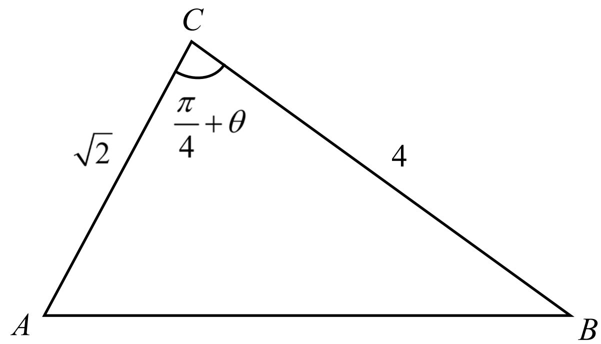
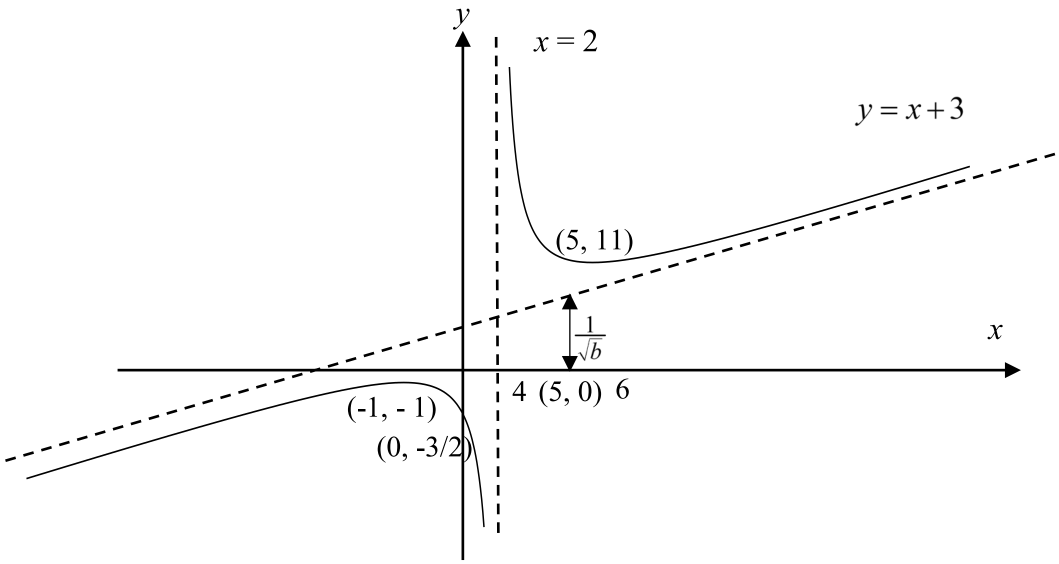
2021 Year 6 H2 Math Prelim Paper 1 Mark Scheme

	Suggested Solution
1	$3x^2+xy=5$ --------- (1) $\frac{\mathrm{d}}{\mathrm{d}t}$: $6x\frac{\mathrm{d}x}{\mathrm{d}t}+x\frac{\mathrm{d}y}{\mathrm{d}t}+y\frac{\mathrm{d}x}{\mathrm{d}t}=0$ ------- (2) When $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}t}=4$: (1): $3+y=5$ $\therefore y=2$ (2): $6\frac{\mathrm{d}x}{\mathrm{d}t}+4+2\frac{\mathrm{d}x}{\mathrm{d}t}=0$ $\therefore \frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{4}{8}=-\frac{1}{2}$ Rate of decrease of x is $\frac{1}{2}$. **Alternative** $3x^2+xy=5$ --------- (1) $\frac{\mathrm{d}}{\mathrm{d}x}$: $6x+x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0$ ------- (2) When $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}t}=4$: (1): $3+y=5$ $\therefore y=2$ (2): $6+\frac{\mathrm{d}y}{\mathrm{d}x}+2=0$ $\therefore \frac{\mathrm{d}y}{\mathrm{d}x}=-8$ Since $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}x}.\frac{\mathrm{d}x}{\mathrm{d}t}$ $\therefore \frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{4}{8}=-\frac{1}{2}$ Rate of decrease of x is $\frac{1}{2}$.

