

Name:	Class:	SOLUTIONS
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中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

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Parent's Signature

**PRELIMINARY EXAMINATION 2021
SECONDARY 4**

MATHEMATICS

4048/01

Paper 1

Wednesday 15 September 2021

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use	
Total	/ 80

This document consists of **19** printed pages and **1** blank page.

[Turn over

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} a b \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum f x}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f} \right)^2}$$

Answer **all** the questions.

1 State which of the following number(s) is/are irrational.

$$0.\dot{5}, \frac{\pi}{2}, 2\sqrt{2}, 3\sqrt{3} \times \sqrt{3}$$

$$\frac{\pi}{2}, 2\sqrt{2}$$

$0.\dot{5}$ can be expressed as $\frac{5}{9}$, so $0.\dot{5}$ is a rational number

Answer [1]

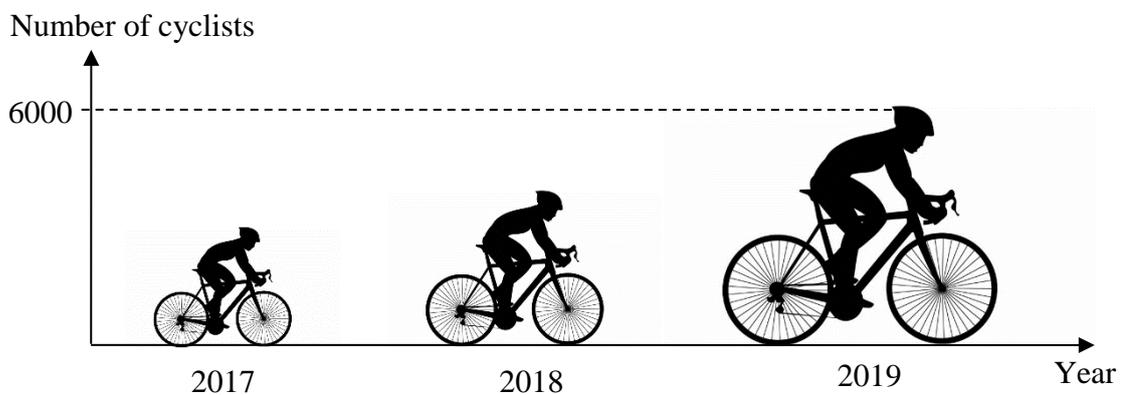
2 x is an integer greater than 1.
Write the following in order of size, starting with the largest.

$$x^0, x^{-\frac{3}{2}}, x^{\frac{3}{4}}, x^{\frac{3}{2}}$$

$$x^{\frac{3}{2}}, x^{\frac{3}{4}}, x^0, x^{-\frac{3}{2}}$$

Answer [1]

3 The graph shows the number of cyclists in an annual cycling marathon for the years 2017 to 2019.



(a) State one aspect of the graph that may be misleading.

The vertical axis is not labelled at the start / No "0" on the y-axis. **OR**
The size of the picture not only increases with height, but also in the width.

(b) Explain how this may lead to a misinterpretation of the graph.

It is difficult to estimate the number of cyclists for 2017 and 2018. **OR**
Unclear as to whether the height or the area is used to determine the number of cyclists

4 (a) Simplify $\left(\frac{2^{a-1}\sqrt{2}}{2^a}\right)^2$.

$$\begin{aligned}\left(\frac{2^{a-1}\sqrt{2}}{2^a}\right)^2 &= \left(\frac{2^a \cdot 2^{-1} \cdot \sqrt{2}}{2^a}\right)^2 \\ &= \left(\frac{\sqrt{2}}{2}\right)^2 \\ &= \frac{2}{4} \\ &= \frac{1}{2}\end{aligned}$$

2^{-1} is considered as not simplified, need to simplify to $\frac{1}{2}$

Answer [2]

(b) If $27m^{3x} = 1$ and $m > 0$, find $m^{2x} + m^{-x}$.

