CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination 2021
Secondary Four

CANDIDATE NAME	Worked Solutions
CLASS	
CLASS INDEX NUMBER	
CENTRE/ INDEX NO	/

MATHEMATICS

4048/01

Paper 1

31 August 2021
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Given that $\frac{4}{64^x} = 1$, find the value of x.

$$4 = 4^{3x}$$
$$3x = 1$$
$$x = \frac{1}{3}$$

Answer $x = \frac{1}{3}$ [1]

---

2 **(a)** Factorise completely $6x^2 + x - 2$.

$$6x^2 + x - 2 = (2x-1)(3x+2)$$

Answer $(2x-1)(3x+2)$ [1]

(b) Hence , factorise completely $6(3m-1)^2 + 3m - 3$.

$$6(3m-1)^2 + 3m - 3 = 6(3m-1)^2 + (3m-1) - 2$$
$$x = 3m-1$$
$$6(3m-1)^2 + (3m-1) - 2 = (2(3m-1)-1)(3(3m-1)+2)$$
$$= (6m-3)(9m-1)$$
$$= 3(2m-1)(9m-1)$$

Answer $3(2m-1)(9m-1)$ [2]

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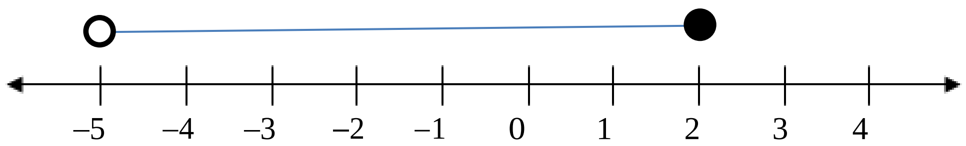
3 A range of values for x is represented on the number line below.



Given that x is an integer, find the smallest value of x^3.

Smallest value = $(-4)^3$ = –64

Answer –64 [1]

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4 **(a)** Show that $y = 5 - x^2 - 4x$ has a maximum point $(-2, 9)$.

Answer

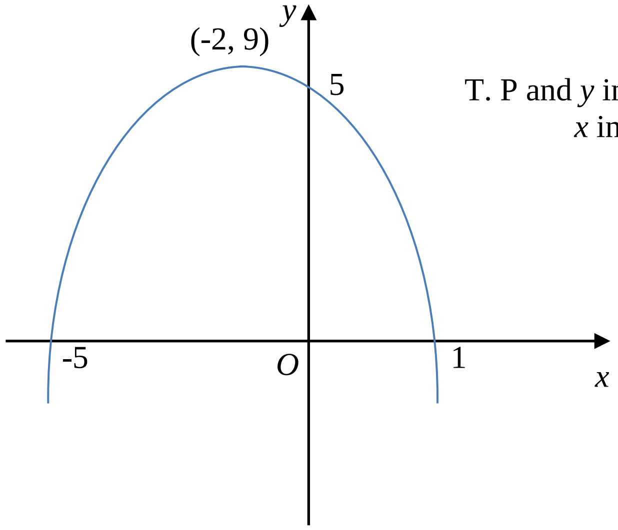
$$y = -\left(x^2 + 4x - 5\right)$$
$$= -\left((x+2)^2 - 5 - (2)^2\right)$$
$$= -\left((x+2)^2 - 9\right)$$
$$= -(x+2)^2 + 9$$

Since coefficient of $x^2 < 0$

Therefore has a maximum turning point at $(-2, 9)$

[3]

(b) Sketch the graph of $y = 5 - x^2 - 4x$ on the axes below.
Indicate clearly the values where the graph crosses the axes and the maximum point on the graph.



Shape
T. P and y intercept
x intercept

[3]

(c) Hence, explain why the equation $x^2 + 4x + 5 = 0$ does not have any solutions. [2]

Answer $x^2 + 4x + 5 = 0$

$-x^2 - 4x - 5 = 0$

$-x^2 - 4x + 5 = 10$

By drawing a line y **= 10**, the line **does not meet** the curve,

Therefore the equation has no solutions.

5 **(a)** Express 396 as the product of its prime factors.

