# CEDAR GIRLS' SECONDARY SCHOOL
## Preliminary Examination
## Secondary Four

| CANDIDATE NAME | Worked Solutions | Sec 4 (  ) Reg. No: (  ) |
|---|---|---|

| CENTRE NUMBER | | | | | INDEX NUMBER | | | | |
|---|---|---|---|---|---|---|---|---|---|

# MATHEMATICS 4048/02

Paper 2 **1 September 2021**

**2 hours 30 minutes**

Candidates answer on the Question Paper.

Additional Materials: Nil

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

| For Examiner's Use |
|---|
| 100 |

## *Mathematical Formulae*

*Compound interest*

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

**1** **(a)** Simplify

**(i)** $\sqrt{\frac{1}{a}} \times b \div \left(\frac{2}{ab}\right)^{-2}$,

$\sqrt{\frac{1}{a}} \times b \div \left(\frac{2}{ab}\right)^{-2}$

$= \frac{b}{\sqrt{a}} \times \frac{4}{a^2b^2}$

$= \boxed{\frac{4}{a^{2.5}b} \text{ or } \frac{4}{a^{\frac{5}{2}}b} \text{ or } \frac{4}{a^{2\frac{1}{2}}b} \text{ or } \frac{4}{b\sqrt{a^5}} \text{ or } \frac{4}{\sqrt{a^5}b}}$

*Answer* ……. [3]

**(ii)** $\frac{4x^2 - 36}{2x^2 - 20x + 42}$.

$\frac{4x^2 - 36}{2x^2 - 20x + 42}$

$= \frac{4(x+3)(x-3)}{2(x-7)(x-3)}$

$= \boxed{\frac{2(x+3)}{x-7} \text{ or } \frac{2x+6}{x-7}}$

*Answer* ……. [3]

**(b)** Solve these simultaneous equations.

$$2x-3y=19$$
$$3x+2y=-4$$

Substitute $x=\frac{19+3y}{2}$ into $3x+2y=-4$,

$$3\left(\frac{19+3y}{2}\right)+2y=-4$$

$$\boxed{y=-5}$$

$$x=\frac{19+3(-5)}{2}$$

$$\boxed{x=2}$$

*Answer* $x =$ ……

$y =$ …… [3]

**(c)** It is given that $4^p = 5$, $5^{2q} = 6$, $6^{3r} = 7$ and $7^{4s} = 8$.
Find the exact value of $pqrs$.

$$4^{24\,pqrs}=8$$
$$2^{48\,pqrs}=2^3$$
$$48\,pqrs=3$$
$$\boxed{pqrs=\frac{1}{16}}$$

*Answer* $pqrs =$ …… [3]

**2** **(a)** The cost of manufacturing a sofa is \$1500.
It is sold to a retailer at a profit of 15% of the cost.

**(i)** Calculate the price the retailer paid for the sofa.

$\$1500 \times 1.15 = \boxed{\$1725}$

*Answer* \$……. [1]

**(ii)** At a furniture fair, the retailer then sold the sofa to a customer at \$2250. Calculate the retailer's profit for the sale of the sofa as a percentage of the selling price.

$$\frac{\$2250 - \$1725}{\$2250} \times 100\%$$
$$= \boxed{23\frac{1}{3}\%}$$

*Answer* ……% [2]

**(b)** John changed S\$ 3500 for US Dollar (US\$) at a bank, for his trip to the United States. Upon his return, he still had US\$ 78 left in his wallet.

The table below shows the exchange rate between Singapore dollar (S\$) and US dollar (US\$) at the bank upon his return.

| | | **Singapore Dollar (S\$)** | |
|---|---|---|---|
| **Currency** | **Unit** | **Selling** | **Buying** |
| **US Dollar (US\$)** | 1 | 1.38 | 1.34 |

Calculate the amount he spent in Singapore dollar (S\$) for his trip to the United States.

US\$ 78
= S\$ (78 × 1.34)
= S\$ 104.52

Amount spent in S\$ for his trip to US
= S\$ 3500 – S\$ 104.52
= $\boxed{\text{S\$ 3395.48}}$

*Answer* S\$……. [2]

**(c)** The cash price of a gaming device is $ 710.
Jolene buys this gaming device on hire purchase and pays a 30% deposit.

The following shows the different hire purchase schemes with a repayment period of 5 years that are offered to her by the finance company.

Scheme *A*: Compound interest of 2.5% per annum

Scheme *B*: Simple interest of 2.6% per annum

Explain and justify, with clear mathematical working, which hire purchase scheme should Jolene take up.