$$27m^{3x} = 1$$

$$m^{3x} = \frac{1}{27}$$

$$m^x = \frac{1}{3}$$

$$\begin{aligned}m^{2x} + m^{-x} &= (m^x)^2 + (m^x)^{-1} \\ &= \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^{-1} \\ &= 3\frac{1}{9}\end{aligned}$$

Some obtained $m^{3x} = 3^{-3}$ and from this step, some went to equate $m = 3$ and $x = -1$ which is not always true as $m = \frac{1}{3}, x = 1$ or $m = 9, x = -\frac{1}{2}$.
As there are 2 unknown variables in this equation so we need two equations to be solved simultaneously in order to solve for the value of m and x .

Answer [2]

5 n is a positive integer.

Show that, for all n , $(5n+3)^2 - (5n-3)^2$ is a multiple of 4. [2]

Answer

$$\begin{aligned}(5n+3)^2 - (5n-3)^2 &= (5n+3+5n-3)(5n+3-(5n-3)) \\ &= (10n)(6) \\ &= 60n \\ &= 4(15n)\end{aligned}$$

Since n is a positive integer and $60n = 4(15n)$, $(5n+3)^2 - (5n-3)^2$ is a multiple of 4.

- 6 Gladys wrote down four numbers.
The mean of these numbers is 13, the median is 12 and the mode is 10.
Find the four numbers.

Let the four numbers be a , b , c and d in ascending order.
Since the mode is smaller than the median, a and b will be 10.
Since median is 12,

$$\frac{10+c}{2} = 12$$

$$c = 24 - 10$$

$$= 14$$

Since mean is 13,

$$10 + 10 + 14 + d = 13 \times 4$$

$$d = 18$$

The four numbers are 10, 10, 14 and 18.

Answer [2]

- 7 (a) Mrs Boey deposited \$20 000 in a bank that pays 0.8% per annum compound interest compounded half-yearly. Find the total amount of money she will receive at the end of 5 years, leaving your answer to the nearest cent.

$$\text{Total amount} = \$20000 \left(1 + \frac{0.8}{2(100)} \right)^{5 \times 2}$$

$$= \$20814.55468$$

$$= \$20814.55 \text{ (nearest cent)}$$

Answer [2]

(b)

Singapore Budget 2018: GST to be raised from 7% to 9%
some time between 2021 and 2025

Shane and Glen came across an article in the newspaper with the headline shown above and made the following comments.

Shane: Oh no, the GST will increase **by** 2% soon!

Glen: No! I disagree. It did not increase by 2%, in fact it is **MORE** than 2%!

Whose statement is correct? Support your answer with mathematical calculations.

$$\frac{9-7}{7} \times 100\% = 28.6\% \text{ (3sf)}$$

Percentage point is defined as the **difference** between two percentages. So the increase from 7% to 9% is two percentage points, and not two percent.

Glen's statement is correct as the GST will increase by 28.6%, which is more than 2%.

Answer [2]

- 8 (a) Find the range of values of x which satisfy the inequalities

$$\frac{17-8x}{3} < \frac{2(3x-1)}{5} \leq 4.$$

$$\begin{array}{l} \frac{2(3x-1)}{5} \leq 4 \\ 3x-1 \leq 10 \\ 3x \leq 11 \\ x \leq 3\frac{2}{3} \end{array} \qquad \text{and} \qquad \begin{array}{l} \frac{17-8x}{3} < \frac{2(3x-1)}{5} \\ 5(17-8x) < 6(3x-1) \\ 85-40x < 18x-6 \\ -58x < -91 \\ x > \frac{91}{58} \\ x > 1\frac{33}{58} \end{array}$$

$$\therefore 1\frac{33}{58} < x \leq 3\frac{2}{3}$$

Answer [3]

- (b) Hence, state the smallest prime number that satisfies the inequalities.

Smallest prime number is 2.