$396 = 2^2 \times 3^2 \times 11$

Answer $2^2 \times 3^2 \times 11$ [1]

(b) Given that $16\ 200 = 2^3 \times 3^4 \times 5^2$, find

(i) the smallest possible integer value of k such that $396k$ is a multiple of 16 200,

$396\,k = 2^2 \times 3^2 \times 11 \times k$

$16\ 200 = 2^3 \times 3^4 \times 5^2$

$k = 2 \times 3^2 \times 5^2$

$= 450$

Answer $k =$ 450 [1]

(ii) the smallest possible integer value of p such that $\frac{16\ 200}{p}$ is a cube number.

$$\frac{16\ 200}{p} = \frac{2^3 \times 3^4 \times 5^2}{3 \times 5^2}$$

$p = 75$

Answer $p =$ 75 [1]

6 The matrix **T** shows the number of training sessions Alyssa and Farah attended for the different training programmes in a year.

$$\begin{array}{cc} & \begin{array}{ccc} \text{Circuit} & \text{Interval} & \text{Long Run} \end{array} \\ \mathbf{T} = & \begin{pmatrix} 50 & 100 & 150 \\ 60 & 100 & 160 \end{pmatrix} \end{array} \begin{array}{l} \text{Alyssa} \\ \text{Farah} \end{array}$$

(a) The duration of each circuit session, interval session and long run is 40 minutes, 15 minutes and 120 minutes respectively. Represent the duration of the training programmes by a 3×1 column matrix **S**.

Answer $\mathbf{S} = \begin{pmatrix} 40 \\ 15 \\ 120 \end{pmatrix}$ [1]

(b) Evaluate the matrix **R =TS.**

$$\mathbf{R} = \begin{pmatrix} 50 & 100 & 150 \\ 60 & 100 & 160 \end{pmatrix} \begin{pmatrix} 40 \\ 15 \\ 120 \end{pmatrix}$$

$$= \begin{pmatrix} 21500 \\ 23100 \end{pmatrix}$$

Answer $\mathbf{R} = \begin{pmatrix} 21500 \\ 23100 \end{pmatrix}$ [1]

(c) State what the elements of **R** represent.

Answer The total duration in minutes on training by Alyssa (21 500 min) and Farah (23100 mins) **in a year** [1]

(d) Evaluate the matrix $\mathbf{P} = \begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{R}$.

$$\mathbf{P} = \begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} 21500 \\ 23100 \end{pmatrix}.$$

$$= \begin{pmatrix} 1600 \end{pmatrix}$$

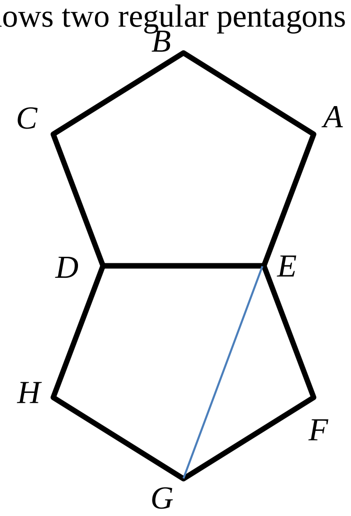
Answer $\mathbf{P} = \begin{pmatrix} 1600 \end{pmatrix}$ [1]

(e) State what the element/s of **P** represent.

Answer The **difference** in the time spent on training between Alyssa and Farah **in a year**.

[1]

7 The diagram shows two regular pentagons *ABCDE* and *DEFGH*.



Show that the points A, E and G are collinear. Justify your answer.

$\angle AED = \frac{3\times 180^\circ}{5}$

$= 108^\circ = \angle EFG$

$\angle FEG = \frac{180^\circ - 108^\circ}{2}$ (base angles of isosceles triangle)

$= 36^\circ$

$\angle GED = 108^\circ - 36^\circ$

$= 72^\circ$

$\therefore \angle AEG = \angle AED + \angle GED$

$= 108^\circ + 72^\circ$

$= 180^\circ$

Therefore the points A, E and G are collinear. [4]

8 A group of students sat for an examination.
50% of the boys and 40% of the girls passed the examination.
Megan commented that 45% of the students passed the examination.
Explain why Megan may be wrong.