*Answer*

Loan
$= \$710 \times 0.7$
$= \$497$

Total Repayment Amount for Scheme *A*
$= \$497 \times \left(1 + \frac{2.5}{100}\right)^5$
$= \$562.31$ (nearest cent)

Total Repayment Amount for Scheme *B*
$= \$497 + \left(\$497 \times \frac{2.6}{100} \times 5\right)$
$= \$561.61$

Jolene should take up scheme *B* as the total repayment amount / interest payable on the hire purchase is lower for scheme *B* than scheme *A*.

……

……

…… [5]

**3** $\overrightarrow{AB} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$, $D$ is the point $(-2,1)$ and $E$ is $(h, 6)$.

**(a)** Express $\overrightarrow{DE}$ as a column vector.

$$\overrightarrow{DE}$$
$$= \overrightarrow{OE} - \overrightarrow{OD}$$
$$= \begin{pmatrix} h \\ 6 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$
$$\boxed{= \begin{pmatrix} h+2 \\ 5 \end{pmatrix}}$$

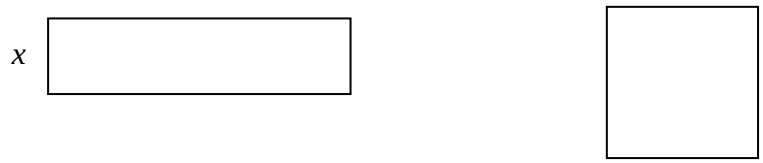
*Answer* ..... [1]

**(b)** $DE = AB$
Find the possible values of $h$.

$$\sqrt{(h+2)^2 + 5^2} = \sqrt{7^2 + 1^2}$$
$$(h+2)^2 = 25$$
$$h + 2 = \pm 5$$
$$\boxed{h = 3 \text{ or } h = -7}$$

*Answer* $h =$ ..... or ..... [2]

**4**




A piece of wire, 44 cm in length, is cut into two parts.
One part is used to make a rectangle and the other a square.
The length of the rectangle is 200% longer than its width.
The width of the rectangle is $x$ centimetres.

**(a) (i)** Write down an expression, in terms of $x$, for the length of the rectangle.

Length of rectangle = $\boxed{3x}$ cm

*Answer* ..... cm [1]

**(ii)** Find, and simplify, an expression, in terms of $x$, for the length of the square.

Length of the square
$= \frac{1}{4}[44 - 2(3x + x)]$
$= \boxed{11 - 2x}$ cm

*Answer* ..... cm [2]

The area of the rectangle is 1 cm$^2$ smaller than the area of the square.

**(b) (i)** Form an equation in $x$ and show that it reduces to $x^2 - 44x + 120 = 0$.

*Answer*
$3x(x) = (11 - 2x)^2 - 1$
$3x^2 = 121 - 44x + 4x^2 - 1$
$x^2 - 44x + 120 = 0$

[3]

**(ii)** Solve the equation $x^2 - 44x + 120 = 0$, giving each solution correct to 5 significant figures.

$$x = \frac{44 \pm \sqrt{(-44)^2 - 4(1)(120)}}{2(1)}$$
$\boxed{x = 2.9212 \text{ or } 41.079}$

*Answer* $x =$ ..... or ..... [3]

**(iii)** Explain why one of the solutions in **(b)(ii)** must be rejected as the width of the rectangle.

*Answer* $x = 41.079$ is rejected as the length of the rectangle becomes 3(41.079)

= 123.237 cm which exceeds the total length of wire from which it is formed.