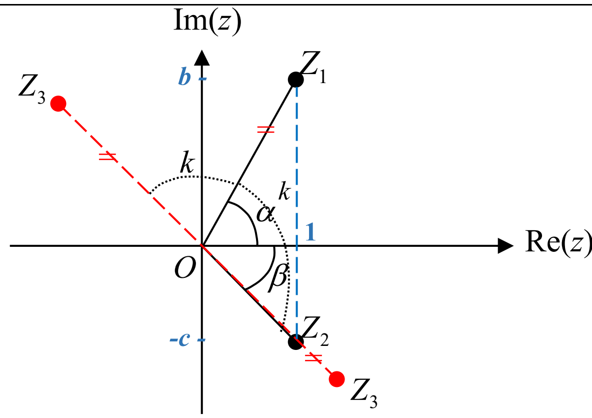
	Suggested Solution
2	 $AB^2=\left(\sqrt{2}\right)^2+4^2-2\sqrt{2}\left(4\right)\cos\left(\frac{\pi}{4}+\theta\right)$ $=2+16-8\sqrt{2}\cos\left(\frac{\pi}{4}+\theta\right)$ $=18-8\sqrt{2}\left(\cos\frac{\pi}{4}\cos\theta-\sin\frac{\pi}{4}\sin\theta\right)$ $=18-8\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos\theta-\frac{1}{\sqrt{2}}\sin\theta\right)$ $=18-8\left(\cos\theta-\sin\theta\right)$ $\approx 18-8\left(1-\frac{\theta^2}{2}-\theta\right)$ (since θ is small) $=18-8+4\theta^2+8\theta$ $=10+8\theta+4\theta^2$

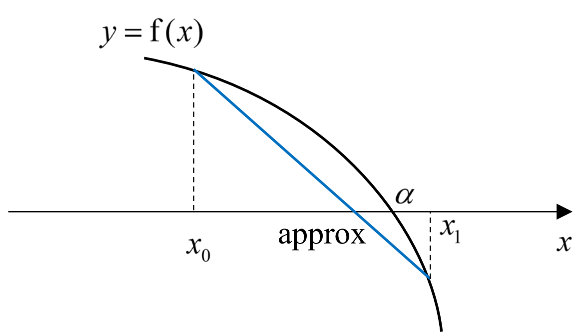
	$AB=\left(10+8\theta+4\theta^2\right)^{\frac{1}{2}}$ $=\sqrt{10}\left[1+\left(\frac{4}{5}\theta+\frac{2}{5}\theta^2\right)\right]^{\frac{1}{2}}$ $=\sqrt{10}\left[1+\frac{1}{2}\left(\frac{4}{5}\theta+\frac{2}{5}\theta^2\right)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{4}{5}\theta+\frac{2}{5}\theta^2\right)^2+\ldots\right]$ $=\sqrt{10}\left[1+\frac{2}{5}\theta+\frac{1}{5}\theta^2-\frac{1}{8}\left(\frac{16}{25}\theta^2+\ldots\right)+\ldots\right]$ $=\sqrt{10}\left(1+\frac{2}{5}\theta+\frac{1}{5}\theta^2-\frac{2}{25}\theta^2+\ldots\right)$ $\approx\sqrt{10}\left(1+\frac{2}{5}\theta+\frac{3}{25}\theta^2\right)$

Qn	Suggested Solution
3(i)	Since $x=2$ is a vertical asymptote, we have $q=-2$. <u>Method 1:</u> $y=x+3+\frac{k}{x-2}$ $=\frac{x^2+x-6+k}{x-2}$ Comparing coefficient of x in given equation, $p=1$ $\therefore y=\frac{x^2+x+3}{x-2}=x+3+\frac{9}{x-2}$ <u>Method 2:</u> $y=\frac{x^2+px+3}{x-2}$ $=(x+p+2)+\frac{7+2p}{x-2}$ (by long division) Comparing with the given oblique asymptote $p+2=3\Rightarrow p=1$