Answer [1]

- 9 (a) Factorise completely $9b^2 - 6ab + a^2 - x^2$.

$$\begin{aligned} 9b^2 - 6ab + a^2 - x^2 &= (3b-a)^2 - x^2 \\ &= (3b-a+x)(3b-a-x) \end{aligned}$$

Or

$$\begin{aligned} 9b^2 - 6ab + a^2 - x^2 &= (a-3b)^2 - x^2 \\ &= (a-3b+x)(a-3b-x) \end{aligned}$$

Answer [2]

- (b) Simplify $\frac{5}{2x^2-7x-4} - \frac{8}{4-x}$.

$$\begin{aligned} \frac{5}{2x^2-7x-4} - \frac{8}{4-x} &= \frac{5}{(2x+1)(x-4)} + \frac{8}{x-4} \\ &= \frac{5+8(2x+1)}{(2x+1)(x-4)} \\ &= \frac{5+16x+8}{(2x+1)(x-4)} \\ &= \frac{16x+13}{(2x+1)(x-4)} \end{aligned}$$

Answer [3]

10 Roy made a model of his yacht with a scale of 1: 40.

(a) The actual length of the yacht is 12.5 m, find the length of the model in cm.

1: 40

1 cm: 0.4 m

$$\begin{aligned} \text{Length of model} &= \frac{12.5}{0.4} \\ &= 31.25 \text{ cm} \end{aligned}$$

The value 31.25 is exact, so it should not be expressed as 31.3 (correct to 3 s.f.)

Answercm [1]

(b) Roy wants to spray paint his yacht. He can select either of the following options to paint his yacht.

Option A: A lump sum payment of \$1 000

Option B: Payment for cost of paint at \$5.50 per m²

[Cost of manpower is included in both options]

If the total surface area of the model to be painted is 937.5 cm², find the surface area of his yacht. Which one of the two options should Roy select to paint his yacht? [Show your workings clearly]

1 cm: 0.4 m

1 cm² : (0.4)² m²

1 cm² : 0.16 m²

$$\begin{aligned} \text{Cost of paint} &= 937.5 \times 0.16 \times \$5.50 \\ &= \$825 \end{aligned}$$

Roy should select Option B as it is cheaper.

Answer
[3]

11 $\xi = \{x : x \text{ is an integer and } 1 \leq x \leq 10\}$

$A = \{x : x \text{ is a factor of } 20\}$

$B = \{x : x \text{ is a perfect square}\}$

Find

(a) $n(B)$,

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 4, 5, 10\}$

$B = \{1, 4, 9\}$

** 1 is a perfect square

$n(B) = 3$

Answer [1]

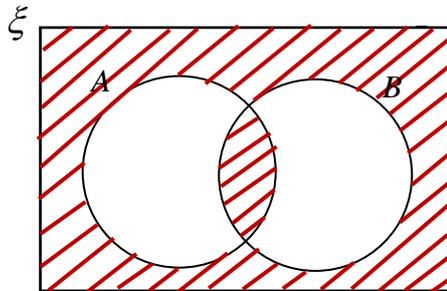
(b) $A' \cap B'$.

$A' \cap B' = \{3, 6, 7, 8\}$

It is a set, so curly brackets must be included.

Answer [1]

(c) On the Venn Diagram, shade the region which represents $(A \cup B)' \cup (A \cap B)$.



