Y6 Prelim Paper 2 Mark Scheme

Qn	Suggested Solutions
1(i)	Given : $w=3-4\mathrm{i}$ $z_1=\mathrm{Im}\left(\frac{w}{w^*}\right)=\mathrm{Im}\left(\frac{w^2}{\lvert w\rvert^2}\right)$ $=\left(\frac{1}{\lvert w\rvert^2}\right)\mathrm{Im}\left(-7-24\mathrm{i}\right)$ $=-\frac{24}{25}$ Or $\frac{w}{w^*}=\frac{3-4\mathrm{i}}{3+4\mathrm{i}}.\frac{3-4\mathrm{i}}{3-4\mathrm{i}}$ $=\frac{9-24\mathrm{i}-16}{9+16}$ $=\frac{-7-24\mathrm{i}}{25}$ $z_1=\mathrm{Im}\left(\frac{w}{w^*}\right)=\mathrm{Im}\left(\frac{-7-24\mathrm{i}}{25}\right)=-\frac{24}{25}$ $z_2=w-w^*=-8\mathrm{i}$
(ii)	Since coefficients of polynomial eqn are all real, by complex conjugate root theorem, $z_3={z_2}^*=8\mathrm{i}$ Hence $\left[z-8\mathrm{i}\right]\left[z+8\mathrm{i}\right]\left[z-\left(-\frac{24}{25}\right)\right]=0$ $\left[z^2+64\right]\left[z+\frac{24}{25}\right]=0$ Simplifying: $25z^3+24z^2+1600z+1536=0$ $\therefore \quad b=25,\ c=24,\ d=1600$

	Suggested Solution
2 **(i)**	$\frac{1}{2^{k+1}-3}-\frac{1}{2^k-3}=\frac{\left(2^k-3\right)-\left(2^{k+1}-3\right)}{\left(2^{k+1}-3\right)\left(2^k-3\right)}$ $=\frac{2^k-2^{k+1}}{\left(2^{k+1}-3\right)\left(2^k-3\right)}$ $=\frac{2^k-2^k\left(2\right)}{\left(2^{k+1}-3\right)\left(2^k-3\right)}$ $=\frac{2^k\left(1-2\right)}{\left(2^{k+1}-3\right)\left(2^k-3\right)}$ $=\frac{-2^k}{\left(2^{k+1}-3\right)\left(2^k-3\right)}$ $\sum_{k=1}^{N}\frac{2^{k+1}}{\left(2^{k+1}-3\right)\left(2^k-3\right)}=-2\sum_{k=1}^{N}\frac{-2^k}{\left(2^{k+1}-3\right)\left(2^k-3\right)}$ $=-2\sum_{k=1}^{N}\left(\frac{1}{2^{k+1}-3}-\frac{1}{2^k-3}\right)$ $=-2\left[\cancel{\frac{1}{2^2-3}}-\frac{1}{2^1-3}\right.$ $+\cancel{\frac{1}{2^3-3}}-\cancel{\frac{1}{2^2-3}}$ $+\cancel{\frac{1}{2^4-3}}-\cancel{\frac{1}{2^3-3}}$ $+\ \vdots$ $+\cancel{\frac{1}{2^{N-1}-3}}-\cancel{\frac{1}{2^{N-2}-3}}$ $+\cancel{\frac{1}{2^N-3}}-\cancel{\frac{1}{2^{N-1}-3}}$ $\left.+\frac{1}{2^{N+1}-3}-\cancel{\frac{1}{2^N-3}}\right]$ $=-2\left[\frac{1}{2^{N+1}-3}-\frac{1}{2^1-3}\right]$ $=-2\left(\frac{1}{2^{N+1}-3}+1\right)$