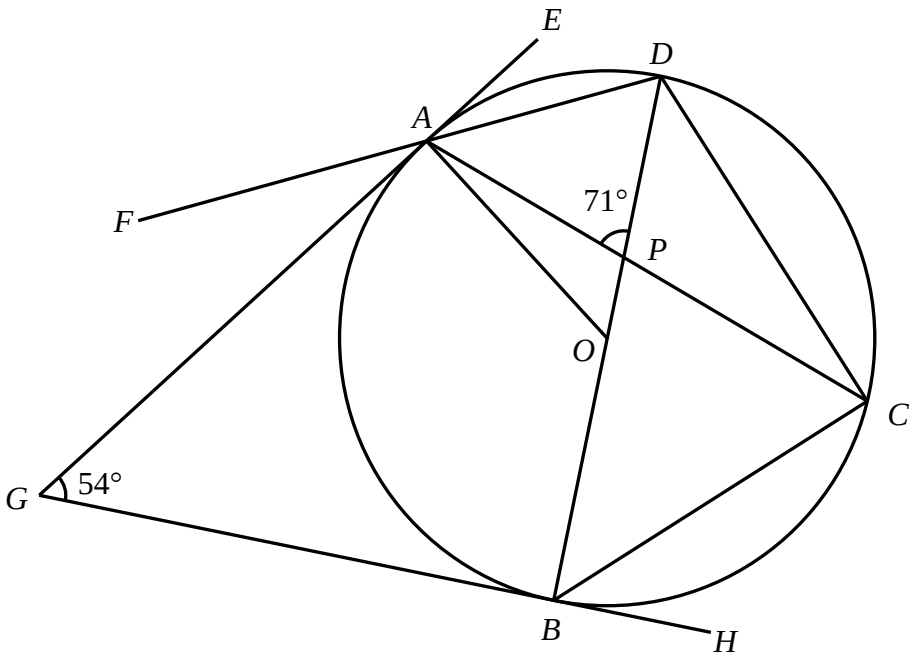
…… [2]

**(iv)** **Hence**, find the perimeter of the rectangle.

Perimeter of rectangle = 8 × (2.9212) = 23.4 cm (to 3 s.f.)

*Answer* …… cm [1]

**5**




In the diagram, *A*, *B*, *C* and *D* are points on the circle with centre *O*. *AG* and *BG* are tangents to the circle. *GAE*, *FAD* and *GBH* are straight lines. Angle *APD* = 71°, angle *AGB* = 54° .

**(a)** Explain why a circle can be drawn passing through the points *A*, *O*, *B* and *G*. State the centre of this circle.

*Answer*
$\angle GAO = 90°$ (tan ⊥ rad)
$\angle GBO = 90°$ (tan ⊥ rad)

A circle with diameter *GO* passes through points *A*, *O*, *B* and *G*. (∠ in a semicircle). Its centre is on the mid-point of *GO*.

[2]

**(b)** Stating your reasons clearly, find

**(i)** angle *AOB*,

Angle *AOB*
= 360° – 90° – 90° – 54° (angle sum of quad.)
= 126°

*Answer* …… [1]

**(ii)** angle *DCA*,

Angle *DCA*
= ½ Angle *AOD* (angle at centre = 2 × angle at circumference)
= ½ (180° – Angle *AOB*) (adj. angles on a str. line)
= ½ (180° – 126°)
= 27°

*Answer* …… [2]

