Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2021

SECONDARY 4

MATHEMATICS 4048/02

Paper 2

Wednesday 1 September 2021

2 hours 30 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer. Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total Marks	

This document consists of **21** printed pages and **1** blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 **(a)** Simplify $\left(\frac{a^6b^3}{8}\right)^{\frac{1}{3}} \div \frac{3b^{-1}}{a^2}$.

$$\left(\frac{a^6b^3}{8}\right)^{\frac{1}{3}} \div \frac{3b^{-1}}{a^2} = \frac{a^2b}{2} \times \frac{a^2}{3b^{-1}}$$

$$= \frac{a^2b}{2} \times \frac{a^2b}{3} \quad [\text{M2} - \text{ Any 2 Laws of Indices}]$$

$$= \frac{a^4b^2}{6} \quad [\text{A1}]$$

Answer .... [3]

(b) Solve $\frac{2}{x-2} + \frac{3}{x+2} = 0$.

$$\frac{2(x+2)+3(x-2)}{(x-2)(x+2)} = 0 \quad [\text{M1} - \text{ Single Fraction}]$$

$$2x+4+3x-6=0$$

$$5x-2=0$$

$$x = \frac{2}{5} \text{ or } 0.4 \quad [\text{A1}]$$

Answer $x =$ .... [2]

(c) Make q the subject in the equation $pq + 9 = p^2 + 3q$ and simplify your answer.

$$pq + 9 = p^2 + 3q$$

$$pq - 3q = p^2 - 9 \quad [\text{M1} - \text{Grouping } q \text{ terms}]$$

$$q(p-3) = (p-3)(p+3) \quad [\text{M1} - \text{Difference of squares}]$$

$$q = \frac{(p-3)(p+3)}{p-3}$$

$$q = p + 3 \quad [\text{A1}]$$

Answer .... [3]

(d) **(i)** Simplify $(4x+3y)^2+(3x-4y)^2$.

$(4x+3y)^2+(3x-4y)^2$
$=16x^2+24xy+9y^2+9x^2-24xy+16y^2$ [M1 – Expansion]
$=25x^2+25y^2$
$=25(x^2+y^2)$ [A1]

Answer .... [2]

(ii) It is given that $x^2+y^2=1$. Using your answer in part **(i)**, explain why the maximum value of $(4x+3y)^2$ is 25.

$(4x+3y)^2+(3x-4y)^2=25(x^2+y^2)$
$=25(1)$ [√M1 – Substitute for $x^2+y^2=1$]
$=25$
$(4x+3y)^2=25-(3x-4y)^2$ [B1]
Since $(3x-4y)^2\geq 0$, $-(3x-4y)^2\leq 0$ [A1 – Recognising that $(3x-4y)^2\geq 0$]
$25-(3x-4y)^2\leq 25$
$(4x+3y)^2\leq 25$
Max value of $(4x+3y)^2=25$ (shown) [AG]

[3]

2 During nuclear fusion, it is observed that when two hydrogen atoms are fused together to form one helium atom, there is a loss in mass, M.

The atomic masses of one hydrogen atom and one helium atom are given in the table below. The masses are given in atomic mass unit (u).

Atom	Hydrogen	Helium
Atomic Mass (u)	2.0141	4.0026

(a) Find M (in u), leaving your answer in standard form.

$$\begin{aligned} M_D &= \mathrm{H}+\mathrm{H}-\mathrm{He} \\ &= 2.0141+2.0141-4.0026 \quad [\text{B1} - \text{ Substituting correct values}] \\ &= 0.0256 \\ &= 2.56\times10^{-2} \quad [\text{B1} - \text{ Correct Answer}] \end{aligned}$$

Answer ………………………….. u [2]

(b) Express M as a percentage of the atomic mass of one helium atom.

$$\begin{aligned} \frac{M}{\mathrm{He}}\times100\% &= \frac{2.56\times10^{-2}}{4.0026}\times100\% \quad [\text{M1} - \text{Subtitution of values}] \\ &= 0.639585 \\ &= 0.640 \ \ (3 \text{ sig. fig.}) \quad [\text{A1}] \end{aligned}$$

Answer …………………………. % [2]

(c) Given that $1\ \mathrm{u} = 1.66\times10^{-27}$ kg, find M in kilograms.

$$\begin{aligned} \text{mass} &= \left(1.66\times10^{-27}\right)M \\ &= \left(1.66\times10^{-27}\right)(0.0256) \\ &= 0.042496\times10^{-27} \\ &= 4.2496\times10^{-29} \\ &= 4.25\times10^{-29}\ \text{kg} \quad (3 \text{ sig. fig.}) \quad [\text{B1 for either}] \end{aligned}$$

Answer ………………………….. kg [1]

3 Town P and Town Q are 40 km apart. A car is travelling from Town P to Town Q at an average speed of x km/h. During its return journey, the car later travelled back to Town P from Town Q at an average speed of $(x+8)$ km/h and the time taken for the return journey was 12 minutes less.