Answer The number of boys and girls in the school may not be equal. [1]

9 The first five terms of a sequence are given below.

$$\frac{3}{2} \quad \frac{7}{8} \quad \frac{11}{18} \quad \frac{15}{32} \quad \frac{19}{50}$$

(a) Write down the next two terms.

Answer $\frac{23}{72}, \frac{27}{98}$ [1]

(b) The kth term is $\frac{47}{288}$. Find k.

Answer $k =$ 12 [1]

(c) Find an expression, in terms of n, for the nth term.

Numerator: -1 (-4) 3 (+4) 7 (+4) 11

$4n-1$

Denominator: 2 8 18 32 50

6 10 14 18

4 4 4

Use mode 3, 2

1 1 1 2

4 2 1 8

9 3 1 18

Using

an^2+bn+c

$2n^2$

Answer $\frac{4n-1}{2n^2}$ [2]

10 **(a)** A new housing estate is represented by an area of 200 cm^2 on a Map *A* drawn to a scale of 1 : *n*. Given that the actual area is 32 km^2, find the value of *n*.

Map	Actual
200 cm^2	32 km^2
100 cm^2	16 km^2
10 cm	4 km
	400 000 cm
1	40 000

Answer $n =$ 40 000 [2]

(b) The scale of another map, Map *B* is 1 : 65 000.
The length of a road on Map *B* is 50 cm.
Find the length of the road on Map *A*.

Maps B	1cm	:	65 000cm
	1 cm		0.65 km
	50 cm		32.5 km
Map A	4 km		10 cm
	32.5 km		81.25 cm

Answer 81.25 cm [2]

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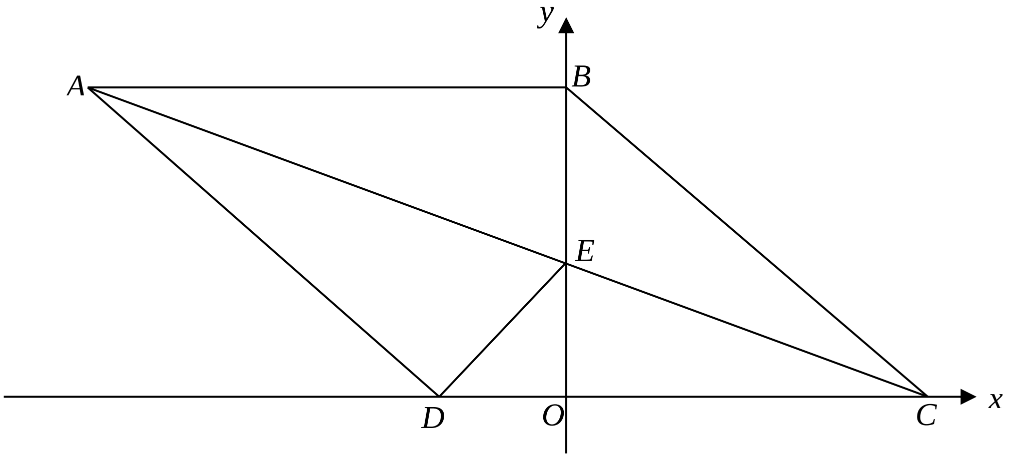
11 In the diagram below, *ABCD* is a rhombus and the diagonal *AC* intersect the *y*-axis at *E*.



Show that the triangle *AEB* is congruent to triangle *AED*. [3]

Answer

$AB = AD$ (side of rhombus)
$AE = AE$ (common side)
$\measuredangle BAE = \measuredangle DAE$ (diagonal bisects angle)

$\Delta AEB \equiv \Delta AED$ (SAS)

12 A box contains 80 paper clips, some of which are grey, some are yellow and the rest are blue.

The probability of drawing a grey clip is $\frac{1}{5}$ and the probability drawing a yellow clip is $\frac{1}{4}$.

(a) Find the number of blue paper clips.

