CANBERRA SECONDARY SCHOOL

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2021 Preliminary Examination

Secondary Four Express

ADDITIONAL MATHEMATICS
4049/01

26 Aug 2021
2 hours 15 minutes
1000h – 1215h

Name: ______________________________________ (   )   **Class:** ________

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number on all work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

FOR MARKER'S USE		
	Marks Awarded	Max Marks
Total		90

This question paper consists of 17 printed pages including the cover page.

Setter: Mrs Irving Long & Mr Ng Chuen Joo

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0,\ a \neq 0,$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}.$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\mathrm{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ⊗ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 \quad 2bc \cos A.$$
$$\otimes = \frac{1}{2} ab \sin C$$

1 Given that $y = p\sin 2x + q$, where p and q are positive integers. The maximum and minimum values of y are 8 and -2 respectively.

(a) Find the value of p and of q. [2]

(b) Hence, sketch the graph of y for $0^\circ \le x \le 360^\circ$. [3]

[Turn Over

2 (**a)** Express each of $4x^2-6x-1$ and $-x^2-10x+3$ in the form $a(x+b)^2+c$,
where *a*, *b* and *c* are constants. [4]

(b) Using your answers in **(a)**, show that the curves $y=4x^2-6x-1$ and $y=-x^2-10x+3$ will intersect. [3]

3 The equation of a curve is given by $y = 5xe^{2x+1}$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Hence, find $\int xe^{2x+1}\ \mathrm{d}x$. [3]

(c) Determine the range of values of x for which y is a decreasing function. [2]

[Turn Over

4 (**a**) A curve has equation $y = x^2 + x + 2k - 4$ and a line has equation $y - 2x = k$,
where k is a constant. Find the set of values of k for which the line and curve do not intersect and represent this set on a number line. [5]

(b) Given that the curve with equation $y = ax^2 + 8x + c$, where a and c are constants, lies completely below the x-axis.
Write down the conditions which must apply to a and c. [3]

5 Given that a point P(1 , 2) lies on the curve $y = \frac{5}{2-x} - 3$, find

(a) **(i)** the value of $\frac{dy}{dx}$ at point P, [2]

(ii) the equation of the normal to the curve at point P. [3]

(b) Explain whether a stationary point will exist on the curve. [1]

[Turn Over

(c) If x is increasing at a rate of 0.2 units per second, find the rate of change of y at the instant when $y =$ 4. [3]

6 **(a)** Given that $\dfrac{\cos\dfrac{\pi}{6}-\tan\dfrac{\pi}{4}}{\sin\dfrac{\pi}{3}+\cos\dfrac{\pi}{3}}=\dfrac{\sqrt{3}+a}{\sqrt{3}+b}$, where a and b are integers.
Find the values of a and b. [3]

[Turn Over

(b) Given that $\sin A = -\frac{5}{13}$ and $\cos B = \frac{4}{5}$, where *A* and *B* are in the same quadrant. Find the exact value of the following:

(i) $\sin(A+B)$ [3]

(ii) $\sec 2B$ [3]

7 A dot on a computer screen moves in a straight line and passes through a fixed point *A*. The distance, *d* metres, that it runs in *t* s after it passes through *A* is given by

$$d = t^3 - 6t^2 + 9t \text{ for } t \geq 0.$$

(i) Find the dot's speed and acceleration 10 seconds after it passes *A*. [6]

(ii) Find the values of *t* at which the dot is instantaneously at rest. [3]

(iii) Find the distance travelled by the dot in the first 4 seconds. [3]

[Turn Over

8 The area of a quadrilateral is $\left(13-\sqrt{48}\right)$ cm^2.

(i) In the case where the quadrilateral is a rectangle with a width of $\left(2-\sqrt{3}\right)$ cm, find, without the use of a calculator, the length of the rectangle in the form of $\left(a+b\sqrt{3}\right)$ cm. [4]

(ii) In the case where the quadrilateral is a square with side $\left(c+2\sqrt{3}\right)$ cm, find, without the use of a calculator, the value of the constant *c*. [3]