[1]

- 12 When written as the product of their prime factors,

$$p \text{ is } 2^2 \times 3 \times 5,$$

$$q \text{ is } 2^6 \times 3^3,$$

$$r \text{ is } 2^2 \times 3^2 \times 11.$$

Find

- (a) the value of the cube root of q ,

$$\begin{aligned} \text{Cube root of } q &= 2^2 \times 3 \\ &= 12 \end{aligned}$$

Answer [1]

- (b) the LCM of p , q and r , giving your answer as the product of its prime factors,

$$\text{LCM of } p, q \text{ and } r = 2^6 \times 3^3 \times 5 \times 11$$

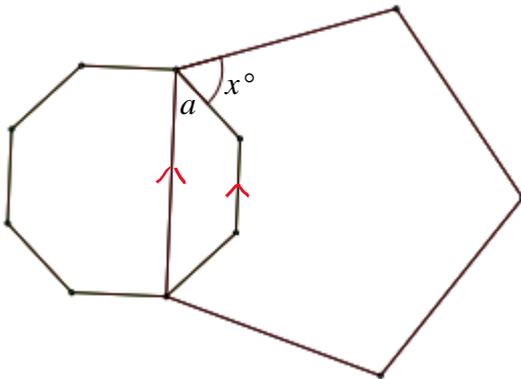
Answer [1]

- (c) the greatest number that will divide p , q and r exactly.

$$\text{The greatest number} = \text{HCF} = 2^2 \times 3 = 12$$

Answer [1]

- 13 The diagram shows a regular pentagon and a regular octagon. Calculate the value of x .



$$\begin{aligned} \text{Each interior angle of pentagon} &= \frac{(5-2)}{5} \times 180^\circ \\ &= 108^\circ \end{aligned}$$

$$\begin{aligned} \text{Each interior angle of octagon} &= \frac{(8-2)}{8} \times 180^\circ \\ &= 135^\circ \end{aligned}$$

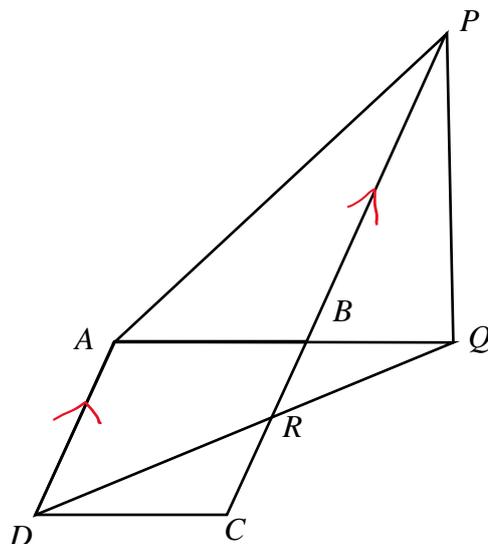
$$\begin{aligned} a &= \frac{1}{2}(360^\circ - 2 \times 135^\circ) \quad (\angle \text{ sum of isosceles trapezium}) \\ &= 45^\circ \end{aligned}$$

$$\begin{aligned} x^\circ &= 108^\circ - 45^\circ \\ &= 63^\circ \end{aligned}$$

Answer $x =$ [3]

14 The diagram shows a rhombus $ABCD$.

ABQ , PBC and DRQ are straight lines and $AQ = BP$.



(a) State the triangle that is similar to triangle BRQ .

Triangle ADQ or Triangle CRD

Answer Triangle [1]

(b) Prove that triangle DQA is congruent to triangle APB .

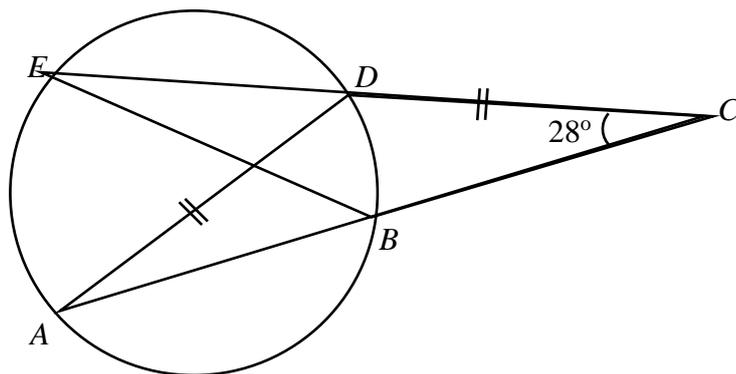
$AD = AB$ (given $ABCD$ is a rhombus)

angle $DAQ =$ angle ABP (alternate angles, AD parallel to BC and BP)

$AQ = BP$ (given)

Therefore triangle DQA is congruent to triangle APB . (SAS)

[2]



A, B, D and E are points on the circle such that $AD = CD$ and angle $BCD = 28^\circ$.
Explain with geometrical reasons, why the length of BC equals to BE .