(ii)	$\frac{16}{(61)(29)}+\frac{32}{(125)(61)}+\frac{64}{(253)(125)}+\ldots+\frac{2^{k-1}}{(2^{k+1}-3)(2^k-3)}+\ldots$ $=\frac{1}{4}\sum_{k=5}^{\infty}\frac{2^{k+1}}{(2^{k+1}-3)(2^k-3)}$ $=\frac{1}{4}\left[-2(0+1)-\sum_{k=1}^{4}\frac{2^{k+1}}{(2^{k+1}-3)(2^k-3)}\right]$ $=\frac{1}{4}\left[-2(0+1)-\left(-\frac{60}{29}\right)\right]$ $=\frac{1}{58}$

Qn	**Solution**
3(i)	Surface area $=2\pi r^2+\pi r^2+2\pi rh$ $=3\pi r^2+2\pi rh$ Since $3\pi r^2+2\pi rh=k$, $h=\frac{k-3\pi r^2}{2\pi r}$ Let the volume be V. $V=\frac{2}{3}\pi r^3+\pi r^2 h$ $=\frac{2}{3}\pi r^3+\pi r^2\left(\frac{k-3\pi r^2}{2\pi r}\right)$ $=\frac{2}{3}\pi r^3+\frac{k}{2}r-\frac{3}{2}\pi r^3$ $=-\frac{5}{6}\pi r^3+\frac{k}{2}r$ $\frac{\mathrm{d}V}{\mathrm{d}r}=\frac{\mathrm{d}}{\mathrm{d}r}\left(-\frac{5}{6}\pi r^3+\frac{k}{2}r\right)$ $=-\frac{15}{6}\pi r^2+\frac{k}{2}$

When V max, $\dfrac{\mathrm{d}V}{\mathrm{d}r}=0$:

$$-\frac{15}{6}\pi r^2+\frac{k}{2}=0$$

$$r^2=\frac{\frac{k}{2}}{\frac{15}{6}\pi}=\frac{6k}{30\pi}=\frac{k}{5\pi} \quad \text{---------------(*)}$$

$$r=\sqrt{\frac{k}{5\pi}}$$

$$\frac{\mathrm{d}V}{\mathrm{d}r}=-\frac{15}{6}\pi r^2+\frac{k}{2}\Rightarrow\frac{\mathrm{d}^2V}{\mathrm{d}r^2}=-5\pi r<0\ \left(\because r>0\right)$$

Hence, V max when $r=\sqrt{\dfrac{k}{5\pi}}$

Alternative

$$\frac{\mathrm{d}V}{\mathrm{d}r}=\frac{5\pi}{2}\left(\frac{k}{5\pi}-r^2\right)=\frac{5\pi}{2}\left(\sqrt{\frac{k}{5\pi}}+r\right)\left(\sqrt{\frac{k}{5\pi}}-r\right)$$

r	$r<\sqrt{\dfrac{k}{5\pi}}$	$r=\sqrt{\dfrac{k}{5\pi}}$	$r<\sqrt{\dfrac{k}{5\pi}}$
$\dfrac{\mathrm{d}V}{\mathrm{d}r}$	$(+)(+)=+$	0	$(+)(-)=-$

(ii)	$V=-\frac{5}{6}\pi r^3+\frac{k}{2}r$ $=-\frac{5}{6}\pi\left(\frac{k}{\sqrt{5\pi}}\right)^3+\frac{k}{2}\left(\sqrt{\frac{k}{5\pi}}\right)$ $=-\frac{k^{\frac{3}{2}}}{6\sqrt{5\pi}}+\frac{k^{\frac{3}{2}}}{2\sqrt{5\pi}}$ $=\frac{k^{\frac{3}{2}}}{3\sqrt{5\pi}}$ From (*), we have $k=5\pi r^2$ Hence, $h=\frac{k-3\pi r^2}{2\pi r}=\frac{5\pi r^2-3\pi r^2}{2\pi r}=\frac{2\pi r^2}{2\pi r}=r$ Ratio of r : h is 1:1.