9 The price of a new car on 1 January 2021 is \$120 888.00. Given that the value of the car depreciates in such a way that, n months after the purchase, the sale price, \$$P$, is determined by the formula

$$P = 120888e^{-0.015n},$$

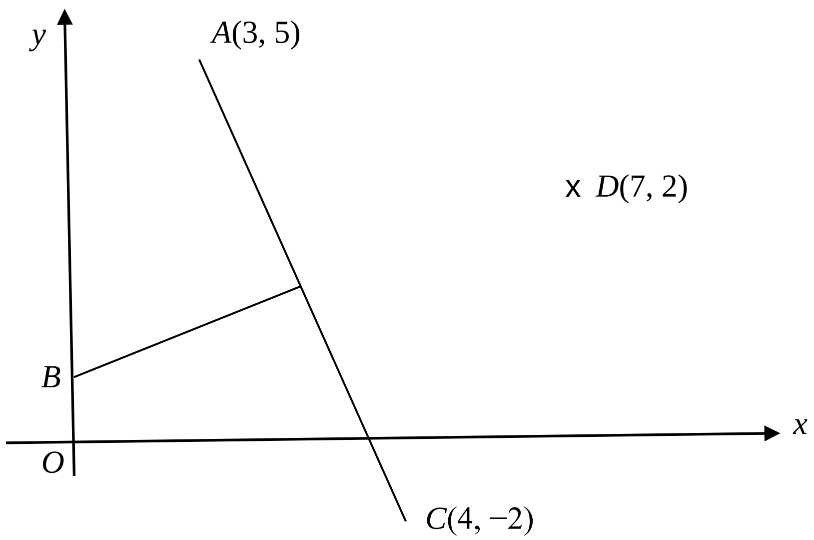
estimate

(i) the sale price of the car after 1 year, giving your answer correct to the nearest \$1000, [2]

(ii) the month and year when the sale price of the car is less than \$50 000. [4]

[Turn Over

10



The figure shows three points A, C and D whose coordinates are (3, 5), (4, −2) and (7, 2) respectively.

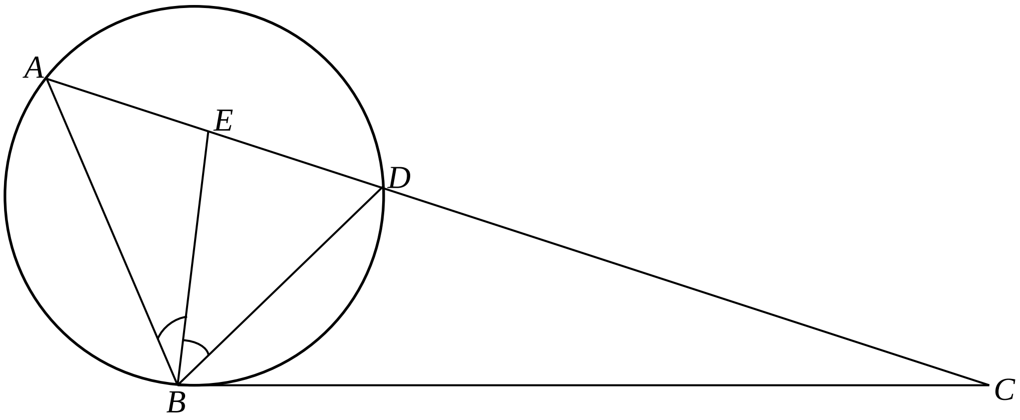
(i) Find the equation of the perpendicular bisector of AC. [2]

(ii) Find the coordinate of B, the point where the perpendicular bisector cuts the y-axis. [1]

(iii) Show that *ABCD* is a square. [3]

[Turn Over

11 In the diagram, AC is a straight line intersecting a circle at A and D.
The point B lies on the circle and BC is a tangent to the circle. The point E lies on AC such that the line BE bisects angle ABD.



(i) State a reason why angle DBC and angle BAD are equal. [1]

(ii) Show that angle EBC = angle BEC. [3]

(iii) Hence name an isosceles triangle and explain why it is so. [2]

12 Find, in ascending powers of x, the first three terms in the expansion of

(i) $(1+4x)^7$, [3]

(ii) $(2-x)^7$. [3]

Hence, find the coefficient of x^2 in the expansion of $(2+7x-4x^2)^7$. [2]

- End of paper -

Answers

1(a)	$= \frac{8-(-2)}{2}$ Amplitude $= 5$ P = 5 q = 3	B1 B1
1(b)	[Graph] Note: B3 for correct and smooth graph M1 for correct amplitude and period seen (follow-thru) M1 for y = 5 sin 2x seen	B3
2(a)	$4x^2-6x-1$ $=4\left(x^2-\frac{3}{2}x-\frac{1}{4}\right)$ $=4\left[x^2-\frac{3}{2}x+\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2-\frac{1}{4}\right]$ $=4\left[\left(x-\frac{3}{4}\right)^2-\frac{13}{16}\right]$ $=4\left(x-\frac{3}{4}\right)^2-\frac{13}{4}$ $-x^2-10x+3$ $=-\left(x^2+10x-3\right)$ $=-\left[x^2+10x+5^2-5^2-3\right]$ $=-\left(x+5\right)^2+28$	M1 A1 M1 A1
(b)	Min turning point of $y=4x^2-6x-1$ is at $\left(\frac{3}{4},-\frac{13}{4}\right)$. Max turning point of $y=-x^2-10x+3$ is at (5, 28). Since the y-coord of the max turning point of $y=-x^2-10x+3$ is greater than the y-coord of the min turning point of $y=4x^2-6x-1$, the 2 curves will intersect.	B1 B1 B1

