the origin O is collinear with Z_2 and Z_3, and $z_3 = \text{e}^{k\text{i}} z_1$, where k is a real constant.

(ii) Indicate the two possible positions of Z_3 on the same Argand diagram drawn in part **(i)**. [2]

(iii) Hence determine the two possible values of k, leaving your answers in terms of α and β. [2]

(iv) For value of $k = \frac{1}{2}\pi$, express the area of triangle $Z_1Z_2Z_3$ in terms of $|z_1|$ and $|z_2|$. [2]

6 **Do not use a calculator in answering this question.**

Let $\text{f}(x) = 2x + 1 - 2\ln(\sec x + \tan x)$.

(i) Find $\text{f}'(x)$, leaving your result in the form which involves one trigonometric function. [2]

(ii) Deduce that $\text{f}(x) < 1$ for $0 < x < \frac{1}{2}\pi$. [2]

The equation $\text{f}(x) = 0$ has a root, α, where $0 < \alpha < \frac{1}{2}\pi$.

(iii) A student wants to approximate the value of α using the x-intercept of the line passing through the points $(p,\ \text{f}(p))$ and $(q,\ \text{f}(q))$, where $0 < p < \alpha < q < \frac{1}{2}\pi$. Obtain an expression for the approximation of α, giving it in terms of p, q, $\text{f}(p)$ and $\text{f}(q)$. [2]

(iv) By finding $\text{f}''(x)$ and by considering the concavity of the graph $y = \text{f}(x)$, explain whether the estimate in part **(iii)** is an over-estimate or an under-estimate. [2]

7 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_{n+2} + au_{n+1} + u_n = b$, where a, b are constants and $n \geq 1$.

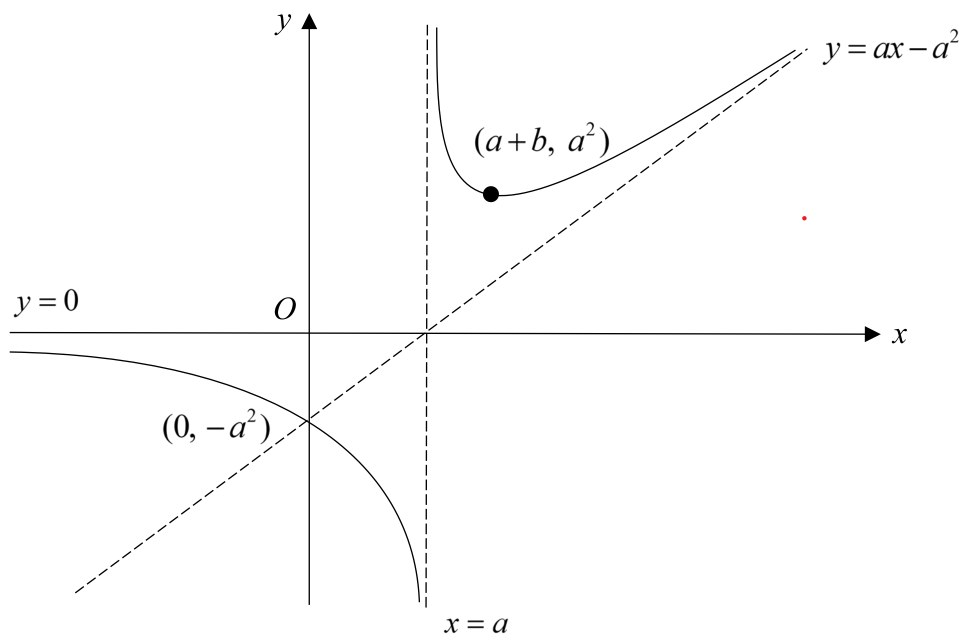
(i) Given that $u_1 = c$, $u_2 = 0.5$, $u_3 = 5$, $u_4 = 14.5$ and $u_5 = 29$, where c is a constant, find a, b and c. [3]

(ii) By using the substitution $v_n = u_{n+1} - u_n$, explain why the sequence $\{v_n\}$ is an arithmetic progression. [2]

(iii) By considering $\sum_{r=1}^{n-1} v_r = \sum_{r=1}^{n-1} (u_{r+1} - u_r)$, find u_n in terms of n. [4]

(iv) Describe the behaviour of the sequence $\{u_n\}$ for large values of n. [1]

8 The diagram below shows the graph of $y = \mathrm{f}(x)$, defined for $x \in \mathbb{R} \setminus \{a\}$, with asymptotes $x = a$, $y = 0$ and $y = ax - a^2$, stationary point at $(a+b,\ a^2)$, and cuts the y-axis at $(0,\ -a^2)$, where a and b are positive constants such that $a > b$.



(i) Sketch the graph of $y = \dfrac{1}{\mathrm{f}(x)}$. [3]

(ii) Sketch the graph of $y = \mathrm{f}'(x)$. [3]

Another function g, is given by

$$\mathrm{g}(x) = \begin{cases} x & \text{for } 0 \leq x < a, \\ a+b & \text{for } x \geq a. \end{cases}$$

(iii) Sketch the graph of g. [2]

(iv) Explain why fg exists. [3]

(v) Find the range of fg, leaving your answer in terms of a and b. [2]

9 Points A and B have position vectors $\begin{pmatrix}1\\q\\2\end{pmatrix}$ and $\begin{pmatrix}-1\\0\\1\end{pmatrix}$ respectively, where q is a positive constant.

