Canberra Secondary School
4E Additional Mathematics Paper 2 Preliminary Examination 2021
Marking Scheme

Question	Marking Scheme	Marks
1	$f(3)=0$ $2(3)^3+p(3)^2+7(3)+q=0$ $9p+q=-75$ ------------------- Equation 1	M1
	$f(2)==0$ $2(2)^3+p(2)^2+7(2)+q=0$ $4p+q=-30$ ------------------Equation 2	M1
	$9p-4p=-75+30$ $5p=-45$ $p=-9$	A1
	$4(-9)+q=-30$ $q=6$	A1
	$2x^3-9x^2+7x+6=(x-3)(x-2)(Ax+B)$ Let $x=0$ $6B=6$ $B=1$	B1
	By inspection, $A=2$ (Or by long division)	B1
	$2x^3-9x^2+7x+6=0$	M1
	$(2x+1)(x-3)(x-2)=0$ $x=-\frac{1}{2}$, $x=3$ or $x=2$	A1

2i	$\text{LHS} = \frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x}$ $= \frac{(1+\sin x)^2 + \cos^2 x}{\cos x(1+\sin x)}$ $= \frac{1+2\sin x+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)}$ $= \frac{1+2\sin x+1}{\cos x(1+\sin x)}$ $= \frac{2(1+\sin x)}{\cos x(1+\sin x)}$ $= \frac{2}{\cos x}$ $= 2\sec x$ $= \text{RHS}$ (proven)	 M1 M1 M1 A1
2ii	$\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 4$ $2\sec x = 4$ $\cos x = \frac{1}{2}$ Basic angle $= \cos^{-1}\left(\frac{1}{2}\right)$ $= \frac{\pi}{3}$ Cosine is positive, 1st and 4th quadrant, $x = \frac{\pi}{3}, \frac{5\pi}{3}$	 M1 M1 M1 A1

3i	$m=\frac{7-3}{6}$ $m=\frac{4}{6}$ $m=\frac{2}{3}$ Equation: $Y=\frac{2}{3}X+3$ or $3y=2x+9$ $x^2y=\frac{2}{3}x+3$ $y=\frac{2}{3x}+\frac{3}{x^2}$	M1 M1 M1 A1
3ii	$x^2y=5$ $5=\frac{2}{3}x+3$ $\frac{2}{3}x=2$ $x=3$	M1 A1
3iii	$y=\frac{2}{3x}+\frac{3}{x^2}$ $xy=\frac{2}{3}+\frac{3}{x}$ $xy=3\left(\frac{1}{x}\right)+\frac{2}{3}$ Plot xy against $\frac{1}{x}$ where the gradient is 3 and the y-intercept is $\frac{2}{3}$, o.e	B1, B1, B1

4i	$\cos\theta = \frac{BX}{10}$ $BX = 10\cos\theta$ $\sin\theta = \frac{XA}{6}$ $XA = 6\sin\theta$ $AB = 6\sin\theta + 10\cos\theta$ (shown)	B1 B1
4ii	$6\sin\theta + 10\cos\theta = R\sin(\theta+\alpha)$ $6\sin\theta + 10\cos\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ $R\cos\alpha = 6$, $R\sin\alpha = 10$ $\frac{\sin\alpha}{\cos\alpha} = \frac{10}{6}$ $\tan\alpha = \frac{10}{6}$ $\alpha = \tan^{-1}\frac{10}{6}$ $\alpha = 59.0°$ $R^2 = 136$ $R = \sqrt{136}$ $6\sin\theta + 10\cos\theta = \sqrt{136}\sin(\theta + 59.0°)$ Maximum *AB* = $\sqrt{136}$ or $2\sqrt{34}$ or 11.7 m $\sin(\theta + 59.0°) = 1$ $\theta = \sin^{-1}(1) - 59.0°$ $\theta = 31.0°$	M1 M1 M1 A1 A1 A1
4iii	For $AB = \sqrt{136} = \sqrt{10^2 + 6^2}$, *AD* must be 0, so that triangle *BCD* becomes a right-angled triangle. This will result that $\angle CBA = \theta$ to be smallest possible.	B2

5i	$M_{AB}=\frac{8-5}{3-6}$ $M_{AB}=\frac{3}{-3}$ $M_{AB}=-1$ Therefore perpendicular gradient = 1 Midpoint $\left(\frac{3+6}{2},\frac{8+5}{2}\right)=\left(\frac{9}{2},\frac{13}{2}\right)$ $=\left(4\frac{1}{2},6\frac{1}{2}\right)$ $=(4.5,6.5)$ Equation of perpendicular bisector of *AB*: $y=mx+c$ $\frac{13}{2}=\frac{9}{2}+c$ $c=2$ $y=x+2$	M1 M1 A1
5ii	$y+x=8$ $y=x+2$ $x+2+x=8$ $2x=6$ $x=3$ $y=5$ Center $(3,5)$ Radius $=\sqrt{(3-3)^2+(8-5)^2}$ $=3$ units	M1 A1 A1
5iii	$(x-3)^2+(y-5)^2=9$	B1
5iv	Distance from center of circle to *C* $=\sqrt{(3-6)^2+(5-3)^2}$ $=\sqrt{13}$ units $\text{Radius}=3$ $=\sqrt{9}$ units $\quad\left(<\sqrt{13}\right)$ Since the distance from the center of the circle is greater than the radius, point *C* is outside of the circle.	M1 A1