**(iii)** angle *PBC*,

Angle *ACB*
= ½ Angle *AOB* (angle at centre = 2 × angle at circumference)
= 63°

Angle *BPC*
= Angle *APD* (vert. opp. angles)
= 71°

Angle PBC
= 180° – 63° – 71° (angle sum of triangle).
= 46°

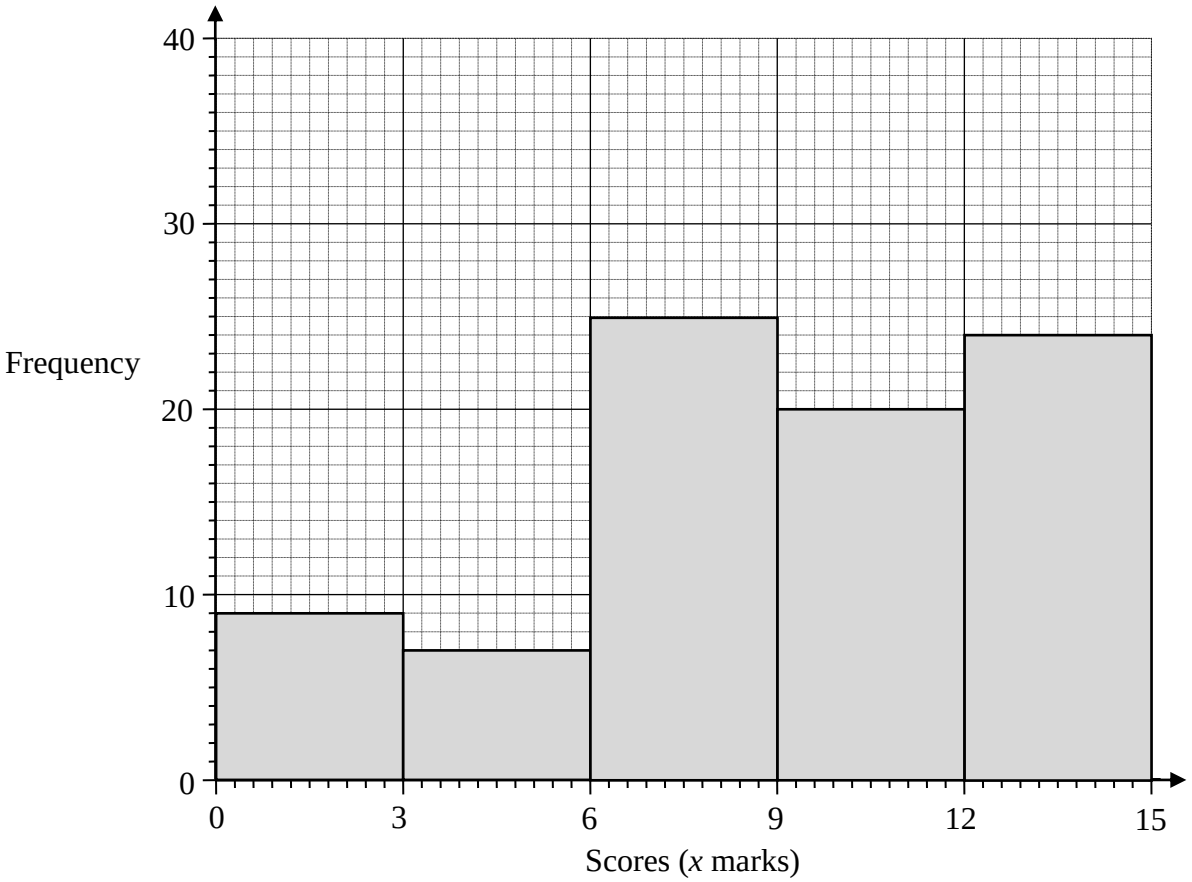
*Answer* …… [2]

**(iv)** angle *CBH*.

Angle *PBH* = 90° (tan ⊥ rad)
Angle *CBH*
= 90° – 46°
= 44°

*Answer* …… [1]

**6** **(a)** The histogram below shows the distribution of the scores of the participants from Potong Pasir Secondary School in a current affairs quiz.




**(i)** Calculate the total number of participants from Potong Pasir Secondary School.

*Answer* 85 [1]

**(ii)** Calculate an estimate of the mean score.

*Answer* 9.02 [1]

**(iii)** Calculate an estimate of the standard deviation.

*Answer* 3.81 [1]

**(iv)** Explain why the mean and standard deviation are estimates.

*Answer* ……

We assumed the mid-value of each interval as the representative value for the scores in the calculation of mean and standard deviation. [1]

**(v)** State the interval that contains the median mark.

*Answer* 9 to 12 **or** 9 – 12 **or** $9 < x \leq 12$ [1]

**(vi)** The organiser then decides to present the awards for the quiz according to the following table.

| **Scores ($x$ marks)** | **Category of Award** |
|---|---|
| $12 < x \leq 15$ | Gold |
| $9 < x \leq 12$ | Silver |
| $6 < x \leq 9$ | Bronze |
| $0 < x \leq 6$ | Certificate of Participation |

Calculate the percentage of students who attained at least a Silver Award.

$\frac{44}{85} \times 100\% = \boxed{51\frac{13}{17}\%}$

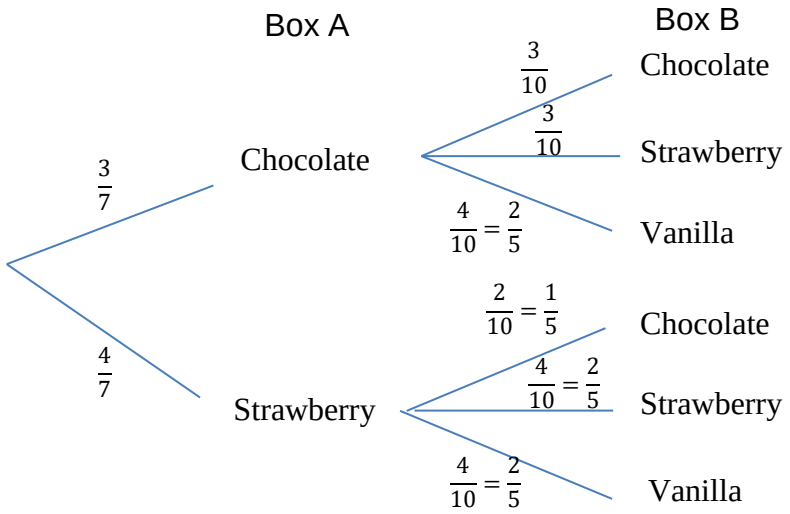
*Answer* ……% [1]

**6 (b)** Box *A* contains 3 cups of chocolate ice-cream and 4 cups of strawberry ice-cream. Box *B* contains 2 cups of chocolate ice-cream, 3 cups of strawberry ice-cream and 4 cups of vanilla ice-cream.