	
(ii)	2 real roots for $(x-5)^2+b\left(\frac{x^2+px+3}{x+q}\right)^2=1$ implies that there are 2 intersection points between curve C and $(x-5)^2+by^2=1 \Rightarrow (x-5)^2+\frac{y^2}{\left(\frac{1}{\sqrt{b}}\right)^2}=1$ (ellipse) Sketch ellipse with centre (5,0) and horizontal axis 1 and vertical axis $\frac{1}{\sqrt{b}}$ $\frac{1}{\sqrt{b}}>11$ $0<\sqrt{b}<\frac{1}{11}$ $0<b<\frac{1}{121}$

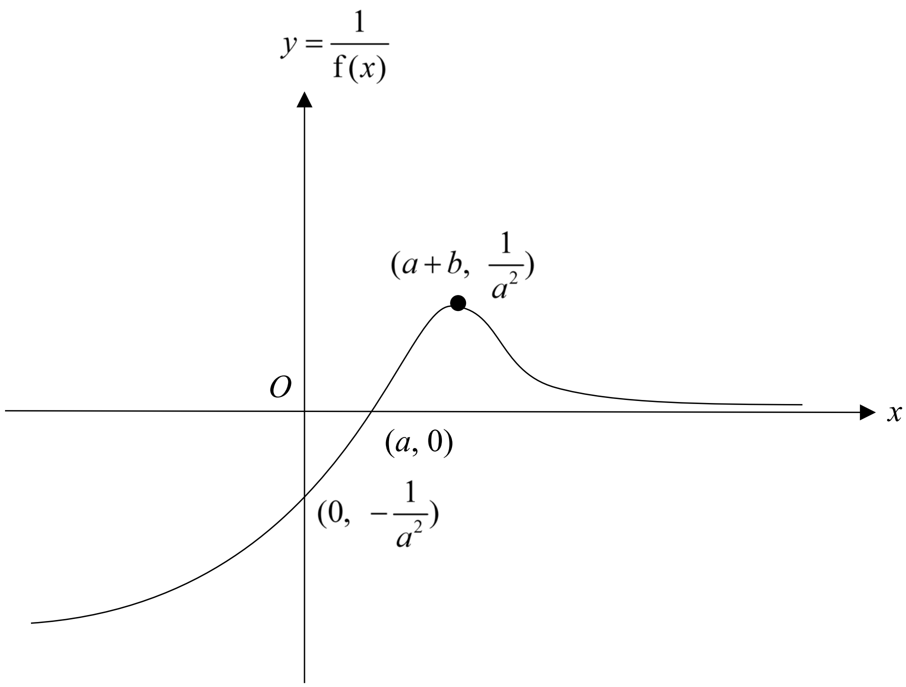
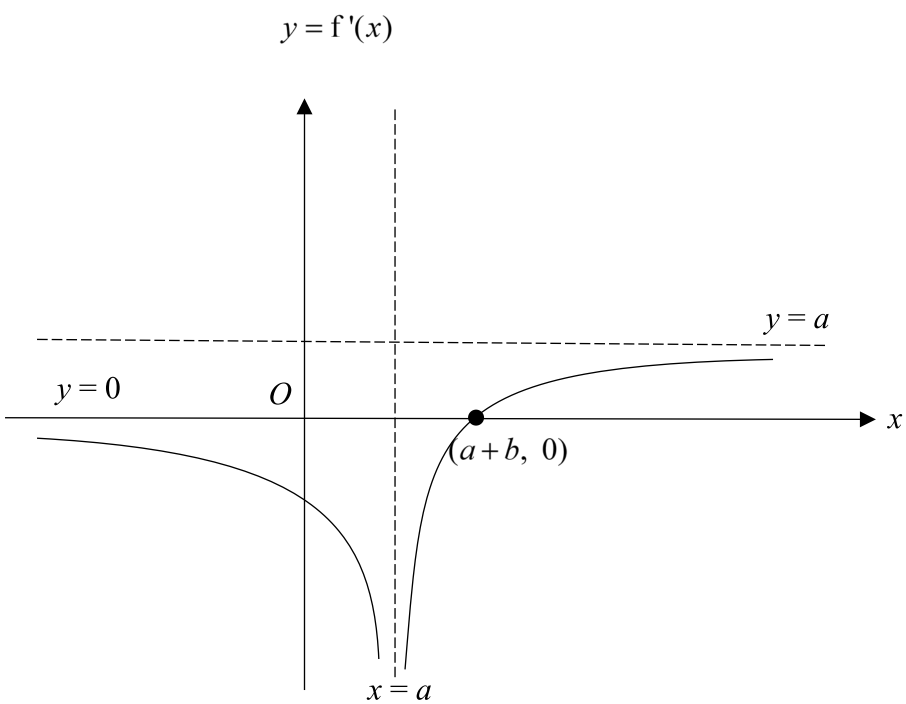
Qn	Solution
4(i)	When $x=2$, $2=\dfrac{1}{\sqrt{1-y^2}}$ $\sqrt{1-y^2}=\dfrac{1}{2}$ $1-y^2=\dfrac{1}{4}$ $y^2=\dfrac{3}{4}$ $y=\dfrac{\sqrt{3}}{2}$ or $y=-\dfrac{\sqrt{3}}{2}$ Volume of solid $=\pi(2^2)(\sqrt{3})-2\pi\int_0^{\frac{\sqrt{3}}{2}}\dfrac{1}{1-y^2}\ \mathrm{d}y$ $=4\sqrt{3}\pi-\dfrac{2\pi}{2}\left[\ln\left(\dfrac{1+y}{1-y}\right)\right]_0^{\frac{\sqrt{3}}{2}}$ $=4\sqrt{3}\pi-\pi\left[\ln\left(\dfrac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}\right)-\ln 1\right]$ $=4\sqrt{3}\pi-\pi\ln\left(\dfrac{2+\sqrt{3}}{2-\sqrt{3}}\right)$
(ii)	Eqn of new curve: $x+2=\dfrac{1}{\sqrt{1-y^2}}$ $\therefore x=\dfrac{1}{\sqrt{1-y^2}}-2$ Volume of solid $=2\pi\int_0^{\frac{\sqrt{3}}{2}}\left(\dfrac{1}{\sqrt{1-y^2}}-2\right)^2\ \mathrm{d}y$ $=3.7213$ $=3.72$ (3sf)