Number of blue paper clips = $\left(1-\frac{1}{5}-\frac{1}{4}\right)$ x 80

= 44

Answer 44 [1]

(b) x blue paper clips are removed from the box so that the probability of drawing a blue clip from the box becomes $\frac{7}{25}$.

Find the value of x.

$$\frac{44-x}{80-x}=\frac{7}{25}$$

$$1100-25x=560-7x$$

$$18x = 540$$

$$x = 30$$

Answer $x =$ 30 [2]

13 $p=\frac{1}{2}\sqrt{\frac{x^2-3y}{x^2}}$

(a) Evaluate p when $x = x=-12$ and $y = 4$, giving your answer correct to two decimal places.

$$p=\frac{1}{2}\sqrt{\frac{(-12)^2-3(4)}{(-12)^2}}$$
$= 0.4787$
$= 0.48$

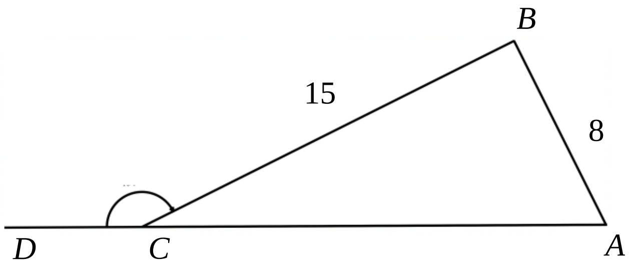
Answer $p =$ 0.48 [1]

(b) Express x in terms of p and y.

$$p=\frac{1}{2}\sqrt{\frac{x^2-3y}{x^2}}$$
$$(2p)^2=\frac{x^2-3y}{x^2}$$
$$4p^2x^2=x^2-3y$$
$$x^2-4p^2x^2=3y$$
$$x^2(1-4p^2)=3y$$
$$x=\pm\sqrt{\frac{3y}{(1-4p^2)}}$$

Answer $x=\pm\sqrt{\frac{3y}{(1-4p^2)}}$ [4]

14 *ABC* is a right-angled triangle with angle $ABC = 90^\circ$, $AB = 8$ cm and $BC = 15$ cm.



Find the value of $\cos \angle BCD$.

$AC = \sqrt{15^2 + 8^2}$
$= 17$

$\cos\angle BCD = -\cos\angle BCA$
$= -\frac{15}{17}$

Answer $\cos \angle BCD =$ $-\frac{15}{17}$ [2]

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15 **(a)** Solve the inequalities $2x + 13 < 4(x+2) \le x + 41$.

$2x + 13 < 4x + 8$ and $4x + 8 \le x + 41$
$2x > 5$ and $3x \le 33$

$x > \frac{5}{2}$ and $x \le 11$

$\frac{5}{2} < x \le 11$

Answer $\frac{5}{2} < x \le 11$ [3]

(b) Hence list all the prime integer values of *x* which satisfy the inequalities $2x + 13 < 4(x+2) \le x + 41$.

Answer 3, 5, 7, 11 [1]

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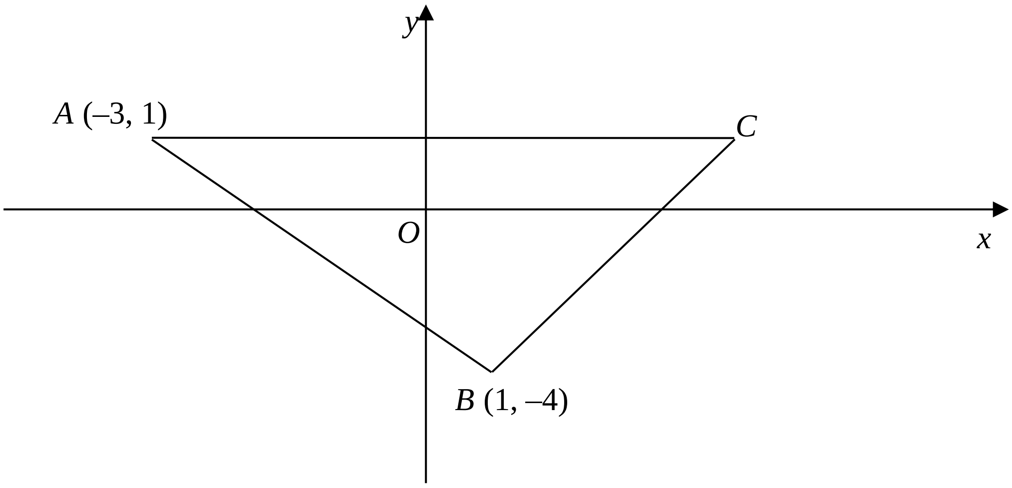
16 In the diagram, A is the point (–3, 1) and B is the point (1, – 4).
The line AC is parallel to the x- axis.