A cup of ice-cream is selected at random from box *A*.
It is then placed in box *B* before a cup of ice-cream is selected at random from box *B*.

**(i)** Draw a tree diagram to show the probabilities of the possible outcomes.

*Answer*




[3]

**(ii)** Find, as a fraction in its simplest form, the probability that

**(a)** the two cups of ice-cream selected are of the same flavour,

$$\left(\frac{3}{7}\times\frac{3}{10}\right)+\left(\frac{4}{7}\times\frac{4}{10}\right)$$
$$=\boxed{\frac{5}{14}}$$

*Answer* ..... [2]

**(b)** the second cup of ice-cream selected is not chocolate.

$$\left(\frac{3}{7}\times\frac{7}{10}\right)+\left(\frac{4}{7}\times\frac{8}{10}\right)$$
$$=\boxed{\frac{53}{70}}$$

*Answer* ..... [2]

**7** **(a)** $\xi = \{\text{integers } x : 2 < x \leq 12\}$
$A = \{\text{prime numbers}\}$
$B = \{\text{factors of } 12\}$
$C = \{\text{ greater than } \sqrt{100}\ \}$

List the elements in

**(i)** $A \cap C'$,

$A \cap C' = \boxed{\{3, 5, 7\}}$

*Answer* ..... [1]

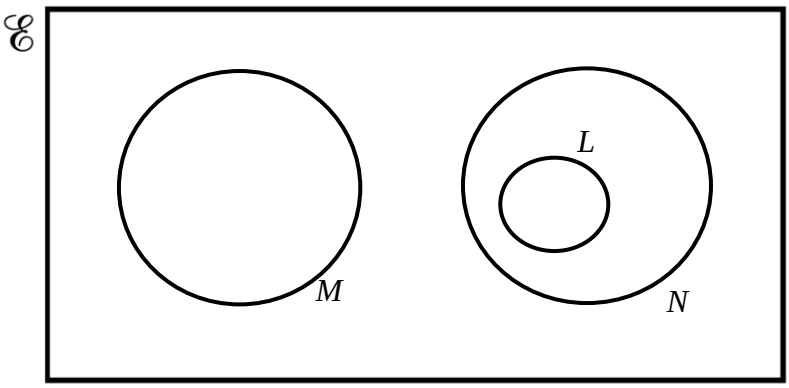
**(ii)** $A' \cup B$.

$A' \cup B = \boxed{\{3, 4, 6, 8, 9, 10, 12\}}$

*Answer* ..... [1]

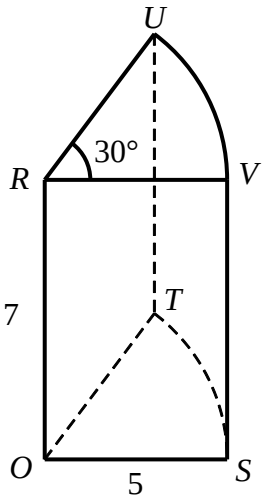
**(b)** It is given that $M \cap N = \phi$ and $L \subset N$.
Complete and label the Venn diagram below for the sets $L$, $M$ and $N$.

*Answer*




[2]

**8**




The figure above shows a solid.
The cross-section of the solid is a sector of a circle of radius 5 cm and angle 30°.
The horizontal cross-sections, *OST* and *RVU*, are 7 cm apart.
*S*, *T*, *U* and *V* lie on the curved surface of the solid.
The lines *OR*, *TU* and *SV* are vertical.

**(a)** Find

**(i)** the area of the curved surface *STUV* in terms of π,

Length of arc $UV = \frac{5\pi}{6}$ cm

Curved surface area $STUV = \frac{5\pi}{6} \times 7$

$= \boxed{\frac{35\pi}{6}}$ cm$^2$

*Answer* ..... cm$^2$ [2]

**(ii)** the angle *UST*.

$\frac{5}{\sin 75°} = \frac{ST}{\sin 30°}$

$ST = 2.5882$ (to 5 s.f.)