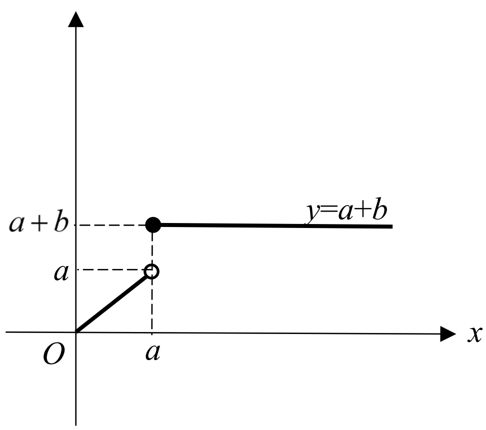
Qn	Solution
5 **(i)** **(ii)**	 Note: (i) $\alpha > 0, \ \ \beta < 0$ (ii) $z_3 = e^{ki} z_1 \Rightarrow \lvert z_3 \rvert = \lvert z_1 \rvert$ and $\arg(z_3) = k + \alpha$
(iii)	From the Argand diagram **Case 1:** z_3 on same side as z_2 (in 4$^{\text{th}}$ quadrant) $\arg(z_3) = k + \alpha$ ie, $k + \alpha = \beta \Rightarrow k = \beta - \alpha$ (note $k < 0$) **Case 2:** z_3 on opposite side as z_2 (in 2$^{\text{nd}}$ quadrant) $-\beta + \alpha + k = \pi$ $\therefore k = \pi + \beta - \alpha$ (Note $k > 0$)
(iv)	For $k = \frac{\pi}{2}$, $OZ_1 \perp OZ_3$, hence base = $\lvert z_3 \rvert + \lvert z_2 \rvert = \lvert z_1 \rvert + \lvert z_2 \rvert$ ht $= \lvert z_1 \rvert$ triangle area = $\frac{1}{2}\left(\lvert z_1 \rvert + \lvert z_2 \rvert \right) \lvert z_1 \rvert$