(i) Write down an equation to represent this information and show that it simplifies to $x^2+8x-1600=0$.

Answer

$$\frac{40}{x}-\frac{40}{x+8}=\frac{1}{5} \quad [\text{M1} - \text{Correct expression for time (either)}]$$

$$\left[\text{B1} - \text{ Forming equation with } \frac{1}{5}\right]$$

$$\frac{40(x+8)-40x}{x(x+8)}=\frac{1}{5} \quad [\text{M1} - \text{ Single fraction}]$$

$$\frac{320}{x^2+8x}=\frac{1}{5} \quad [\text{M1} - \text{ Simplifying}]$$

$$x^2+8x=1600$$

$$x^2+8x-1600=0 \ (\textit{shown}) \quad [\text{AG}]$$

[4]

(ii) Solve the equation $x^2+8x-1600=0$, giving your solutions correct to two decimal places.

$$x^2+8x-1600=0$$

$$x=\frac{-8\pm\sqrt{8^2-4(1)((-1600))}}{2(1)} \quad [\text{M1} - \text{Formula or completing the square}]$$

$$=\frac{-8\pm\sqrt{6464}}{2}$$

$$x=36.199 \text{ or } x=-44.199$$

$$x=36.20 \text{ or } x=-44.20 \quad (2 \text{ dec. places}) \quad [\text{A1 each}]$$

Note: -1 mark if left in 3 sig. fig.

Answer $x=$ ……….. or ………….. [3]

(iii) Given that the car left Town P at 0900, find the time when the car reached Town Q. Give your answer correct to the nearest minute.

Since $x > 0, x = 36.199$

$$\text{Time taken} = \frac{40}{36.199} \qquad [\text{M1} - \text{ Time formula}]$$

$$= 1.10498 \text{ hours}$$

$$\text{Time } = 1006 \qquad [\text{A1}]$$

Answer ………………………….. [2]

4 The variables x and y are connected by the equation $y=\frac{3}{2}x+\frac{5}{x}-7$.

Some corresponding values of x and y are given in the table below.

x	0.5	1	1.5	2	3	4	5	6	7
y	3.75	–0.5	–1.42	p	–0.83	0.25	1.5	2.83	4.21

(a) Find the value of p. [1]

$-1.5 \quad [B1]$

Answer $p =$ ....

(b) On the grid opposite, draw the graph of $y=\frac{3}{2}x+\frac{5}{x}-7$ for $0.5 \le x \le 7$.

Use the scale of 2 cm to represent 1 unit on both axes. [3]

(c) By drawing a tangent, find the gradient of the curve at $x=2.5$.

From graph,

Drawing tangent $[M1]$

Gradient $=0.7 \quad [0.6-0.8] \quad [A1]$

Answer .... [2]

(d) **(i)** On the grid in part **(b)**, draw the line $2y+x=2$ for $0 \le x \le 7$.

$2y+x=2$

$2y=-x+2 \quad$ [M1 – Make y the subject]

$y=-\frac{1}{2}x+1 \quad$ [G1 – Draw y][G2 – If draw graph only]

[2]

(ii) Write down the x-coordinates of the points where the line intersects the curve.

$x=0.75\ [0.7-0.8],\ x=3.25\ [3.2-3.3] \quad [A1]$

Answer $x =$ .... or .... [2]

(iii) Show that the points of intersection of the line and the curve give the solutions to the equation $2x^2-8x+5=0$.