Angle $DAC = 28^\circ$ (base angles of isosceles triangle ACD)

Angle $DEB =$ Angle DAC (angles in same segment)

$$= 28^\circ$$

$$= \text{Angle } BCD$$

So triangle BCE is an isosceles triangle.

Therefore the length of BC equals to BE .

Many students are not able to spell the word **isosceles** correctly.

[3]

- 16 The masses of 20 bags, in kg, are measured.
The results are shown on the stem-and-leaf diagram.

Mass of 20 bags

Stem	Leaf
0	9
1	2
2	0 6 6 6
3	0 3 3 4 4 8 8
4	1 2 9 9 9
5	0 1

Key: 0|9 means 0.9 kg

- (a) Find the mean mass of the bags.

$$\begin{aligned} \text{Mean mass} &= \frac{69}{20} \text{ kg} \\ &= 3.45 \text{ kg} \end{aligned}$$

A few forgot to check the key and did not realise the decimal point, hence giving the answer as 34.5 kg which is incorrect.

Answerkg [1]

- (b) Find the standard deviation of the masses of the bags.

$$\begin{aligned} \text{Standard deviation} &= \sqrt{\frac{266.8}{30} - 3.45^2} \text{ kg} \\ &= 1.20 \text{ kg (3 s.f.)} \end{aligned}$$

Answerkg [1]

- (c) It was found later that the weighing machine has an error.
Each bag was actually 0.08 kg heavier.
Explain how this will affect the mean and standard deviation.

Answer

The mean will increase by 0.08 kg.
The standard deviation will remain the same.

[2]

- 17 y is inversely proportional to the square of x , $x > 0$.

- (a) If x is increased by 25%, find the percentage change in y .

$$y = \frac{k}{x^2} \text{ where } k \text{ is a constant}$$

$$\text{New } y = \frac{k}{\left(\frac{125}{100}x\right)^2}$$

$$= \frac{16}{25} \left(\frac{k}{x^2}\right)$$

$$= \frac{16}{25} y$$

Quite a few missed the negative sign as it is a **d**ecrease in y .

Percentage change = $\frac{\text{new} - \text{original}}{\text{original}} \times 100\%$

$$\text{Percentage change in } y = \frac{\frac{16}{25}y - y}{y} \times 100\%$$

$$= -36\%$$

Answer% [2]

- (b) Given that $y = 16$ when $x = \frac{1}{2}$, find

- (i) the equation connecting y and x ,

$$k = yx^2$$

$$= 16 \left(\frac{1}{2}\right)^2$$

$$= 4$$

$$\therefore y = \frac{4}{x^2}$$

Answer [2]

- (ii) the value of x when $y = 100$.

When $y = 100$,

$$x^2 = \frac{4}{100}$$

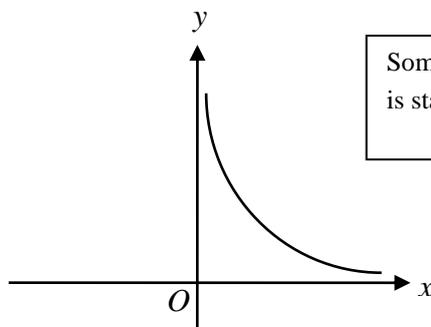
$$= \frac{1}{25}$$

$$x = \frac{1}{5}$$

Some forgot to reject the negative value as it is given at the beginning of the question that $x > 0$.