(a) The equation of the line BC is $y - 2x = -6$. Find the coordinates of point C.

Let $C(x, 1)$

when $y = 1$, $\quad 1 - 2x = -6$

$2x = 7$

$x = \frac{7}{2}$

Answer $C\left(\frac{7}{2}, 1\right)$ [1]

(b) The line l is parallel to AB and passes through point C.
Find the equation of the line l.

Gradient of AB = $\frac{1+4}{-3-1}$

$= -\frac{5}{4}$

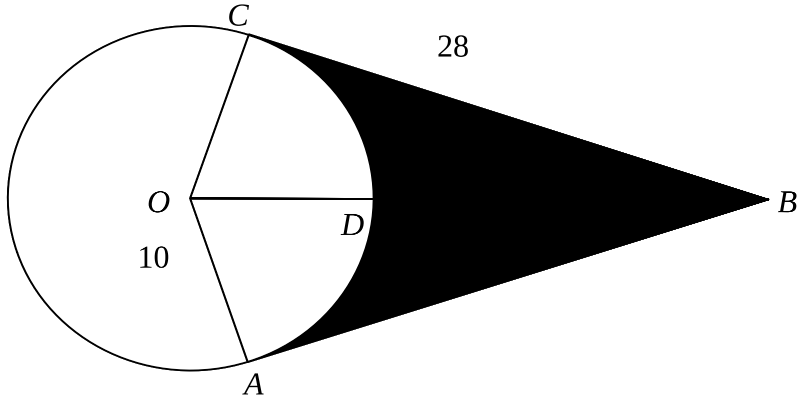
Equation of l : $1 = -\frac{5}{4}\left(\frac{7}{2}\right) + c$

$c = 5\frac{3}{8}$

$y = -\frac{5}{4}x + 5\frac{3}{8}$

Answer $y = -\frac{5}{4}x + 5\frac{3}{8}$ [2]

17 In the diagram, *BA* and *BC* are tangents to the circle with centre *O*. *BO* meets the circle at *D*, *OA* = 10 cm and *BC* = 28cm, find



(a) *BD*

$BD = \sqrt{10^2 + 28^2} - 10$
$= 19.732$
$= 19.7$

Answer 19.7 cm [2]

(b) The area of the shaded region *ABCD*.

Area of Quad = $2 \times \frac{1}{2}(10)(28)$
$= 280$ cm^2

$\tan \angle COD = \frac{28}{10}$

$\angle COD = 1.2278$ rad or 70.346^o
$\angle COA = 2.4556$ rad or 140.692^o

Area of Sector *OADC* $= \frac{1}{2}(10)^2(2.4556)$ or $\frac{140.692}{360}(\pi)(10)^2$
$= 122.78$

Area of shaded region = 280 – 122.78
$= 157.02$
$= 157$ cm^2

Answer 157 cm^2 [4]

18 21 girls took a 40-metre shuttle run test.
The timings are shown in the stem-and-leaf diagram.

Stem	Leaf
10	3 4 5 5
10	6 7 7 8 9
11	0 2 2 2 4 5
11	6 8 9
12	2 3
12	5

Key: 10|3 means 10.3 seconds

(a) Find the median time of the distribution.