Qn	Solution
4(a)	$\int \sin^{-1}4x\ \mathrm{d}x = x\sin^{-1}4x-\int\frac{4x}{\sqrt{1-16x^2}}\ \mathrm{d}x$ $=x\sin^{-1}4x+\frac{1}{8}\int -32x(1-16x^2)^{-\frac{1}{2}}\ \mathrm{d}x$ $=x\sin^{-1}4x+\frac{1}{8}\frac{\sqrt{1-16x^2}}{0.5}+C$ $=x\sin^{-1}4x+\frac{1}{4}\sqrt{1-16x^2}+C$
(b) (i)	$\frac{\mathrm{d}x}{\mathrm{d}t}=a(1-t^2),\ \frac{\mathrm{d}y}{\mathrm{d}t}=2at$ Tangent at P: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2at}{a(1-t^2)}=\frac{2t}{1-t^2}$ $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{t=p}=\frac{2p}{1-p^2}$ $y-ap^2=\frac{2p}{1-p^2}\left(x-a\left(p-\frac{p^3}{3}\right)\right)$

	Alternative $y = mx + c$ $ap^2 = \dfrac{2pa}{1-p^2}\left(p - \dfrac{p^3}{3}\right) + c$ $c = ap^2 - \dfrac{2pa}{1-p^2}\left(p - \dfrac{p^3}{3}\right)$ $\therefore y = \dfrac{2p}{1-p^2}x + ap^2 - \dfrac{2pa}{1-p^2}\left(p - \dfrac{p^3}{3}\right)$ $y = \dfrac{2p}{1-p^2}x + ap^2 - \dfrac{2p^2a}{1-p^2} + \dfrac{2p^4a}{3(1-p^2)}$
(ii)	At Q, $y = 0$: $-ap^2 = \dfrac{2p}{1-p^2}\left(x - a\left(p - \dfrac{p^3}{3}\right)\right)$ $\dfrac{-ap(1-p^2)}{2} = x - ap + \dfrac{ap^3}{3}$ $x = ap - \dfrac{ap^3}{3} - \dfrac{ap}{2} + \dfrac{ap^3}{2} = \dfrac{ap}{2} + \dfrac{ap^3}{6}$ $\therefore OQ = \dfrac{ap}{2} + \dfrac{ap^3}{6}$ Arc length OP $= \int_0^p \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\ dt$ $= \int_0^p \sqrt{a^2\left(1-t^2\right)^2 + \left(2at\right)^2}\ dt$ $= a\int_0^p \sqrt{1-2t^2+t^4+4t^2}\ dt$ $= a\int_0^p \sqrt{(t^2+1)^2}\ dt$ $= a\int_0^p (t^2+1)\ dt$ $=a\left[\dfrac{t^3}{3} + t\right]_0^p$ $= a\left(\dfrac{p^3}{3} + p\right)$ $OP = a\left(\dfrac{p^3}{3} + p\right) = 2\left(\dfrac{ap}{2} + \dfrac{ap^3}{6}\right) = 2OQ$