$\tan \angle UST = \frac{7}{2.5882}$

$\angle UST = 69.708°$ (to 3 d.p.)

$\angle UST = \boxed{69.7°}$ (to 1 d.p.)

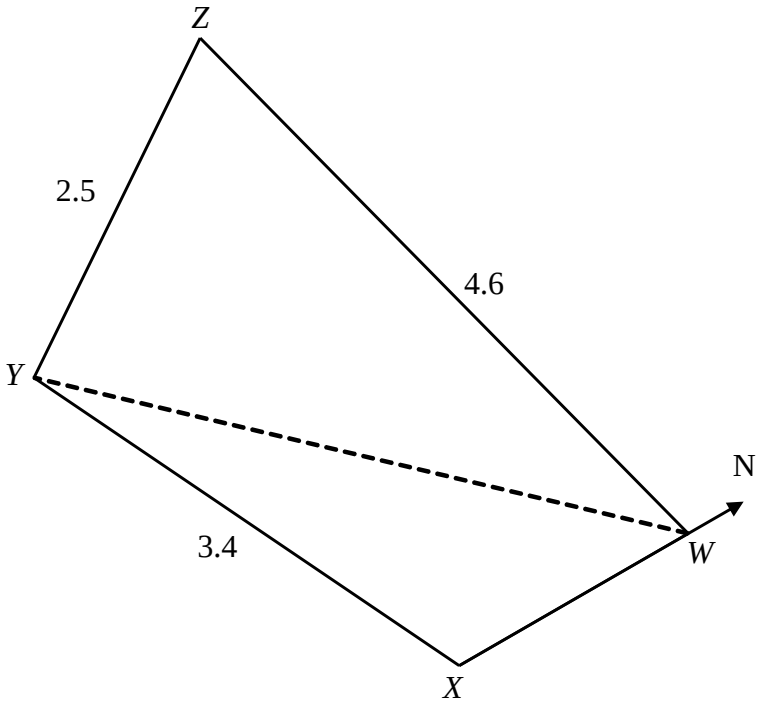
*Answer* ..... [3]

**(b)** Another solid geometrically similar to the given figure has a base radius of 3 cm. Find the ratio of the volume of the smaller solid to the volume of the larger one.

$\left(\frac{3}{5}\right)^3 =$ 27 : 125

*Answer* …… [1]

**9**




The diagram shows part of a map of a small town.
Joel's house is located at point *W*, the childcare centre at point *X*, the park at point *Y* and the shopping mall at point *Z*.

*WZ* = 4.6 km, *YZ* = 2.5 km and *XY* = 3.4 km
The bearing of *W* from *Y* is 043.4° and the bearing of *X* from *Y* is 064.1°.

**(a)** Find the bearing of park *Y* from childcare centre *X*.

*Answer* 244.1° [1]

**(b)** Find the distance of park *Y* from Joel's house *W*.

Angle *YWX* = 43.4° (alt. angles, // lines)

$$\frac{3.4}{\sin 43.4°} = \frac{WY}{\sin 115.9°}$$

*WY* = 4.4514 (to 5 s.f.)
= 4.45 km (to 3 s.f.)

*Answer* …… km [3]

**(c)** Find the bearing of the shopping mall *Z* from Joel's house *W*.

$$\cos \angle YWZ = \frac{(4.6)^2 + (4.4514)^2 - (2.5)^2}{2(4.6)(4.4514)}$$
$\angle YWZ = 32.013°$ (to 3 d.p.)
Bearing of *Z* from *W*
= 180° + 43.4° + 32.013°
= $\boxed{255.4°}$ (to 1 d.p.)

*Answer* …… [3]

**(d)** Find the area of the triangle *WYZ*.

Area of triangle *WYZ*
$= \frac{1}{2}(4.6)(4.4514)\sin 32.013°$
$= 5.4274$
= $\boxed{5.43}$ $km^2$ (to 3 s.f.)

*Answer* …… $km^2$ [2]

**(e)** The **smallest** possible angle of depression of a point on the path *WY* from the top of the shopping mall *Z* is 25°.

Find the height of shopping mall *Z*, giving your answer to the nearest metre.