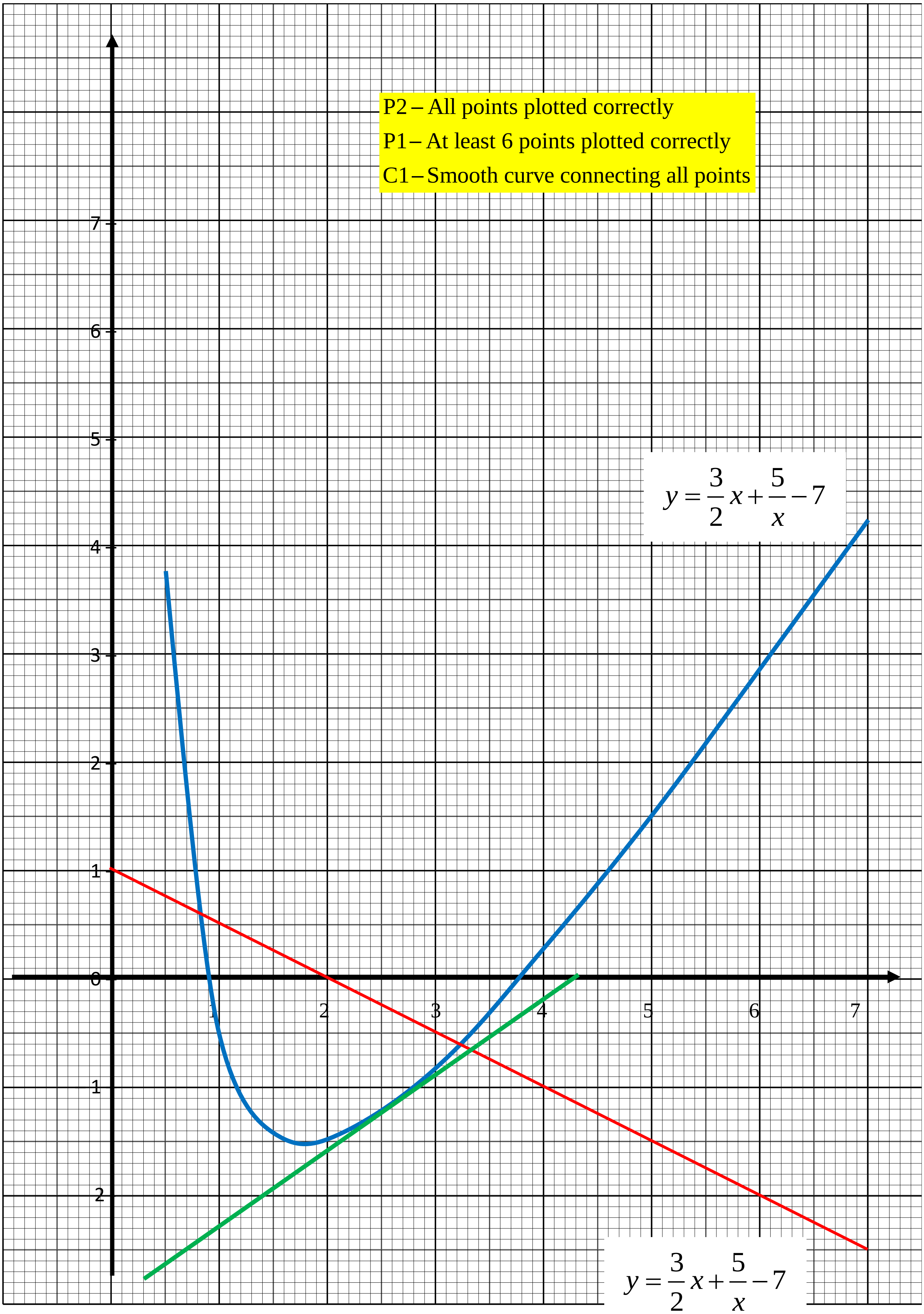
$\frac{3}{2}x+\frac{5}{x}-7=-\frac{1}{2}x+1 \quad$ [M1 – Equating]

$3x^2+10-14x=-x^2+2x$

$4x^2-16x+10=0 \quad [A1]$

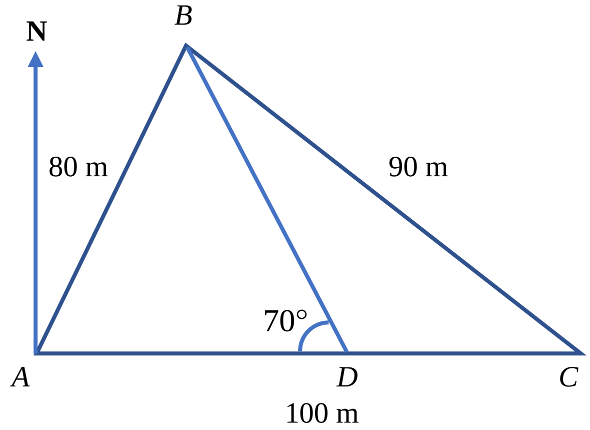
$2x^2-8x+5=0 \quad [AG]$

Answer .... [2]


P2 – All points plotted correctly
P1 – At least 6 points plotted correctly
C1 – Smooth curve connecting all points
$y = \frac{3}{2}x + \frac{5}{x} - 7$
$y = \frac{3}{2}x + \frac{5}{x} - 7$
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
1
2

5 The diagram below shows three points, A, B and C on flat ground.

AB = 80 m, AC = 100 m and BC = 90 m. C is due east of A.



(a) **(i)** Show that angle $ACB = 49.5°$, correct to one decimal place.

$$\cos\angle ACB = \frac{90^2+100^2-80^2}{2(90)(100)} \quad [\text{M1} - \text{ Cosine Rule or other method}]$$

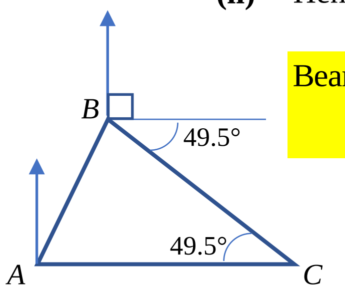
$$\angle ACB = \cos^{-1}\left(\frac{90^2+100^2-80^2}{2(90)(100)}\right) \quad [\text{M1} - \text{ Cos}\,C \text{ the subject}]$$

$$= 49.458° \quad [\text{A1}]$$

$$= 49.5° \quad (\text{shown})$$

[3]