Answer $x =$ [2]

- (c) Sketch the graph which represents the relation between y and x . [1]



Some still include the negative part even though it is stated at the beginning of the question that $x > 0$.

18 In the following sequence,

$$(1 \times 2) - 2 = 0$$

$$(2 \times 3) - 4 = 2$$

$$(3 \times 4) - 6 = 6$$

$$(4 \times 5) - 8 = 12$$

.....

.....

$$(a \times 12) - b = c$$

.....

.....

$$(d \times e) - f = g$$

(a) Find the values of a , b and c .

$$a = 11 \qquad b = 2 \times 11 \qquad c = 11 \times 12 - 22$$

$$\qquad \qquad = 22 \qquad \qquad = 110$$

Answer $a = \dots\dots\dots, b = \dots\dots\dots, c = \dots\dots\dots$ [2]

(b) Express g in terms of d .

$$e = d + 1$$

$$f = 2d$$

$$g = d(d + 1) - 2d$$

$$g = d^2 + d - 2d$$

$$g = d^2 - d$$

$$g = d(d - 1)$$

Some left out the “g =” part in their answer blank which is part of the required answer.

Answer $\dots\dots\dots$ [2]

(c) Explain why 279 cannot be the result of an equation in this sequence.

Answer $\dots\dots\dots$ [1]

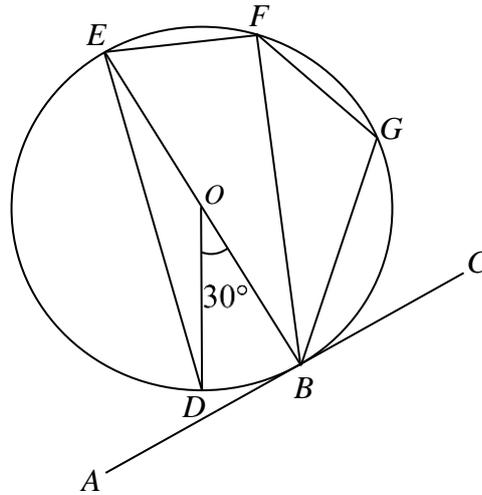
g is the product of an odd and an even number. Therefore g must be even. Since 279 is odd, 279 cannot be the result of an equation.

Alternatively,

$$279 = 9 \times 31$$

From (b), g is the product of 2 consecutive integers which is an even number. 279 is odd, therefore 279 cannot be the result of an equation in this sequence.

- 19 The diagram shows a circle $BDEFG$, centre O and diameter BE . The line AC is a tangent to the circle at B and angle $BOD = 30^\circ$. The ratio of angle EBF to angle FBG to angle GBC is $1 : 1 : 2$.



- (a) State a fact about the arcs EF and FG .

Answer Arc $EF = \text{Arc } FG$ [1]

- (b) Showing all reasons clearly, find

- (i) angle DEB ,

angle $DEB = 15^\circ$ (angle at centre = $2 \times$ angle at circumference)

Answer [1]

- (ii) angle EFG .

angle $EBC = 90^\circ$ (tangent perpendicular to radius)

$$\begin{aligned} \text{angle } EBG &= \frac{2}{4} \times 90^\circ \\ &= 45^\circ \end{aligned}$$

$$\begin{aligned} \text{angle } EFG &= 180^\circ - 45^\circ \text{ (angles in opposite segments)} \\ &= 135^\circ \end{aligned}$$

Some are still not using the standardised geometrical reasons/statements for stating the reasons.

Students must use only the given standardised geometrical properties/reasons.

Some are using very long-winded methods even though it is only 2 marks.