Height = 4.6 × tan 25°
= 2.14502 km
= 2145.02 m
= $\boxed{2145}$ m (nearest m)

*Answer* …… m [2]

**10** **(a)** Complete the table of values for $y = 10 - \frac{x^2}{2} - \frac{4}{x}$.

| $x$ | 0.5 | 0.7 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 1.9 | 4.0 | 5.5 | 6 | 4.2 | 1 | – 3.3 | – 8.7 |

[1]

**(b)** On the grid opposite, draw the graph of $y = 10 - \frac{x^2}{2} - \frac{4}{x}$ for $0.5 \le x \le 6$. [3]

**(c)** By drawing a tangent, find the gradient of the curve at (2, 6).

Draw tangent correctly at (2, 6)

Gradient of the curve at (2, 6) = – 1 (Accepted –0.8 to –1.2)

*Answer* ..... [2]

**(d)** By drawing suitable straight lines, find the $x$-coordinate of the point(s) on the curve at which the gradient of the tangent is 3, in the range $0.5 \le x \le 6$.

Drawing of // lines with gradient 3

$x = 1$ (Accepted 0.9 to 1.1)

*Answer* $x =$ ..... [2]

**(e)** Use your graph to find the solutions of the equation $x^3 - x^2 - 14x + 8 = 0$ in the range $0.5 \le x \le 6$.

$$-\frac{x^2}{2} + \frac{x}{2} + 7 - \frac{4}{x} = 0$$

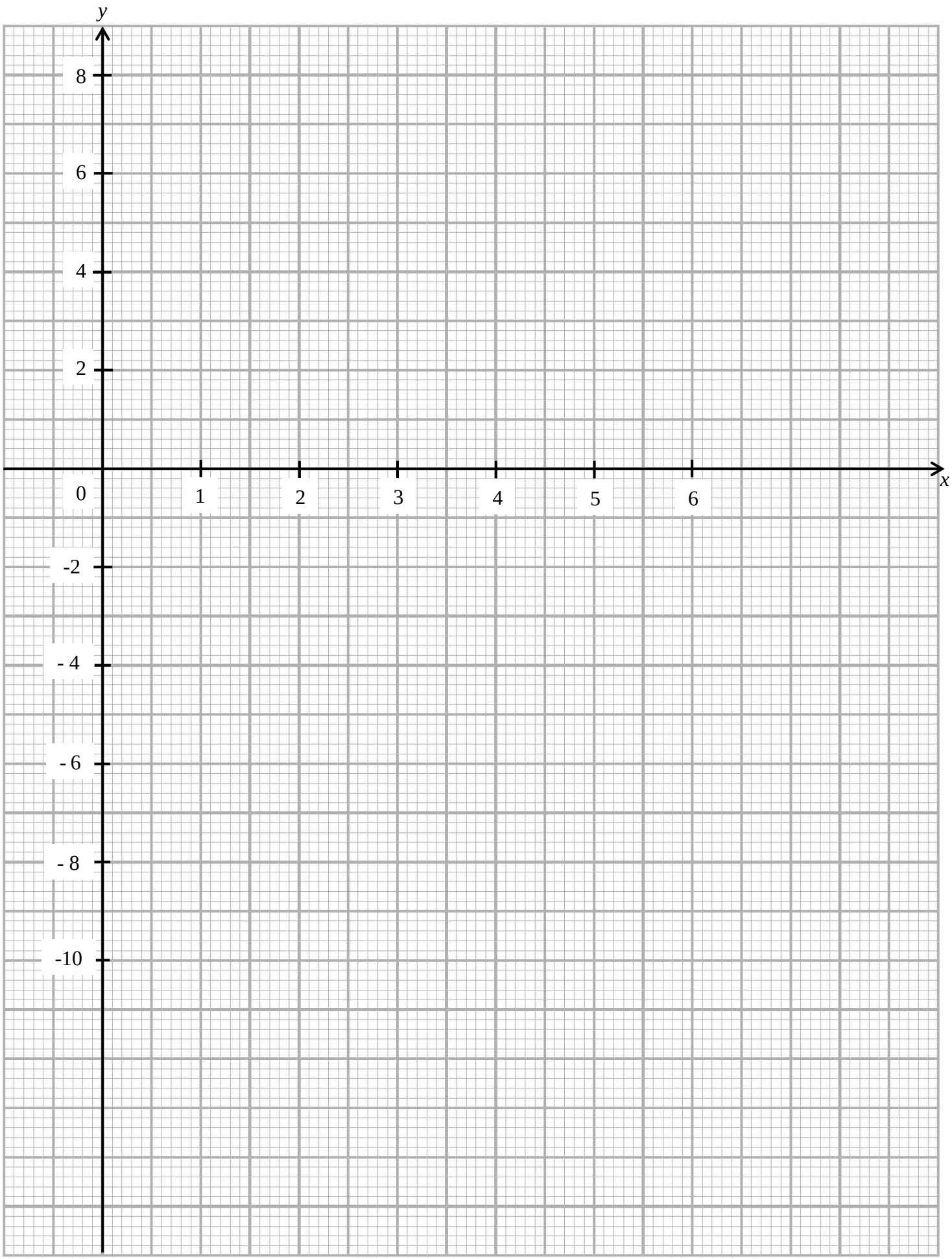
$$-\frac{x^2}{2} - \frac{4}{x} = -\frac{x}{2} - 7$$