(ii) Hence, calculate the bearing of C from B.



$$\text{Bearing of } C \text{ from } B = 90° + 49.458° \quad [\text{M1} - 90° \text{ due east}]$$

$$= 139.5° \quad (1 \text{ dec. place}) \quad [\text{A1}]$$

Answer .... ° [2]

(iii) Point D lies along AC such that angle BDA = 70°. Find the distance CD.

$$\angle BDC = 180° - 70° \quad (\text{Adjacent } \angle\text{s on a straight line})$$

$$= 110°$$

$$\angle CBD = 180° - 110° - 49.458°$$

$$= 20.542° \quad [\text{M1} - \text{Finding } \angle CBD]$$

$$\frac{90}{\sin 110°} = \frac{CD}{\sin 20.542°} \quad [\text{M1} - \text{Sine Rule}]$$

$$CD = \frac{90\sin 20.542°}{\sin 110°}$$

$$= 33.606$$

$$= 33.6 \text{ m } (3 \text{ sig. fig.}) \quad [\text{A1}]$$

Answer .... m [3]

(iv) Find the ratio of area of triangle *ABD* to area of triangle *BDC*.

Give your answer correct to one decimal place.

Since they share a common height,

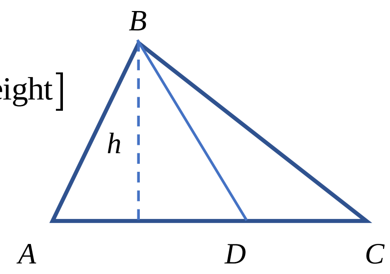
$$\frac{\text{Area of }\Delta ABD}{\text{Area of }\Delta BDC}=\frac{\frac{1}{2}(AD)(h)}{\frac{1}{2}(DC)(h)}\left[\text{M1} - \text{Compare ratio with common height}\right]$$

$$=\frac{AD}{CD}$$

$$=\frac{100-33.606}{33.606}$$

$$=1.975$$

$$=2.0\ \left(1\text{ dec. place}\right)\quad\left[\text{A1} - \text{Accept }\ 2.0:1.0\right]$$

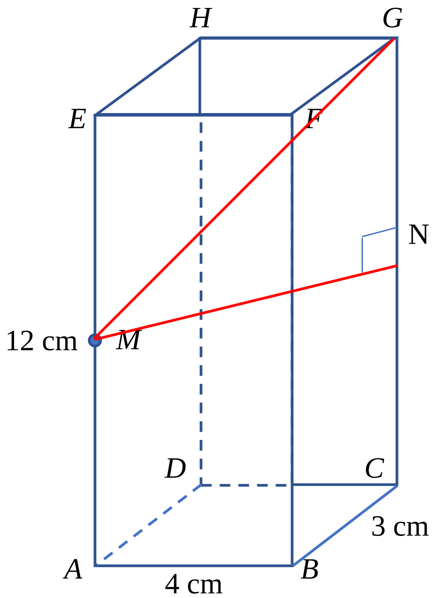


Answer .... [2]

(b) The diagram below shows a cuboid with rectangular base *ABCD*.

AB = 4 cm, *BC* = 3 cm and *AE* = 12 cm. *M* is the midpoint of *AE*.

Find the angle of elevation of *G* from *M*.



Let N be midpoint of GC.