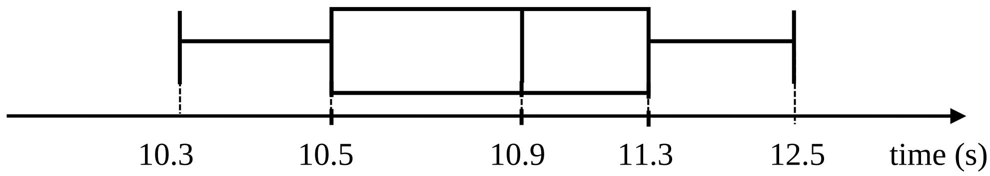
Answer ....11.2............ s [1]

(b) Find the interquartile range.

IQR = 11.7 – 10.65
= 1.05

Answer 1.05 .... s [2]

(c) The box-and-whisker plot shows the distribution of the timings obtained by the same group of girls in July 2021.



The teacher claims that the performance has improved and is more consistent in July 2021 than in January 2021.
Explain if this statement is true.

The statement is true. The median in July (10.9) is faster than in Jan (11.2)
And the IQR in July is smaller (0.8) than in Jan (1.05)

[2]

19 (a) The air resistance, R newtons, is directly proportional to the square of the speed, V m/s, of an object when it is falling.
The air resistance is 24 newtons at a certain speed.
Find the air resistance when the speed is increased by 50%.

$$\frac{R_1}{(V_1)^2} = \frac{R_2}{(V_2)^2}$$

$$\frac{R}{(1.5V)^2} = \frac{24}{V^2}$$

$$R = \frac{24}{V^2} \times 1.5V^2$$

$$= 54$$

Answer 54 newtons [3]

(b) 16 workers can tile 2 rooms in 60 hours.
How many workers are needed if 5 rooms are to be tiled in 72 hours?

Workers x Time			Rooms
16 men	x	60 hrs	2
W men	x	72 hrs	5

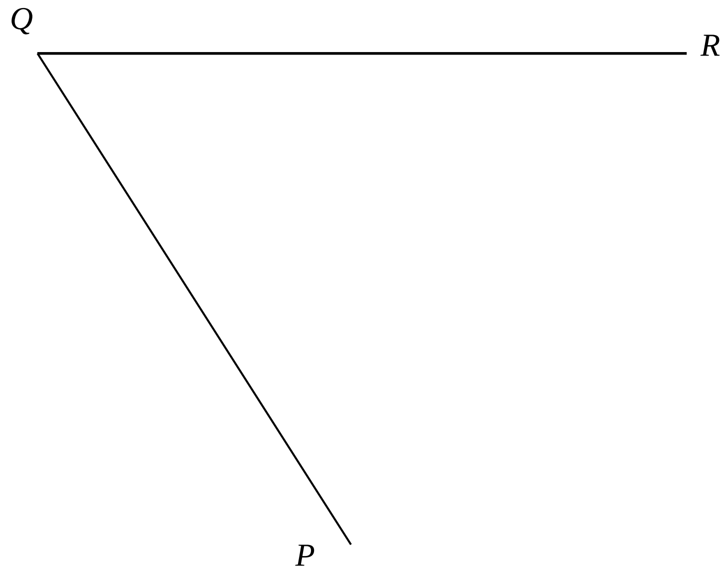
$$\frac{72W}{5} = \frac{16 \times 60}{2}$$

$$W = 33\frac{1}{2}$$

Answer 34 workers [2]

20 **(a)** In the space below, construct a quadrilateral such that $PS = 7$ cm , angle $QRS = 110°$ and angle PSR is an acute angle.
QR and QP have already been drawn. [2]

Answer

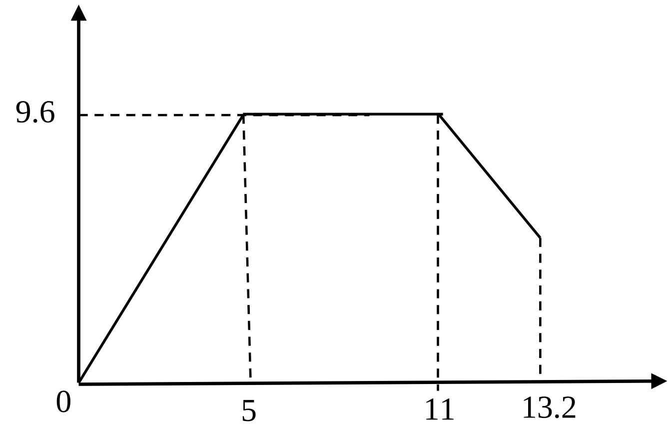


(b) Construct the perpendicular bisector of PQ. [1]

(c) The perpendicular bisector in **(b)** intersects the line QR at T.
Measure the angle QTP.