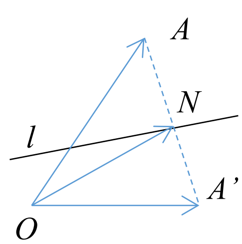
Qn	Solution
6(i)	$\mathrm{f}(x) = 2x + 1 - 2\ln(\sec x + \tan x)$ $\mathrm{f}'(x) = 2 - 2\dfrac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$ $\quad\quad = 2 - 2\sec x$
(ii)	For $0 < x < \frac{1}{2}\pi$, $0 < \cos x < 1$ $\sec x > 1$ $\therefore \mathrm{f}'(x) = 2 - 2\sec x < 0$ Together with the fact that $\mathrm{f}(0) = 20 + 1 - 2\ln(\sec 0 + \tan 0) = 1$ $\mathrm{f}(x) < 1$ for $0 < x < \frac{1}{2}\pi$
(iii)	Line passing through $(p,\ \mathrm{f}(p))$ and $(q,\ \mathrm{f}(q))$: $y - \mathrm{f}(p) = \dfrac{\mathrm{f}(q) - \mathrm{f}(p)}{q - p}(x - p)$ $\alpha \approx p - \mathrm{f}(p)\dfrac{q - p}{\mathrm{f}(q) - \mathrm{f}(p)}$ **Alternative:** $y - \mathrm{f}(q) = \dfrac{\mathrm{f}(q) - \mathrm{f}(p)}{q - p}(x - q)$ $\alpha \approx q - \mathrm{f}(q)\dfrac{q - p}{\mathrm{f}(q) - \mathrm{f}(p)}$
(iv)	$\mathrm{f}''(x) = -2\sec x \tan x < 0$ for $0 < x < \frac{1}{2}\pi$ I.e. curve is concave downwards  From curve, the approximation in part **(iii)** is an under-estimate.

Qn	Suggested Solution
7(i)	$u_3 + au_2 + u_1 = b \rightarrow 5 + a(0.5) + c = b \rightarrow 0.5a - b + c = -5$ $u_4 + au_3 + u_2 = b \rightarrow 14.5 + a(5) + 0.5 = b \rightarrow 5a - b = -15$ $u_5 + au_4 + u_3 = b \rightarrow 29 + a(14.5) + 5 = b \rightarrow 14.5a - b = -34$ Solving, $a = -2, b = 5, c = 1$
(ii)	$v_{n+1} - v_n$ $= (u_{n+2} - u_{n+1}) - (u_{n+1} - u_n)$ $= u_{n+2} - 2u_{n+1} + u_n$ From (i), $u_{n+2} + au_{n+1} + u_n = b$ where $a = -2,\ b = 5$ Hence, $u_{n+2} - 2u_{n+1} + u_n = 5$ Thus, $v_{n+1} - v_n = 5$ (constant) Therefore, $\{v_n\}$ is an arithmetic progression. **ALTERNATIVE** $v_n = u_{n+1} - u_n$ $= (5 + 2u_n - u_{n-1}) - u_n$ $= 5 + u_n - u_{n-1}$ $= 5 + v_{n-1}$ $v_n - v_{n-1} = 5$ Since the difference between consecutive terms is a constant, $\{v_n\}$ is an arithmetic progression with common difference 5.

(iii)	$\sum_{r=1}^{n-1} v_r = \sum_{r=1}^{n-1}\left(u_{r+1}-u_r\right)$ LHS$=\sum_{r=1}^{n-1} v_r$ $=\frac{n-1}{2}\left(2(-0.5)+(n-2)5\right)$ $=\frac{n-1}{2}(5n-11)$ **Alternative:** LHS$=\sum_{r=1}^{n-1} v_r$ $=\sum_{r=1}^{n-1}(5r-5.5)$ $=\frac{n-1}{2}(-0.5+5n-10.5)$ $=\frac{n-1}{2}(5n-11)$ RHS $=\sum_{r=1}^{n-1}\left(u_{r+1}-u_r\right)$ $=\left\{\begin{array}{l} \cancel{u_2}-u_1 \\ \cancel{u_3}-\cancel{u_2} \\ \cancel{u_4}-\cancel{u_3} \\ \quad\ldots \\ \cancel{u_{n-1}}-\cancel{u_{n-2}} \\ u_n-\cancel{u_{n-1}} \end{array}\right.$ $=u_n-u_1$ Thus, $u_n-u_1=\frac{n-1}{2}(5n-11)$ $u_n=\frac{n-1}{2}(5n-11)+1$ $=\frac{5n^2-16n+13}{2}$
(iv)	Possible responses: -Sequence **increases** and **diverges** for large values of *n*. -Sequence **increases to infinity** for large values of *n*.