Answer [2]

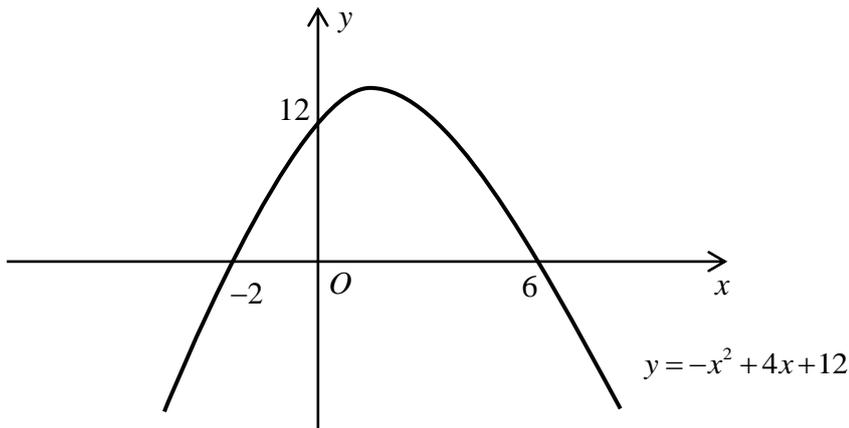
- 20 (a) Express $-x^2 + 4x + 12$ in the form $-(x + p)(x - q)$.

$$\begin{aligned} -x^2 + 4x + 12 &= -(x^2 - 4x - 12) \\ &= -(x + 2)(x - 6) \end{aligned}$$

Answer [1]

- (b) Sketch the graph of $y = -x^2 + 4x + 12$ on the axes below.

Indicate clearly the values where the graph crosses both axes. [2]



- (c) Find the coordinates of the turning point.

Turning point is (2, 16).

Answer (.....,) [1]

- (d) Without solving the equation algebraically, explain why $-x^2 + 4x + 12 = 18$ has no solution.

Since the maximum value of the curve y is 16, there is no point of intersection between the curve and the horizontal straight line $y = 18$.

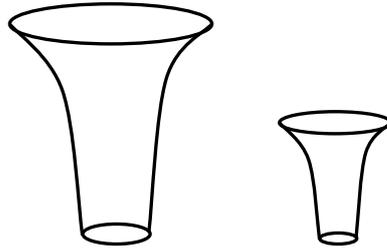
Or

The line $y = 18$ is always above the curve, so there is no point of intersection between the curve and the straight line $y = 18$.

Answer [1]

Some are still using algebraic method to solve despite the instruction saying otherwise.

- 21 The diagram shows two **geometrically similar** containers. The cost of painting the base **area** of the smaller container is $\frac{25}{64}$ of the cost of painting the base of the larger container.



- (a) The top of the larger container has a circumference of 24 cm.
Find the circumference of the top of the smaller container.

$$\left(\frac{l_s}{l_l}\right) = \sqrt{\frac{25}{64}} = \frac{5}{8}$$

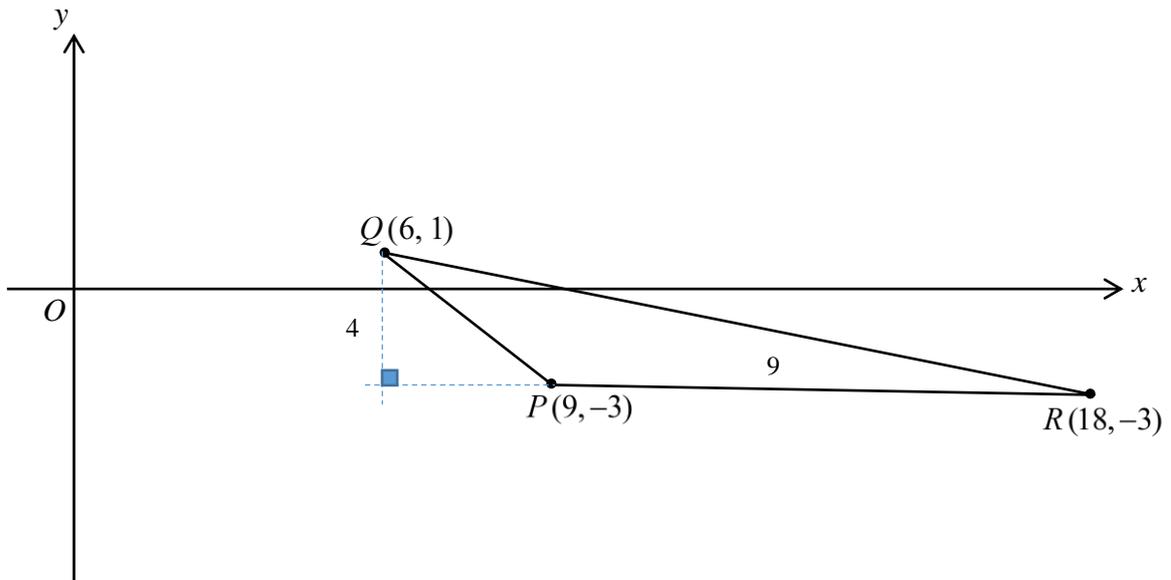
$$\begin{aligned} \text{Circumference of the top of smaller container} &= \frac{5}{8} \times 24 \\ &= 15 \text{ cm} \end{aligned}$$