$$AC=\sqrt{4^2+3^2}$$

$$=5\text{ cm}\qquad\left[\text{M1} - \text{ Finding }AC\right]$$

$$GN=\frac{12}{2}=6\text{ cm}$$

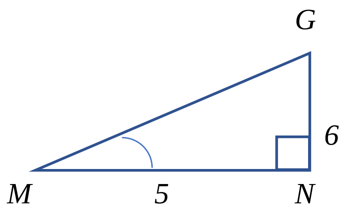
$$\tan\angle GMN=\frac{GN}{MN}$$

$$=\frac{6}{5}\qquad\left[\text{M1} - \text{Using tangent}\right]$$

$$\angle GMN=\tan^{-1}\left(\frac{6}{5}\right)$$

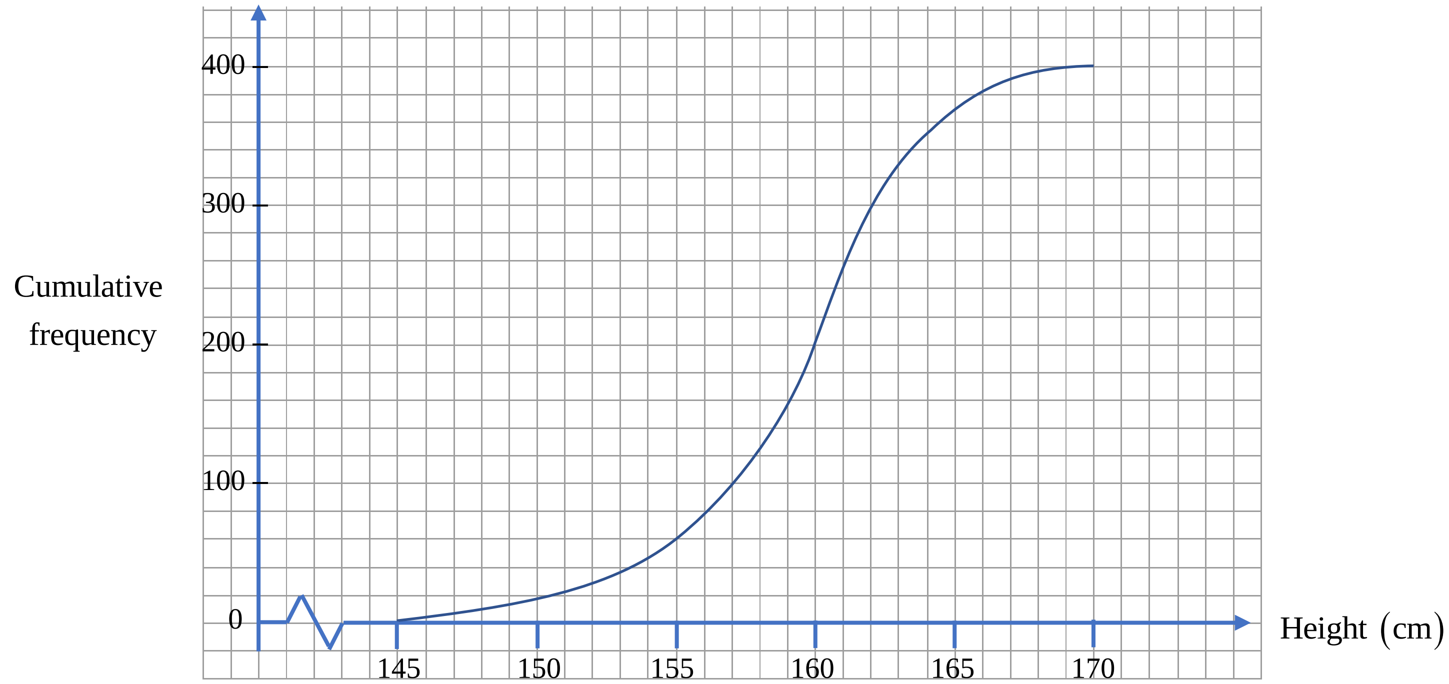
$$=50.194°$$

$$=50.2°\ \left(1\text{ dec. place}\right)\ \left[\text{A1}\right]$$



Answer .... [3]

6 **(a)** The heights of 400 students in School *A* are recorded. The cumulative frequency curve below shows the distribution of their heights.



(i) State the median.

160 cm [B1]

Answer ………………………….. cm [1]

(ii) Find the interquartile range of the distribution.

$Q_1 = 157,\ Q_3 = 162$

$\text{IQR} = 162 - 157$ [B1]

$= 5$ cm [B1]

Answer ………………………….. cm [2]

(iii) In School *B*, the median and interquartile range of the heights of another 400 students are 165 cm and 8 cm respectively. Make two comments comparing the heights of students in School *A* and School *B*.

Answer

The students in School *B* are generally taller as they have a higher median of 165 cm as compared to School *A* (160 cm).

The height of students in School *B* are generally more widespread as they have a higher interquartile range of 8 cm as compared to School *A* (5 cm). [B1 each]

[2]

(b) John flipped a coin five times and obtained five successive heads. Is it conclusive that the coin is biased? Explain your answer clearly.

Answer

No, given that John only flipped the coin 5 times, there is a 1 in 32 chance that of obtaining 5 successive heads which is still possible. The sample size is **too small** for any logical conclusion that the coin is biased.

.... [2]

(c) A two-digit number is formed by drawing a random card from Bag *A* followed by another random card from Bag *B*. Bag *A* contains three cards labelled '1', '2' and '3'. Bag *B* contains three cards labelled '4', '5' and '7'. For example, drawing a '1' and "5" from Bag *A* and Bag *B* respectively will form the number '15'.

(i) Complete the possibility diagram below.

Any 1 row correct [B1]
All rows correct [B2]

Bag *B* \ Bag *A*	1	2	3
4	**14**	24	34
5	15	**25**	35
7	17	27	37

[2]

Find, as a fraction in its simplest form, the probability that the two-digit number formed is

(ii) odd,

$\frac{2}{3}$ [B1]

Answer .... [1]

(iii) a perfect square,

$$\text{P(Number is a perfect square)} = \text{P(Number is 25)}$$
$$= \frac{1}{9} \quad [\text{B1}]$$

[1]

(iv) a prime number,

$$\text{P(Number is prime)} = \text{P(Number is 17, 37)}$$
$$= \frac{2}{9} \quad [\text{B1}]$$

Answer .... [1]

(v) a factor of 135.