3(a)	$y = 5xe^{2x+1}$ $\frac{dy}{dx} = 5x\frac{d}{dx}(e^{2x+1}) + e^{2x+1}\frac{d}{dx}(5x)$ $= 5x(2e^{2x+1}) + 5e^{2x+1}$ $= 5e^{2x+1}(2x+1)$	M1 A1
(b)	$\int(10xe^{2x+1} + 5e^{2x+1})dx = 5xe^{2x+1}$ $\int 10xe^{2x+1}\,dx = 5xe^{2x+1} - \int 5e^{2x+1}\,dx$ $\int xe^{2x+1}\,dx = \frac{1}{10}\left[5xe^{2x+1} - \frac{5}{2}e^{2x+1}\right] + c$ $= \frac{1}{2}xe^{2x+1} - \frac{1}{4}e^{2x+1} + c$	M1 M1 A1
(c)	For decreasing function of *x*, $\frac{dy}{dx} < 0$ $5e^{2x+1}(2x+1) < 0$ Since $5e^{2x+1} > 0, \therefore (2x+1) < 0$ $\therefore x < -\frac{1}{2}$	 M1 A1
4(a)	$y = x^2 + x + 2k - 4$ ------(1) $y - 2x = k$ -----(2) Subst (1) into (2): $x^2 + x + 2k - 4 - 2x = k$ $\therefore x^2 - x + k - 4 = 0$ For the line and curve do not intersect, $b^2 - 4ac < 0$ $(-1)^2 - 4(1)(k-4) < 0$ $1 - 4k + 16 < 0$ $-4k < -17$ $k > 4\frac{1}{4}$ [Number line – A1]	 M1 M1 M1 A1 A1

(b)	$y = ax^2 + 8x + c$ For the curve lies completely below the *x*-axis, it must be a "n" shape curve. $\therefore a < 0$ Also, $8^2 - 4ac < 0$ $-4ac < -64$ $\therefore ac > 16$	B1 M1 A1
5(a)(i)	$\frac{dy}{dx} = -5(2-x)^{-2}(-1)$ $= \frac{5}{(2-x)^2}$ When x = 1, $\frac{dy}{dx} = 5$	M1 A1
(a)(ii)	Grad of tangent = 5 Grad of normal = $-\frac{1}{5}$ Eqn of normal is: $y - 2 = -\frac{1}{5}(x-1)$ $5y - 10 = -x + 1$ $5y + x - 11 = 0$	M1 M1 A1
(b)	$Since\ (2-x)^2 \geq 0,$ $\frac{dy}{dx} = \frac{5}{(2-x)^2} \geq 0$ There are no point on the curve whereby $\frac{dy}{dx} = 0$. ∴ a stationary point will not exist on the curve.	B1
(c)	Given: $\frac{dx}{dt} = 0.2\ units/\sec$ When y = -4, $\frac{5}{2-x} - 3 = -4$ $x = 7$	 M1

	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $=\frac{5}{(2-x)^2}\times 0.2$ When x = 2, $\frac{dy}{dt}=\frac{5}{(2-7)^2}\times 0.2$ $=\frac{1}{25}$ (*Accept* 0.04)	M1 A1
6(a)	$\frac{\cos\frac{\pi}{6}-\tan\frac{\pi}{4}}{\sin\frac{\pi}{3}+\cos\frac{\pi}{3}}=\frac{\frac{\sqrt{3}}{2}-1}{\frac{\sqrt{3}}{2}+\frac{1}{2}}$ $=\frac{\frac{\sqrt{3}-2}{2}}{\frac{\sqrt{3}+1}{2}}$ $=\frac{\sqrt{3}-2}{\sqrt{3}+1}$ a = -2, b = 1	M1 A1, A1
(b)(i)	$\sin A=-\frac{5}{13}$ and $\cos B=\frac{4}{5}$ $\cos A=\frac{12}{13}$, $\sin B=-\frac{3}{5}$ $\sin(A+B)=\sin A\cos B+\cos A\sin B$ $=\left(-\frac{5}{13}\right)\left(\frac{4}{5}\right)+\left(\frac{12}{13}\right)\left(-\frac{3}{5}\right)$ $=-\frac{56}{65}$	M1 M1 A1