Answer ……………………………… ° [1]

21 The diagram shows the speed-time graph for Sriya's 100 metre run during her school's sports day.



In the first 5 seconds, Sriya's accelerated uniformly to a speed of 9.6 m/s. She maintained her speed for the next 6 seconds and slowed down over the last 2.2 seconds. She crossed the finishing line after 13.2 seconds.

(a) Calculate Sriya's acceleration 3 seconds after the race started.

$$\text{Acceleration} = \frac{9.6}{5}$$
$$= 1.92$$

Answer 1.92 m/s^2 [1]

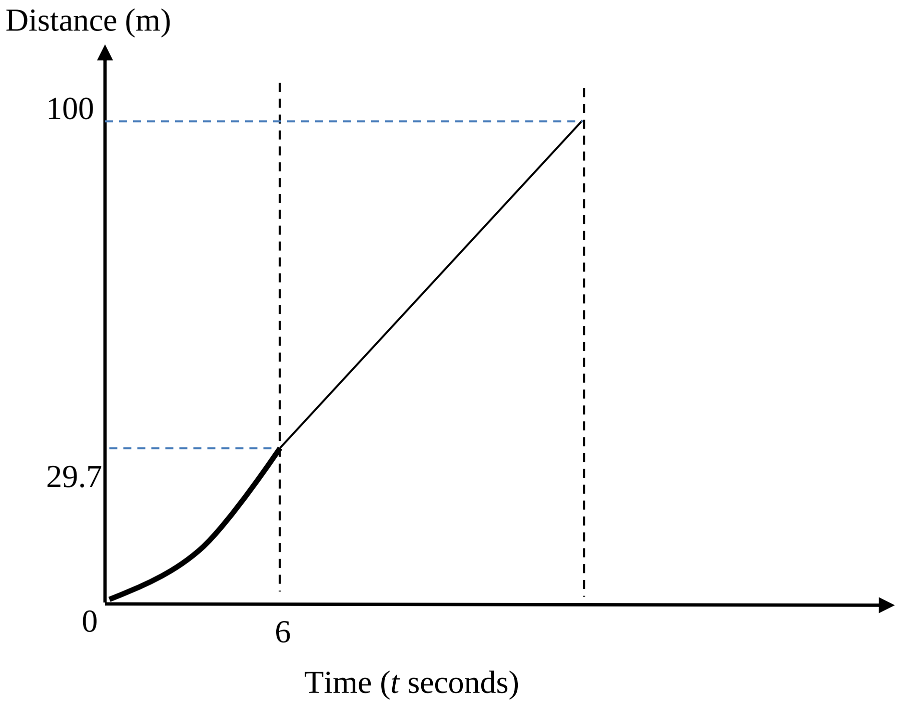
(b) Calculate the speed when she crossed the finishing line..

Let the speed be x m/s

$$100 = \frac{1}{2}(11+6)(9.6)+\frac{1}{2}(9.6+x)(2.2)$$
$$18.4 = \frac{1}{2}(9.6+x)(2.2)$$
$$x = 7.1273$$
$$= 7.13$$

Answer 7.13 m/s [2]

(c) The distance-time graph for another runner, Ella, in the same race is shown on the grid below.
Ella accelerated uniformly to a speed of 10.2 m/s and then maintained her speed until she crossed the finishing line
She ran a distance of 29.7 m in the first 6 seconds.



[3]

Who do you think won the race ? Justify you answer.

Time taken to finish the last 70.3 m = $\frac{70.3}{10.2}$
= 6.8921 sec

Therefore total time taken = 6 +6.8921
= 12.8921 sec
= 12.9 sec
Ella won the race as her time(12.9) is faster than Sriya (13.2)

End of Paper

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