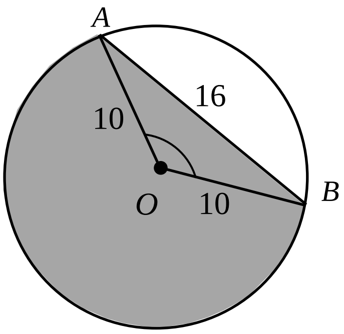
$$135 = 3\times3\times3\times5 \quad [\text{M1} - \text{Prime factorisation of 135}]$$

Factors of 135 are: 1, 3, 5, 9, 15, 27, 45, 135

$$\text{P(Factor of 135)} = \text{P(15, 27)}$$
$$= \frac{2}{9} \qquad [\text{A1}]$$

Answer .... [2]

7 The diagram shows a circle, centre O, radius 10 cm. AB is a chord of length 16 cm.



(a) Show that angle AOB = 1.855 radians, correct to four significant figures.

Answer

$$\sin\frac{\angle AOB}{2}=\frac{8}{10}$$

$$\angle AOB = 2\sin^{-1}\frac{8}{10} \qquad [\text{M1} - \text{Any method}]$$

$$= 1.8545 \qquad [\text{A1}]$$

$$= 1.855 \text{ rad } (\text{shown}) \quad [\text{AG}]$$

[2]

Hence, find the

(b) perimeter of the shaded segment,

$$\text{Perimeter} = (2\pi - 1.8545)(10) + 16 \quad [\text{M1, M1}]$$

$$= 60.28$$

$$= 60.3 \text{ cm } (3 \text{ sig. fig.}) \quad [\text{A1}]$$

Answer ………………………… cm [3]

(c) area of the shaded segment.

$$\text{Area of shaded segment} = \frac{1}{2}(10)^2(2\pi - 1.8545) + \frac{1}{2}(10)^2 \sin 1.8545$$

$$[\text{M1} - \text{Area of sector or Area of triangle}]$$

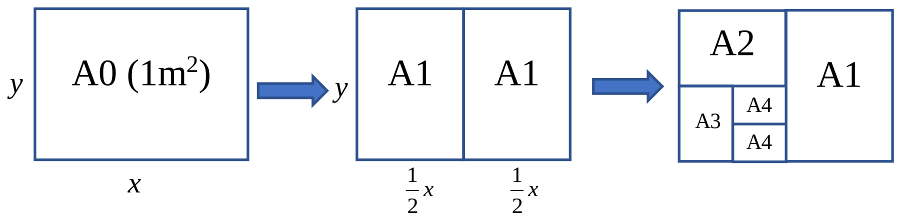
$$= 269.43$$

$$= 269 \text{ cm}^2 \ (3 \text{ sig. fig.}) \qquad [\text{A1}]$$

Answer ……………………….. cm^2 [2]

8 A piece of A0-sized paper has a total surface area of 1 m^2. The length and breadth of the A0-sized paper are x m and y m respectively.

An A1 paper is an A0 paper folded in half, length-wise. An A2 paper is an A1 paper folded in half, so on and so forth.



(i) Write down the relationship between x and y.

$xy = 1 \quad$ [B1 oe]

Answer .... [1]

(ii) Given that all A-sized papers are geometrically similar, show that $\frac{x}{y} = \sqrt{2}$.

Since the papers are geometrically similar, ratio of corresponding sides are equal.

$$\frac{length_{A0}}{breadth_{A0}} = \frac{length_{A1}}{breadth_{A1}}$$

$$\frac{x}{y} = \frac{y}{\left(\frac{1}{2}x\right)} \qquad \text{[M1 or similar]}$$

$$\frac{x^2}{y^2} = 2 \qquad \text{[A1]}$$

$$\frac{x}{y} = \sqrt{2} \quad \text{(Shown)} \quad \text{[AG]}$$

[2]

(iii) Hence, using parts **(i)** and **(ii)**, find the values of x and y.

$$xy = 1$$

$$y = \tfrac{1}{x} \quad ---(1) \qquad [\text{M1} - \text{Making } x \text{ or } y \text{ the subject}]$$

$$\tfrac{x}{y} = \sqrt{2} \quad ---(2)$$

Substitute (1) into (2):

$$\frac{x}{y} = \sqrt{2}$$

$$\frac{x}{\left(\frac{1}{x}\right)} = \sqrt{2} \qquad [\text{M1} - \text{Substitution}]$$

$$x^2 = \sqrt{2}$$

$$x = \sqrt{\sqrt{2}} \qquad [\text{M1} - \text{Solving}]$$

$$= 1.1892..$$

$$= 1.19$$

$$y = 0.841 \quad (3 \text{ sig. fig}) \quad [\text{A1} - \text{Both answers correct}]$$