5v	Since the *x*–coordinate of the center of the circle is equal to the radius, the *y*–axis is tangent to the circle.	B2
6a	$y = \sec x$ $= (\cos x)^{-1}$ $\frac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$ $= \frac{\sin x}{\cos^2 x}$ $= \sec x \tan x$ (shown) $y^4 - y^2 = \left(\frac{dy}{dx}\right)^2$ LHS $= y^4 - y^2$ $= \sec^4 x - \sec^2 x$ $= \sec^2 x(\sec^2 x - 1)$ $= \sec^2 x(\tan^2 x)$ $= (\sec x \tan x)^2$ $= \left(\frac{dy}{dx}\right)^2$ $=$ RHS (shown)	M1 A1 M1 M1 A1
6b	$\frac{6x+1}{2x+1} = A + \frac{B}{2x+1}$ $\frac{6x+1}{2x+1} = \frac{2Ax + A + B}{2x+1}$ $2A = 6$ $A = 3$ $A + B = 1$ $B = -2$ $\int \frac{6x+1}{2x+1} dx = \int 3 - \frac{2}{2x+1} dx$ $= 3x - \ln(2x+1) + c$	M1 A1 A1 B1, B1

7i	$r^2 = 5^2 - (h-5)^2$ $r^2 = 25 - (h^2 - 10h + 25)$ $r^2 = 10h - h^2$ $r = \sqrt{10h - h^2}$	M1 A1
7ii	$V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\pi (10h - h^2) h$ $V = \frac{\pi h^2 (10-h)}{3}$ (shown)	 M1 A1
7iii	$V = \frac{\pi h^2 (10-h)}{3}$ $V = \frac{10\pi}{3}h^2 - \frac{\pi}{3}h^3$ $\frac{dV}{dh} = \frac{20\pi}{3}h - \pi h^2$ $\frac{20\pi}{3}h - \pi h^2 = 0$ $h\left(\frac{20\pi}{3} - \pi h\right) = 0$ $h = 0$ or $\frac{20\pi}{3} - \pi h = 0$ $h = 0$ or $\pi h = \frac{20\pi}{3}$ (reject) $h = 6\frac{2}{3}$ cm	 M1 M1 A1
7iv	$V = \frac{\pi h^2 (10-h)}{3}$ $V = \frac{\pi\left(\frac{20}{3}\right)^2\left(10 - \frac{20}{3}\right)}{3}$ $= 155.14$ $= 155 \text{ cm}^3$ (3 s.f.) $\frac{dV}{dh} = \frac{20\pi}{3}h - \pi h^2$	 M1 A1

	$\frac{d^2V}{dh^2}=\frac{20\pi}{3}-2\pi h$ $\frac{d^2V}{dh^2}=\frac{20\pi}{3}-2\pi\left(\frac{20}{3}\right) \quad (<0)$ Therefore area is maximum.	M1 A1
8i	$x^3-12x^2+20x=0$ $x\left(x^2-12x+20\right)=0$ $x=0$ or $x^2-12x+20=0$ $x=0$ or $(x-2)(x-10)=0$ $x=0$ or $x=2$ or $x=10$ (reject) Therefore, $A(2,0)$ and $B(10,0)$	M1 A1, A1
8ii	$y=x^3-12x^2+20x$ $\frac{dy}{dx}=3x^2-24x+20$ when x = 8 $\frac{dy}{dx}=3(8)^2-24(8)+20$ $=20$ $y=mx+c$ $-96=20(8)+c$ $c=-256$ $y=20x-256$ $0=20x-256$ $x=12.8$ $C(12.8,0)$	M1 M1 A1 A1
8iii	Area A $=\int_0^2\left(x^3-12x^2+20x\right)dx$ $=\left[\frac{1}{4}x^4-4x^3+10x^2\right]_0^2$ $=12$ units^2 Area B $=\left[\frac{1}{4}x^4-4x^3+10x^2\right]_2^8$ $=-384-12$ $=-396$ units^2	M1

	Therefore Area B = 396 units^2 Area C $=\frac{1}{2}(12.8-8)(96)$ $= 230.4$ units^2 Total area = 12 + 396 + 230.4 = 638.4 units^2	M1 M1 A1
9a	$9^{x+1}-3^{x+2}=2\left(3^{x+1}-2\right)$ $3^{2x+2}-3^{x+2}=2\left(3^{x+1}-2\right)$ Let $y=3^x$, $9y^2-9y=2(3y-2)$ $9y^2-15y+4=0$ $(3y-1)(3y-4)=0$ $y=\frac{1}{3}$ or $y=\frac{4}{3}$ $3^x=3^{-1}$ or $3^x=\frac{4}{3}$ $x=-1$ or $x=\frac{\lg\frac{4}{3}}{\lg 3}$ $x=-1$ or $x\approx 0.262$	M1 M1 A1, A1
9b	$\log_3(2x-1)=\log_9(2x-1)+1$ $\log_3(2x-1)=\frac{\log_3(2x-1)}{\log_3 9}+1$ $\log_3(2x-1)=\frac{\log_3(2x-1)}{2}+1$ $2\log_3(2x-1)=\log_3(2x-1)+2$ $\log_3(2x-1)=2$ $2x-1=3^2$ $2x=10$ $x=5$	M1 M1 A1
9c	$\log_a xy=m$ $\log_a x+\log_a y=m$ $\log_a \frac{x^3}{y^2}=n$ $3\log_a x-2\log_a y=n$ $2\log_a x+2\log_a y=2m$	M1 M1 M1

	$5\log_a x = 2m + n$ $5\log_a x + 5\log_a y = 5m$ $5\log_a y = 5m - 2m - n$ $5\log_a y = 3m - n$ $5\log_a\left(\frac{x}{y}\right) = 2m + n - 3m + n$ $5\log_a\left(\frac{x}{y}\right) = 2n - m$	M1 A1