Qn	Solution
8(i)	
(ii)	

(iii)	$y=\mathrm{g}(x)$ 
(iv)	Range of g $=[0,a)\cup\{a+b\}$ Since range of g is subset of domain of f, $\mathbb{R}\setminus\{a\}$, Thus fg exists. (explained)
(v)	To find range of fg: $\mathrm{D_{fg}}=\mathrm{D_g}\xrightarrow{\mathrm{g}}\mathrm{R_g}=[0,a)\cup\{a+b\}\xrightarrow{\mathrm{f}}(-\infty,-a^2]\cup\{\mathrm{a}^2\}=\mathrm{R_{fg}}$

Qn	Suggested Solutions
9(i)	$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}-2\\-q\\-1\end{pmatrix}$ $l: r=\begin{pmatrix}\frac{15}{4}\\0\\\frac{5}{2}\end{pmatrix}+\lambda\begin{pmatrix}-\frac{1}{2}\\1\\0\end{pmatrix}, \lambda\in\mathbb{R}$ $\frac{\lvert\overrightarrow{AB}\bullet\mathbf{d}_l\rvert}{\lvert\mathbf{d}_l\rvert}=\frac{8}{\sqrt{5}}$ $\frac{\left\lvert\begin{pmatrix}-2\\-q\\-1\end{pmatrix}\bullet\begin{pmatrix}-\frac{1}{2}\\1\\0\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}-\frac{1}{2}\\1\\0\end{pmatrix}\right\rvert}=\frac{8}{\sqrt{5}}$ $\lvert 1-q\rvert=\frac{8}{\sqrt{5}}\left(\frac{\sqrt{5}}{2}\right)$ $1-q=4$ or $1-q=-4$ $q=-3$ (reject $\because q>0$) or $q=5$
(ii)	 $\overrightarrow{ON}=\begin{pmatrix}\frac{15}{4}-\frac{1}{2}\lambda\\\lambda\\\frac{5}{2}\end{pmatrix}$, for some $\lambda\in\mathbb{R}$

	$\overrightarrow{AN}=\overrightarrow{ON}-\overrightarrow{OA}=\begin{pmatrix}\frac{15}{4}-\frac{1}{2}\lambda\\ \lambda\\ \frac{5}{2}\end{pmatrix}-\begin{pmatrix}1\\5\\2\end{pmatrix}=\begin{pmatrix}\frac{11}{4}-\frac{1}{2}\lambda\\ \lambda-5\\ \frac{1}{2}\end{pmatrix}$ $\overrightarrow{AN}\cdot\mathbf{d}_l=0$ $\begin{pmatrix}\frac{11}{4}-\frac{1}{2}\lambda\\ \lambda-5\\ \frac{1}{2}\end{pmatrix}\cdot\begin{pmatrix}-\frac{1}{2}\\1\\0\end{pmatrix}=0$ $\lambda=\frac{51}{10}$ $\therefore\overrightarrow{ON}=\begin{pmatrix}\frac{15}{4}-\frac{1}{2}\left(\frac{51}{10}\right)\\ \frac{51}{10}\\ \frac{5}{2}\end{pmatrix}=\begin{pmatrix}\frac{12}{10}\\ \frac{51}{10}\\ \frac{5}{2}\end{pmatrix}=\frac{1}{10}\begin{pmatrix}12\\51\\25\end{pmatrix}$ Using ratio theorem, $\overrightarrow{ON}=\frac{\overrightarrow{OA}+\overrightarrow{OA'}}{2}$ $\overrightarrow{OA'}=2\overrightarrow{ON}-\overrightarrow{OA}=\frac{2}{10}\begin{pmatrix}12\\51\\25\end{pmatrix}-\begin{pmatrix}1\\5\\2\end{pmatrix}=\frac{1}{10}\begin{pmatrix}14\\52\\30\end{pmatrix}=\frac{1}{5}\begin{pmatrix}7\\26\\15\end{pmatrix}$ Coordinates of A' $=\left(\frac{7}{5},\frac{26}{5},3\right)$
(iii)	Since the point $\left(\frac{15}{4},0,\frac{5}{2}\right)$ lies on the p_1, $2\left(\frac{15}{4}\right)+\frac{5}{2}k=1$ $k=-\frac{13}{5}$ Normal of $p_2=\begin{pmatrix}1\\s\\1\end{pmatrix}\times\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}s+1\\1\\-1-2s\end{pmatrix}$