[4]

(iv) Write down the dimensions of an A4-sized paper in millimetres.

$$x = \frac{1.1892 \times 1000}{4}$$
$$= 297\ (3\ sig\ \ fig)\ \left[\text{M1} - \text{ Converting to mm }\right]$$

$$y = \frac{0.84089 \times 1000}{4}$$
$$= 210\ \left(3\ sig\ \ fig\right)\left[\text{A1} - \text{Both correct}\right]\left[\text{B2} - \text{Both correct}\right]$$

Answer ……….. mm × ……….. mm [2]

(v) Given that N is an integer, express the number of pieces of AN-sized paper that can be cut out from an A0-sized paper in terms of N.

$$\text{Number of pieces} = 2^{N} \quad \left[\text{B1}\right]$$

Answer ……………………………… [1]

9 Robert has 3 options to get to school every weekday morning.

Option **A**: He takes the bus.

Option **B**: He takes the MRT.

Option **C**: He takes a taxi.

The matrices **A**, **B** and **C** represent his journey to school if he were to choose option **A**, **B** or **C** respectively.

$$\mathbf{A}=\begin{array}{c}\begin{matrix}\text{Bus} & \text{MRT} & \text{Taxi}\end{matrix}\\ \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\end{array},\quad \mathbf{B}=\begin{array}{c}\begin{matrix}\text{Bus} & \text{MRT} & \text{Taxi}\end{matrix}\\ \begin{pmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}\end{array},\quad \mathbf{C}=\begin{array}{c}\begin{matrix}\text{Bus} & \text{MRT} & \text{Taxi}\end{matrix}\\ \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}\end{array}.$$

(a) In a particular week, Robert took the bus 3 times. He took the MRT and taxi in each of the other two days. The above information can be represented by a matrix **S** where $\mathbf{S}=3\mathbf{A}+\mathbf{B}+\mathbf{C}$. Find **S**.

$$\mathbf{S}=3\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}+\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}+\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{pmatrix} \qquad [\text{M1}]$$

$$=\begin{pmatrix}3 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} \qquad [\text{A1}]$$

Answer .... [2]

(b) The table below shows the travelling time taken, in minutes, for each of the 3 options that Robert can take to get to school.

Option	Bus	MRT	Taxi
Time taken (in minutes)	30	20	15

(i) Write down a column matrix **N** to represent the information in the table.

$$\mathbf{N}=\begin{pmatrix}30\\ 20\\ 15\end{pmatrix} \qquad [\text{B1}]$$

Answer **N** = ... [1]

(ii) Evaluate the matrix **SN**.

$$\mathbf{SN} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 30 \\ 20 \\ 15 \end{pmatrix} = \begin{pmatrix} 3(30) \\ 1(20) \\ 1(15) \end{pmatrix} \quad [\text{M1}]$$

$$= \begin{pmatrix} 90 \\ 20 \\ 15 \end{pmatrix} \quad [\text{A1}]$$

.

Answer **SN** = ... [2]

(iii) State what the elements in matrix **SN** represent.

Answer

The elements represent the total time taken in minutes, in that particular
....
week by Robert to go to school by via option *A*, *B* and *C* respectively.
....

....

.... [1]

(c) Write down a matrix **J** such that product of **J** and the answer in part **(b)(ii)** will give the average travelling time taken by Robert in a particular week.

$\mathbf{J} = \begin{pmatrix} 0.2 & 0.2 & 0.2 \end{pmatrix} \quad [\text{B1} - \text{Accept fractions}]$

Answer .... [1]

(d) Hence, find the average travelling time taken by Robert per day in a particular week.

$$\mathbf{J}(\mathbf{SN}) = \begin{pmatrix} 0.2 & 0.2 & 0.2 \end{pmatrix}\begin{pmatrix} 90 \\ 20 \\ 15 \end{pmatrix} \quad [\text{M1} - \text{ FT but order must be correct}]$$

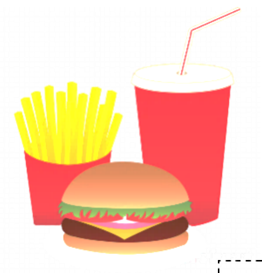
$$= (18 + 4 + 3)$$

$$= (25) \quad [\text{B1} - \text{ with brackets}]$$

Average time taken is 25 minutes

Answer .... minutes [2]

10 The diagram below shows the menu from a fast food restaurant.



Combo A
- Burger
- Medium Fries
- Medium Drink

$5.90 only!

***Upsize** your drink and fries to Large-size for only $1.10*

Ala carte	**Medium ($)**	**Large ($)**
Burger (1 size)	3.20	
Fries	2.20	2.90
Drink	2.80	3.60

(a) Albert wants to buy a burger for both himself and his brother Bart. His brother wants at least a medium-sized fries. Albert wants a large drink while Bart wants a medium drink.