$$10 - \frac{x^2}{2} - \frac{4}{x} = -\frac{x}{2} + 3$$

Draw the line $y = -\frac{x}{2} + 3$ to find the intersections

$x = 0.55$ or $x = 4$ (Accepted $\pm 0.1$)

*Answer* $x =$ ..... or ..... [3]


y
8
6
4
2
0
1
2
3
4
5
6
x
-2
- 4
- 6
- 8
-10

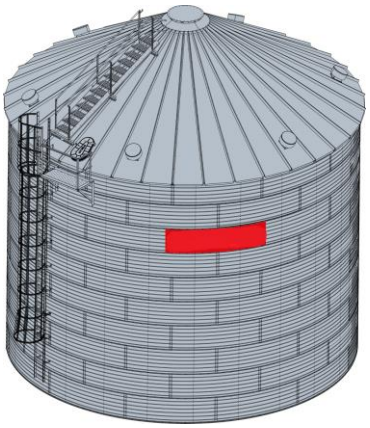
**11** Here is some information about a grain storage bin.

**Grain Storage Bin**

Height ($h$): 6880 mm

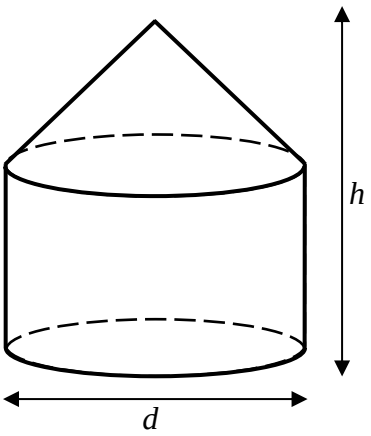
Diameter ($d$): 4550 mm

Mass: 1100 kg



Safety information: The bin can be filled to a maximum of 85% of its total volume.

In this question, the grain storage bin can be modelled as a right cylinder with a right conical top. The height of the conical top is half the radius of the bin.




**(a)** Work out the area, in square metres, of the base of the grain storage bin.

$$\pi\left(\frac{4.550}{2}\right)^2 = 16.260 = \boxed{16.3}\ \text{m}^2 \text{ (to 3 s.f.)}$$

*Answer* ..... $\text{m}^2$ [1]

**(b)** Work out the volume, in cubic metres, of the grain storage bin.

$$\left[\frac{1}{3}\pi\left(\frac{4.550}{2}\right)^2 \times \left(\frac{4.550}{4}\right)\right] + \left[\pi\left(\frac{4.550}{2}\right)^2 \times \left(6.880 - \frac{4.550}{4}\right)\right]$$

$= 99.536$ (to 5 s.f.)

$= \boxed{99.5}\ \text{m}^3$ (to 3 s.f.)

*Answer* ..... $\text{m}^3$ [3]

**(c)**

**Useful information**

- Density of grain stored: 410 kg/m$^3$
- 1000 kg is equivalent to 9.81 kN

The storage bin is never filled to more than its safe volume.
It will need a special load-bearing support structure if its total weight per square metre, on the ground beneath, is greater than 20 kN/m$^2$.

Given that the model is an underestimation of the actual storage capacity of the bin, does the bin need a special load-bearing support structure?
Justify your decision with calculations.

*Answer*

Safe volume
$= 0.85 \times 99.536$
$= 84.606 \text{ m}^3$

Mass of grains stored
$= 84.606 \times 410$
$= 34688 \text{ kg}$

Total load of grains and bin
$= \frac{34688 + 1100}{1000} \times 9.81$
$= 351.08 \text{ kN}$

Load per square metre on the ground beneath
$= \frac{351.08}{16.260}$
$= 21.6 \text{ kN/m}^2$

Since 21.6 kN/m$^2$ > 20 kN/m$^2$ and the model gives an underestimation, so a special load-bearing support structure is needed.

……

……

…… [6]

**End of Paper**

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