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Since line l lies on p_2,

$\mathbf{d}_l \bullet \mathbf{n}_{p_2} = 0$

$$\begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} s+1 \\ 1 \\ -1-2s \end{pmatrix} = 0$$

$$-\frac{1}{2}(s+1)+1=0$$

$s=1$

ALT 1:

Alternatively, solve SLE

$$\begin{pmatrix} \frac{15}{4} \\ 0 \\ \frac{5}{2} \end{pmatrix} = \mu \begin{pmatrix} 1 \\ s \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

to obtain $\mu = \gamma = \mu s = \frac{5}{4}$

so $s=1$

ALT 2:

Can also use $\begin{pmatrix} \frac{15}{4} \\ 1 \\ \frac{5}{2} \end{pmatrix} \bullet \begin{pmatrix} s+1 \\ 1 \\ -1-2s \end{pmatrix} = 0$

since origin must lie on p2.
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<tr><td>(iv)</td><td>
For no points of intersection between the three planes, p_3 is parallel to line l, and line l does not lie on p_3.

$\mathbf{d}_l \bullet \mathbf{n}_{p_3} = 0$

$$\begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ t \\ 1 \end{pmatrix} = 0$$

$-1+t=0$

$t=1$

and

$$\begin{pmatrix} \frac{15}{4} \\ 0 \\ \frac{5}{2} \end{pmatrix} \bullet \mathbf{n}_{p_3} \neq u$$

$$u \neq \begin{pmatrix} \frac{15}{4} \\ 0 \\ \frac{5}{2} \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \frac{15}{2} + \frac{5}{2} = 10$$

$u \neq 10$
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Qn	Solution
10(i)	Loan amount = $\$M$ Loan amount after 6 yrs = $M(1+r)^6$ For amount of compounded interest to not exceed 20% of the initial loan, $M(1+r)^6 - M \le 0.2M$ $(1+r)^6 \le 1.2$ $r \le 1.2^{\frac{1}{6}} - 1$ $r \le 0.030853$ $\therefore r \le 0.0308$
(ii)	Scheme A amount $S_A = 10 + (10+0.3) + \ldots$ (for n terms) $= \frac{n}{2}\left(2(10) + (n-1)(0.3)\right)$ $= 9.85n + 0.15n^2$ Scheme B amount $S_B = 10 + 10(1.02) + 10(1.02)^2 + \ldots + 10(1.02)^{n-1}$ $= \frac{10(1.02^n - 1)}{1.02 - 1}$ $= 500(1.02^n - 1)$ Total amt = $9.85n + 0.15n^2 + 500(1.02^n - 1)$
(iii)	$9.85n + 0.15n^2 < 500(1.02^n - 1)$ From GC, $n \ge 58$ $\therefore \{n \in \mathbb{Z} : 58 \le n \le 72\}$
(iv)	At end of 6 years, total payout $= 9.85(72) + 0.15(72)^2 + 500(1.02^{72} - 1)$ $= 3067.3701$ $= 3067.37$ bank interest $= 20000(1.02^6) - 20000 = 2523.25 <$ total payout Thus it is worthwhile for Mr Safe to take up the bank loan for the investment.