Answer cm [2]

- (b) The capacity of the smaller container is 0.45 litres.
Find the capacity of the larger container, giving your answer to 2 decimal places.

$$\begin{aligned} \text{Capacity of the larger container} &= \left(\frac{8}{5}\right)^3 \times 0.45 \text{ litres} \\ &= 1.8432 \text{ litres} \\ &= 1.84 \text{ litres (2 decimal places)} \end{aligned}$$

Answer litres [1]



In the diagram, the points P , Q and R have coordinates $(9, -3)$, $(6, 1)$ and $(18, -3)$ respectively.

(a) Find the length of PQ .

$$\begin{aligned}
 PQ &= \sqrt{(9-6)^2 + (-3-1)^2} \\
 &= \sqrt{3^2 + (-4)^2} \\
 &= 5 \text{ units}
 \end{aligned}$$

Answerunits [1]

(b) Find the value of $\cos \angle QPR$, giving your answer as a fraction in its simplest form.

$$\cos \angle QPR = -\frac{3}{5}$$

Some could not identify the right - angled triangle and some forget that $\cos A = -\cos(180^\circ - A)$

Answer [1]

(c) Find the area of triangle PQR .

$$\begin{aligned}
 \text{Area of triangle } PQR &= \frac{1}{2} \times 9 \times 4 \text{ units}^2 \\
 &= 18 \text{ units}^2
 \end{aligned}$$

Some are using other more time-consuming methods such as “shoelace”, $\frac{1}{2}ab \sin C$ even though the question is only 1 mark.

Answer [1]

- (d) Find the equation of the line PQ .

$$\begin{aligned}\text{Gradient of } PQ &= \frac{1 - (-3)}{6 - 9} \\ &= -\frac{4}{3}\end{aligned}$$

$$\begin{aligned}\text{Equation of } PQ \text{ is } y - 1 &= -\frac{4}{3}(x - 6) \\ y - 1 &= -\frac{4}{3}x + 8 \\ y &= -\frac{4}{3}x + 9 \quad \text{or } 4x + 3y = 27\end{aligned}$$

Answer [2]

- (e) The equation of the line passing through the point R is $4x + 21y = 9$.

Find the coordinates of the point of intersection of this line and the line PQ .

$$4x + 3y = 27 \quad \text{..... (1)}$$

$$4x + 21y = 9 \quad \text{..... (2)}$$

$$(2) - (1):$$

$$18y = -18$$

$$y = -1$$

Sub $y = -1$ into (1):

$$4x + 3(-1) = 27$$

$$4x = 30$$

$$x = 7\frac{1}{2}$$

\therefore The point of intersection is $\left(7\frac{1}{2}, -1\right)$.

Students must remember to put the brackets for the coordinates on their answer blank.

Answer [3]

~ End of Paper ~

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