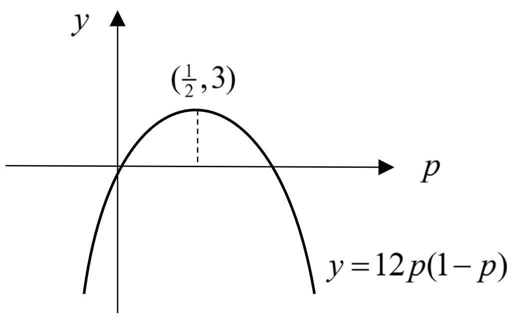
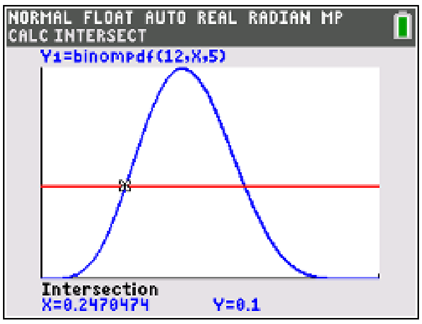
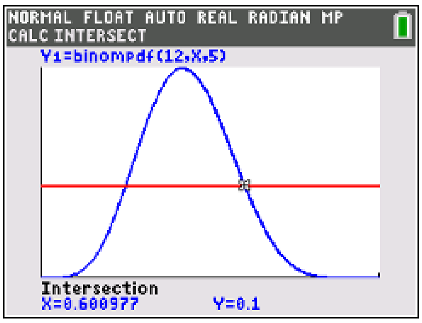
Qn	Suggested Solutions
5(a)	$\overrightarrow{OD}=-\overrightarrow{OC}$ $\mathbf{d}=-\left(\frac{2}{7}\mathbf{a}-\frac{3}{7}\mathbf{b}\right)$ $=\frac{1}{7}(3\mathbf{b}-2\mathbf{a})$ By ratio theorem, $\overrightarrow{OD}=\frac{4\overrightarrow{OR}+3\overrightarrow{OB}}{7}$ $4\overrightarrow{OR}=7\overrightarrow{OD}-3\overrightarrow{OB}$ $\overrightarrow{OR}=\frac{1}{4}\left(7\left(\frac{3}{7}\mathbf{b}-\frac{2}{7}\mathbf{a}\right)-3\mathbf{b}\right)$ $=-\frac{1}{2}\mathbf{a}=-\frac{1}{2}\overrightarrow{OA}$ Since $\overrightarrow{OR}$ can be expressed as a scalar multiple of $\overrightarrow{OA}$ and O is a common point, the points A, O and R are collinear. The ratio of AO to OR is $1:\frac{1}{2}=2:1$
(b)(i)	Area of triangle AOB $=\frac{1}{2}\left\|\overrightarrow{OA}\times\overrightarrow{OB}\right\|$ $=\frac{1}{2}\left\|\begin{pmatrix}1\\3\\2\end{pmatrix}\times\begin{pmatrix}-1\\5\\3\end{pmatrix}\right\|$ $=\frac{1}{2}\left\|\begin{pmatrix}-1\\-5\\8\end{pmatrix}\right\|$ $=\frac{1}{2}\sqrt{90}$ $=\frac{3}{2}\sqrt{10}$
(ii)	$\mathbf{r}\cdot(\mathbf{a}\times\mathbf{b})=0$ refers to the collection of all points on the plane that is perpendicular to $\mathbf{a}\times\mathbf{b}$ and containing the origin.
(iii)	$\mathbf{r}\cdot\begin{pmatrix}-1\\-5\\8\end{pmatrix}=0$ Distance required $=\frac{\left\|0-\begin{pmatrix}-8\\-2\\9\end{pmatrix}\cdot\begin{pmatrix}-1\\5\\8\end{pmatrix}\right\|}{\left\|\begin{pmatrix}-1\\5\\8\end{pmatrix}\right\|}=\frac{90}{\sqrt{90}}=\sqrt{90}$

	Suggested Solution
6(a)(i)	# of selections $= {}^{2}C_{1} \cdot {}^{3}C_{1} = 6$. **Alt:** 3C1 +3C1 = 6
(ii)	Case 1: 2 of the same colour $$\underbrace{\left({}^{3}C_{1} \cdot {}^{3}C_{2}\right)}_{\text{Exactly 1 pair same colour}} + \underbrace{{}^{3}C_{2}}_{\text{2 pairs same colour}}$$ Case 2: 3 of the same colour (answer from (a)) $${}^{2}C_{1} \cdot {}^{3}C_{1}$$ Case 3: 4 (all) the same colour $${}^{1}C_{1}$$ $$\begin{aligned}\text{Total \# of selections} &= \left[\left({}^{3}C_{1} \cdot {}^{3}C_{2}\right) + {}^{3}C_{2}\right] + \left({}^{2}C_{1} \cdot {}^{3}C_{1}\right) + {}^{1}C_{1} \\ &= (9+3)+6+1 \\ &= 19\end{aligned}$$
(iii)	$$\text{Total \# of selections w/o restrictions} = \underbrace{19}_{\text{Ans from (b)}} + \underbrace{1}_{\text{All different}}$$ $$= 20$$ $$\text{\# of selections w/o green and yellow} = \underbrace{4}_{\substack{\text{2 blue, 2 black} \\ \text{3 bllue, 1 black} \\ \text{3 black, 1 blue} \\ \text{4 black}}}$$ # of selections required $= 20 - 4 = 16$
(b)	Total # of arrangements $= \dfrac{7!}{4!2!}\left({}^{8}C_{3}\right) = 5880$