By considering **three options**, suggest how Albert should place his order.

Justify any decisions that you make and show your calculations clearly.

Answer

Choose 3 out of the 4 possible options

Option A: $(\text{Combo A} + \text{Upsize}) + (\text{Burger} + \text{Medium Drink})$

$\text{Total cost} = \$\,5.90 + 1.10 + 3.20 + 2.80$

$= \$13.00$ [B1]

Option B: Combo A + Combo A + Upsize

$\text{Total cost} = \$\,5.90 + 5.90 + 1.10$

$= \$12.90$ [B1]

Option C: $2 \times \text{Burger} + \text{M.Fries} + \text{M.Drink} + \text{L.Drink}$

$\text{Total cost} = \$2(3.20) + 2.20 + 2.80 + 3.60$

$= \$15.00$ [B1]

Option D: $\text{ComboA} + \text{Burger} + \text{L.Drink}$

$\text{Total cost} = \$\,5.90 + 3.20 + 3.60$

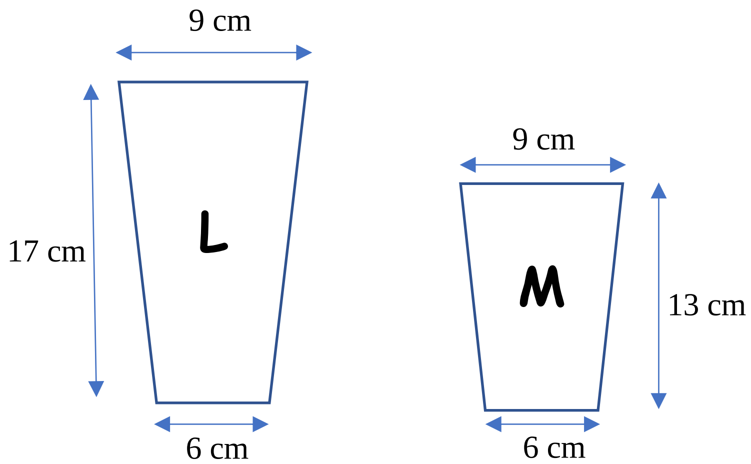
$= \$12.70$ [B1]

He should choose Option D to minimise his spending

[B1 – Correct Option with logical and reasonable explanation] [4]

OR He should choose Option C as for $0.20 more, he can get an extra packet of medium fries so it is the most value – for – money option.

(b) The diagram below shows the cross-sectional area of the large (L) and medium (M) drink cups sold by the fast food restaurant in part **(a)**. The Large and Medium cups have a height of 17 cm and 13 cm respectively. The top and bottom **diameters** of both cups are 9 cm and 6 cm respectively



The volume of a cup is given by $V = \frac{1}{3}\pi h\left(R^2 + Rr + r^2\right)$ where h is the height, R and r are the radii of the top and bottom of the cup respectively. Using the information from part **(a)**, determine whether the medium cup or large cup gives better value for money.

Answer

$$V_L = \frac{1}{3}\pi(17)\left[\left(\frac{9}{2}\right)^2 + \frac{9}{2}\left(\frac{6}{2}\right) + \left(\frac{6}{2}\right)^2\right]$$
$$= 242.25\,\pi \text{ cm}^3 \quad or\ 761\,cm^3\,(3sf) \qquad [\text{B1}]$$

$$V_M = \frac{1}{3}\pi(13)\left[\left(\frac{9}{2}\right)^2 + \frac{9}{2}\left(\frac{6}{2}\right) + \left(\frac{6}{2}\right)^2\right]$$
$$= 185.25\,\pi \text{ cm}^3 \quad \text{or } 582\text{cm}^3\,(3sf) \qquad [\text{B1}]$$

For L cup:

$$\text{Volume per \$1} = \frac{357\pi}{3.60}$$
$$= 3.11541\ldots \text{ cm}^3$$
$$[\text{M1} - \text{Find cost/unit volume}]$$

For M cup:

$$\text{Volume per \$} = \frac{273\pi}{2.80}$$
$$= 306.305\,cm^3$$

$\therefore$ Large cup gives more value for money $[\text{A1}]$

OR

$$\frac{\text{Price of L}}{\text{Price of M}} = \frac{P_L}{P_M} = \frac{3.6}{2.8} = \frac{9}{7}$$

$$\frac{V_L}{V_M} = \frac{17}{13}$$

By comparing ratio:

$$\frac{P_L}{P_M} = \frac{9}{7} \times \frac{13}{13} = \frac{117}{91} \qquad [\text{M1} - \text{Comparing ratio}]$$

$$\frac{V_L}{V_M} = \frac{17}{13} \times \frac{7}{7} = \frac{119}{91}$$

Since $\frac{V_L}{V_M} > \frac{P_L}{P_M}$,

buying a Large drink gives more value for money.

$[\text{A1} - \text{Conclusion}]$

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