(b)(ii)	$\sec 2B = \dfrac{1}{\cos 2B}$ $= \dfrac{1}{2\cos^2 B - 1}$ $= \dfrac{1}{2\left(\dfrac{4}{3}\right)^2 - 1}$ $= \dfrac{1}{\left(\dfrac{7}{25}\right)}$ $= \dfrac{25}{7}$	M1 M1 A1
7i	$d = t^3 - 6t^2 + 9t$ for $t \geq 0$. $v = 3t^2 - 12t + 9$ $\dfrac{dv}{dt} = 6t - 12$ When t = 10, $v = 3(10)^2 - 12(10) + 9$ = 189 m/s When t = 10, $a = 6(10) - 12$ = 48 m/s^2	M1 M1 M1 A1 M1 A1
7ii	$3t^2 - 12t + 9 = 0$ $t^2 - 4t + 3 = 0$ $(t-1)(t-3) = 0$ $t = 1$ or $t = 3$	M1 M1 A1
7iii	$d = t^3 - 6t^2 + 9t$ When t = 0, d = 0 m When t = 1, d = 4 m When t = 3, d = 0 m When t = 4, d = 4 m Distance travelled = 4×3 = 12 metres	M2 A1
8i	$\dfrac{13 - \sqrt{48}}{2 - \sqrt{3}} \times \dfrac{2 + \sqrt{3}}{2 + \sqrt{3}}$	M1 M1

	$= \dfrac{\left(13-4\sqrt{3}\right)\left(2+\sqrt{3}\right)}{4-3}$ $= 26-13\sqrt{3}-8\sqrt{3}-12$ $= \left(14+5\sqrt{3}\right)$ cm	M1 A1
8ii	$\left(c+2\sqrt{3}\right)\left(c+2\sqrt{3}\right)=13-4\sqrt{3}$ $c^2+12+4c\sqrt{3}=13-4\sqrt{3}$ Comparing coefficients, $4c = -4$ $c = -1$	M1 M1 A1
9i	After 1 year, $P=120888e^{-0.015(12)}$ P = \$ 100 974.15 = \$ 101 000 (to the nearest \$1000)	M1 A1
9ii	$120888e^{-0.015t}<50000$ $e^{-0.015t}<\dfrac{50000}{120888}$ $-0.015t<\ln\left(\dfrac{50000}{120888}\right)$ $t>\ln\left(\dfrac{50000}{120888}\right)\div(-0.015)$ t > 58.856 months t > 4 years 11 months The month and year is November 2025	M1 M1 M1 A1
10i	Midpoint of AC = $\left(\dfrac{3+4}{2},\dfrac{5-2}{2}\right)=(3.5,1.5)$ Gradient of AC = $\dfrac{5+2}{3-4}=-7$ Equation of perpendicular bisector is $y=\dfrac{1}{7}x+c$ $1.5=\dfrac{1}{7}(3.5)+c$ c = 1 Hence equation of perpendicular bisector is $y=\dfrac{1}{7}x+1$	M1 A1
10ii	B has the coordinates (0, 1)	B1
10iii	Gradient of AB = Gradient of CD = $\dfrac{4}{3}$	

	Gradient of *AD* = Gradient of *BC* = $-\frac{3}{4}$ The sides are parallel and adjacent sides intersect at 90° Length of *AB* = Length of *AD* = $\sqrt{3^2+4^2}=5$ units The sides are of equal lengths Since the sides are parallel and are of equal length and $AB \perp BC$ and $AD \perp CD$, *ABCD* is a square.	M1 M1 A1
12i	$(1+4x)^7$ $=1+28x+\binom{7}{2}(4x)^2$ $= 1 + 28x + 336x^2 + \ldots$	M1 A2
12ii	$(2-x)^7$ $=2^7-7(2^6)(x)+\binom{7}{2}(2^5)(x^2)$ $= 128 - 448x + 672x^2 + \ldots$	M1 A2
12iii	$(2+7x-4x^2)^7=(1+4x)^7(2-x)^7$ $= (1 + 28x + 336x^2 + \ldots)(128 - 448x + 672x^2 + \ldots)$ Coefficient of x^2 $= 336\times128 + 28\times(-448) + 672\times1$ $= 31136$	M1 A1