	Solution
7i)	$\sum \mathrm{P}(X=x)=1$ $k+2k+3k+\dfrac{3k}{3}+\dfrac{4k}{3}+\dfrac{5k}{3}=1$ $k=\dfrac{1}{10}$ (shown)
ii)	$\mathrm{E}(X)$ $=\dfrac{1}{10}+2\left(\dfrac{2}{10}\right)+3\left(\dfrac{3}{10}\right)+4\left(\dfrac{1}{10}\right)+5\left(\dfrac{2}{15}\right)+6\left(\dfrac{1}{6}\right)$ $=\dfrac{52}{15}\approx 3.46666.....$ Expected net gain = E(X – 3) = E(X) – 3 $=\dfrac{52}{15}-3=0.46666...\approx 0.47$ (2dp) or 0.467 (3sf) Variance of net gain = Var(X – 3) = Var(X) $= 1.5860^2 = 2.5153 \approx 2.52$ (3sf) (or $\frac{566}{225}$)
iii)	Let $Y = X - 3$ which denotes the net gain of one game. Since n = 30 is large, by Central Limit Theorem, $Y_1+Y_2+\cdots+Y_{30} \sim \mathrm{N}\big(30(0.46666), 30(2.5153)\big)$ approximately $\mathrm{P}\left(Y_1+Y_2+\cdots+Y_{30}>20\right)=0.24486 = 0.245$ (3sf)

Qn	Solution
8	Let R and L be the number of right and left steps taken respectively.
(i)	Note that since counter starts at 0, $R-L$ gives the number of the ending position. For the counter to end at $k=7$, $R-L=7 \Rightarrow R=L+7$ Also, since game is played for 10 stages, $R+L=10$. Hence, $(L+7)+L=10 \Rightarrow L=\frac{3}{2}$ (Or show that $R=\frac{17}{2}$) But L must be integer, hence it's not possible for counter to end at $k=7$. **Alternative 1** Case 1: R is odd, L is odd (since must add up 10) $\Rightarrow$ $R-L$ is even. Case 2: R is even, L is even $\Rightarrow$ $R-L$ is even Hence, one can never end at an odd numbered position with 10 steps starting at 0. **Alternative 2** Any combination of 9 right steps and 1 left step leads to $k=8$. Any combination of 8 right steps and 2 left steps leads to $k=6$. Hence $k \neq 7$. **Alternative 3** Any combination of 9 right steps and 2 left step with a total of 11 steps leads to $k=7$. Any combination of 8 right steps and 1 left step with a total of 9 steps leads to $k=7$. Hence it is not possible to get to $k=7$ in exactly 10 steps.
(ii)	For the counter to end at $k=6$, $R-L=6$ and $R+L=10$ Hence $R=8$ and $L=2$. Any combination of 8 right steps and 2 left steps occurs with probability p^8q^2. Number of such combinations is $\binom{10}{8}=45$. Hence the probability that the counter ends at $k=6$ is $45p^8q^2$.
(iii)	$R \sim \mathrm{B}(10, p)$ For the most probable end-point to be $k=6$, the mode of R is 8. As R is binomial, it suffices to ensure the two inequalities below are satisfied: $\binom{10}{7}p^7q^3 < \binom{10}{8}p^8q^2 \quad \ldots(1)$ $\binom{10}{9}p^9q^1 < \binom{10}{8}p^8q^2 \quad \ldots(2)$ From (1), $8(1-p)<3p \Rightarrow p>\frac{8}{11}$ From (2) $2p<9(1-p) \Rightarrow p<\frac{9}{11}$ Hence we have $\frac{8}{11}<p<\frac{9}{11}$ i.e. $p_1=\frac{8}{11}$, $p_2=\frac{9}{11}$.