	Suggested Solution
11(i) (a)	$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2}+\beta\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2=0$ Let $v=\frac{\mathrm{d}x}{\mathrm{d}t}$, $\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$, we have $\frac{\mathrm{d}v}{\mathrm{d}t}+\beta v^2=0$ $\frac{\mathrm{d}v}{\mathrm{d}t}=-\beta v^2$
(i)(b)	$v^{-2}\frac{\mathrm{d}v}{\mathrm{d}t}=-\beta$ $\int v^{-2}\mathrm{d}v=-\beta\int 1\,\mathrm{d}t$ $-\frac{1}{v}=-\beta t+C$ When $t=0$, $v=100$, $C=-\frac{1}{100}$. $-\frac{1}{v}=-\beta t-\frac{1}{100}$ $\frac{1}{v}=\beta t+\frac{1}{100}$ $v=\frac{1}{\beta t+0.01}$ --- (1) $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{\beta t+0.01}$ $x=\int\frac{1}{\beta t+0.01}\,\mathrm{d}t$ $x=\frac{1}{\beta}\ln\left(\beta t+0.01\right)+D$ (since $\beta>0$) When $t=0$, $x=0$, $0=\frac{1}{\beta}\ln\left(0.01\right)+D$ $D=-\frac{1}{\beta}\ln\left(0.01\right)$

	$x=\frac{1}{\beta}\ln\left(\beta t+0.01\right)-\frac{1}{\beta}\ln\left(0.01\right)$ $x=\frac{1}{\beta}\left[\ln\left(\frac{\beta t+0.01}{0.01}\right)\right]$ $x=\frac{1}{\beta}\ln\left(100\beta t+1\right)$ --- (2)
(i)(c)	Let $t=T$ when the bullet leaves plate of material X. When $t=T$, $v=80$, from (1), $80=\frac{1}{\beta T+0.01}$ $\beta T+0.01=\frac{1}{80}$ $\beta T=\frac{1}{400}=0.0025$ $\beta=\frac{1}{400T}$ When $t=T$, $x=0.1$ and $\beta=\frac{1}{400T}$, from (2), $0.1=\frac{1}{\left(\frac{1}{400T}\right)}\ln\left(100\left(\frac{1}{400T}\right)T+1\right)$ $0.1=400T\ln\left(\frac{5}{4}\right)$ $T=\frac{0.1}{400\ln\left(1.25\right)}$ $=0.00112035502$ $=0.00112$ seconds
(ii)	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=\frac{k}{\mathrm{e}^{10000t}}=k\mathrm{e}^{-10^4t}$ When $t=0$, $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-10^6$, $-10^6=k\mathrm{e}^{-10^4(0)}$ $k=-10^6$ Hence, $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-10^6\mathrm{e}^{-10^4t}$

	$\frac{dx}{dt}=\frac{-10^6}{-10^3}e^{-10^4 t}+E$ $=100e^{-10^4 t}+E$ When $t = 0$, $\frac{dx}{dt} = 100$, $100=100e^{-10^4(0)}+E$ $E=0$ $\frac{dx}{dt}=100e^{-10^4 t}$ --- (3) $x=-\frac{100}{10^4}e^{-10^4 t}+F$ $x=-0.01e^{-10^4 t}+F$ When $t = 0, x = 0$, $0=-0.01e^{-10^4(0)}+F$ $F=0.01$ $x=0.01-0.01e^{-10^4 t}$ $x=0.01\left(1-e^{-10^4 t}\right)$ --- (4)
(iii)	**Method 1** When $t = 0.0011204$, from (4), $x=0.01\left(1-e^{-10^4(0.0011204)}\right)$ $\approx 0.001<0.1$ In the same amount of time it takes for the bullet to penetrate the entire plate of material P of 0.1m, it only travels 0.01m through plate of material Q. Hence, the company's claim is valid. **Method 2** From (4), as $t\to\infty$, $x\to 0.01$. This means the bullet will not be able to penetrate the plate of material Q which is 0.1m thick, unlike what it has done to material P. Hence, the company's claim is valid. **Method 3** When $t = 0.0011204$, from (3), $\frac{dx}{dt}=100e^{-10^4(0.0011204)}=0.00136\approx 0$.

	In the same amount of time it takes for the bullet to penetrate the entire plate of material *P* to exit with a velocity of 80 ms^{-1}, the bullet that travels through the plate of material *Q* has its velocity reduced to almost 0 ms^{-1}. Hence, the company's claim is valid.