The equation of the line l is $\dfrac{4x-15}{-2}=y,\ z=\dfrac{5}{2}$.

(i) Given that the length of projection of vector $\overrightarrow{AB}$ onto line l is $\dfrac{8}{\sqrt{5}}$, show that $q=5$. [3]

(ii) Find the coordinates of the point of A', where A' is the image of point A reflected in the line l. [4]

(iii) The planes p_1 and p_2 have equations $\mathbf{r}\cdot\begin{pmatrix}2\\1\\k\end{pmatrix}=1$ and $\mathbf{r}=\mu\begin{pmatrix}1\\s\\1\end{pmatrix}+\gamma\begin{pmatrix}2\\-1\\1\end{pmatrix}$ respectively, for constants k, s and parameters μ, γ. Given that the two planes p_1 and p_2 intersect at l, determine the values of k and s. [3]

(iv) The plane p_3 has equation $2x+ty+z=u$. Given that the planes p_1, p_2 and p_3 have no point in common, what can be said about the values of t and u? [2]

10 Mr Safe takes up a bank loan of $\$M$ for an investment plan. The loan amount compounds at an annual interest rate of $100r\%$, where $0<r<1$.

(i) By considering the loan amount at the end of 6 years, find the range of values of r such that the amount of compounded interest will not exceed 20% of the initial loan. [3]

Take M = 20 000 for the rest of the question.

Mr Safe invests \$10 000 and \$10 000 in Scheme A and Scheme B respectively for 6 years, with the two schemes starting at the same time.

Scheme A:
To receive payouts at the end of every month, starting from \$10 and increasing by \$0.30 every month.

Scheme B:
To receive payouts at the end of every month, starting from \$10 and increasing by 2% every month.

At the end of 6 years, the principal sum of \$20 000 is returned to Mr Safe.

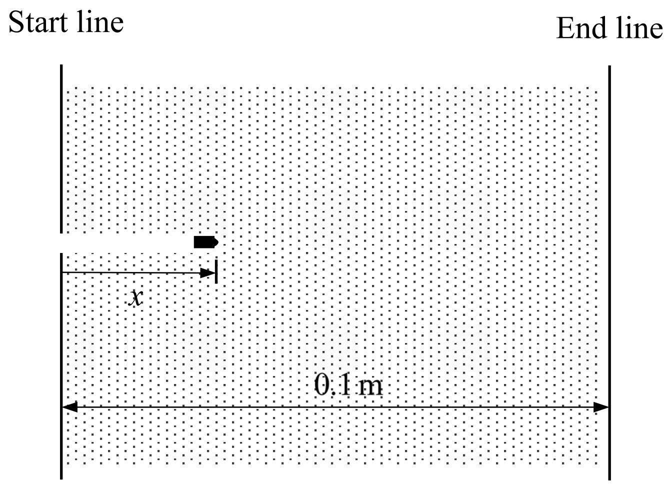
(ii) Find the total monthly payouts Mr Safe receives after n months, leaving your answer in exact form. [4]

(iii) Find the set of values of n such that the total monthly payouts from Scheme B exceeds that of Scheme A. [2]

(iv) Given that the annual interest rate of the bank is 2%, determine whether it is worthwhile for Mr Safe to take up the bank loan for the investment. [3]

11 A company conducts experiments to investigate the resistive power of armoured plates of different materials against a bullet fired from a rifle. The plates used for the experiment have a consistent thickness of 0.1 m. Once the plates are positioned, a bullet is fired at the start line of the plate with a fixed velocity of 100 m s^{-1}.

For a duration of t s after it is fired, the bullet is x m away from the start line and travels at a velocity $v\text{ m s}^{-1}$. The setup is shown below.



(i) When a plate of material P is used, v and x are modelled by the following differential equations:

$$\text{(A)} \qquad \frac{\mathrm{d}x}{\mathrm{d}t} = v,$$

$$\text{(B)} \qquad \frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \beta\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 = 0,$$

where β is a positive constant.

(a) By substituting equation (A), show that equation (B) can be written as $\frac{\mathrm{d}v}{\mathrm{d}t} = -\beta v^2$. [1]

(b) Find v in terms of β and t and hence find x in terms of β and t. [5]

(c) Given that it is found that the bullet exits the end line of the plate with a velocity of 80 m s^{-1}, find the time taken for the bullet to pass through the plate of material P, leaving your answer to 5 decimal places. [3]

(ii) The company develops another plate made of a new material Q. When the experiment is conducted with a plate of material Q, it is found that the bullet is slowed down in such a way that $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ is inversely proportional to $\mathrm{e}^{10\,000t}$. It is given that $\frac{\mathrm{d}x}{\mathrm{d}t} = 100$ and $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -10^6$ when $t = 0$. Write down a differential equation to model the bullet as it travels through the plate of material Q. Solve this differentiation equation to find x in terms of t for this model. [4]

(iii) The company claims that material Q provides better protection against bullets than material P. Explain, with justification, whether the company's claim is valid. [1]