(iv)	Note that when $p = p_1$, the modes of R are 7 and 8 i.e. most probable end-points for the counter are $k = 4$ and $k = 6$. When $p = 1 - p_1$ (complement probability), most probable end-points for the counter will be $k = -4$ and $k = -6$. (by symmetry since interchange right with left) **Alternative** $p = 1 - \frac{8}{11} = \frac{3}{11}$ $R \sim \mathrm{B}\left(10, \frac{3}{11}\right)$ Modes of $R = 2$ and 3 The two most probable end-points for the counter will be $k = -4$ and $k = -6$.

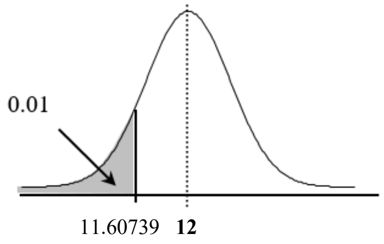
Qn	Suggested Solutions
9(i)	1. The probability of a randomly chosen sweet being mint is a constant p for each sweet in a bag. 2. The event that a sweet in the bag is mint is independent of another sweet being a mint sweet.
(ii)	Let X = number of mint sweets in a bag of 12 $X \sim \mathrm{B}(12, p)$ $\mathrm{Var}(X) = 12p(1-p)$  From the graph, we can see that maximum $\mathrm{Var}(X)$ is 3 (when $p = \frac{1}{2}$) Hence, variance of X does not exceed 3. **Alternative** $\mathrm{Var}(X) = 12p(1-p) = -12(p - \frac{1}{2})^2 + 3$ Since $(p - \frac{1}{2})^2 \geq 0$, $-12(p - \frac{1}{2})^2 \leq 0$ $-12(p - \frac{1}{2})^2 + 3 \leq 3$ $\therefore \mathrm{Var}(X) \leq 3$
(iii)	$\mathrm{P}(X = 5) < 0.1$ 

	From GC, $p < 0.24704$ or $p > 0.60097$ $\therefore p < 0.24$ or $p > 0.60$ (2 d.p.)
(iv)	$X \sim B(12, 0.65)$ Required probability = $P(X \geq 6) = 1 - P(X \leq 5)$ $= 0.91536$ $= 0.915$ (3sf)
(v)	Required probability = $(0.91536)^5 = 0.643$ (3sf)
(vi)	Let Y be the number of mint sweets in a box. $Y \sim B(60, 0.65)$ Required probability = $P(Y \geq 30) = 1 - P(Y \leq 29)$ $= 0.994$ (3sf)
(vii)	The event in part **(v)** a subset of the event in part **(vi)**.

Qn	**Suggested Solutions**
10 (i)	Let X denote queue time. If $X \sim \text{N}(15, 10^2)$, then $\text{P}(X < 0) = 0.0668$, not close to zero. Since queue time cannot be negative, a normal model would not be suitable.
(ii)	Let Q denote haircut duration of a randomly chosen customer at Qcut (in min). $Q \sim N(9.2, \sigma^2)$ $\text{P}(Q > 10) \leq 0.35$ $\text{P}\left(Z > \frac{10 - 9.2}{\sigma}\right) \leq 0.35$ $\frac{0.8}{\sigma} \geq 0.38532$ $\sigma \leq 2.0761$ $\sigma \leq 2.08$ (3sf)
(iii)	Let Q denote haircut duration of a randomly chosen customer at Qcut (in min). $Q \sim N(9.2, 1.5^2)$ Given $\text{P}(\lvert Q - 9.2 \rvert > k) = 0.95$, $9.2 + k = 9.29406$ $k = 0.0941$

	Alternative By complement, $\mathrm{P}\left(\lvert Q-9.2\rvert < k\right)=0.05$ $\mathrm{P}\left(\lvert Z\rvert < \frac{k}{1.5}\right)=0.05$ $\frac{k}{1.5}=0.062707$ $k=0.0941$
(iv)	Let S denote haircut duration of a randomly chosen customer at SPcut (in min). $S \sim \mathrm{N}\left(20.7, 3.1^2\right)$ Let $\mathrm{M}=\frac{Q_1+Q_2+S_1+S_2+S_3}{5}$ $\sim \mathrm{N}\left(\frac{2\times 9.2+3\times 20.7}{5}, \frac{2\times 1.5^2+3\times 3.1^2}{25}\right)$ i.e. $M \sim \mathrm{N}\left(16.1, 1.3332\right)$ $\mathrm{P}\left(M<15\right)=0.17037=0.170\ \left(3 \text{ s.f.}\right)$
(v)	The haircut duration of each customer must be independent of each other / All the haircut durations are independent.
(vi)	$10+2S \geq 6(3+0.5Q)$ $2S-3Q \geq 8$ $2S-3Q \sim \mathrm{N}(13.8, 58.69)$ $P(2S-3Q \geq 8)=0.77550=0.776\ (3 \text{ sf})$

Qn	Suggested Solution
11(i)	Billy is interested in the stress levels of employees in the research department, and surveyed **all 300 employees** in the research department, thus the 300 employees constitute a population.
(ii)	Probability of selecting an employee from research department $=\frac{100}{300}=\frac{1}{3}$ Probability of selecting an employee from manufacturing department $=\frac{100}{330}=\frac{10}{33}$ Since not every employee has the same chance of being selected, the method does not give us a random sample.

(iii)	$\overline{x} = \dfrac{\sum(x-12)}{80} + 12 = \dfrac{-12.8}{80} + 12 = 11.84$ $s^2 = \dfrac{1}{80-1}\left[\sum(x-12)^2 - \dfrac{\left(\sum(x-12)\right)^2}{80}\right]$ $= \dfrac{1}{79}\left[16.19 - \dfrac{(-12.8)^2}{80}\right]$ $= 0.17901 = 0.179$ (3 s.f.)
(iv)	$H_0 : \mu = 12$ $H_1 : \mu < 12$ where μ is the population mean time taken Under H_0, $\overline{X} \sim N\left(12, \dfrac{0.17901}{80}\right)$ approximately by Central Limit Theorem since sample size of 80 is large. From GC, p-value = 0.000359 (3 s.f.) Since the ***p*-value is very small,** there is very strong evidence to reject H_0 and conclude that the **manager's claim is valid.**
(v)	Given, $\overline{x} = k,\ n = 80$ $s^2 = \dfrac{n}{n-1}(\text{sample variance}) = \dfrac{80}{79}\left(1.5^2\right)$ Under H_0, $\overline{X} \sim N\left(12, \dfrac{1.5^2}{79}\right)$ approximately by Central Limit Theorem since $n = 80$ is large. Manager's claim not supported $\Rightarrow$ we do not reject H_0 **<u>Method 1 (critical value)</u>**  Critical value is 11.60739 at 1% significance level. $k > 11.608$ (3 d.p) or $k \geq 11.608$ (3 d.p) **<u>Method 2 (p-value)</u>** p-value > 0.01 $P(\overline{X} \leq k) > 0.01$ $k > 11.608$ (3 d.p) or $k \geq 11.608$ (3 d.p)