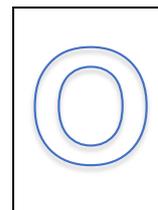




CANBERRA SECONDARY SCHOOL



## 2021 Preliminary Examination

### Secondary Four Express

#### ADDITIONAL MATHEMATICS

4049/01

26 Aug 2021

2 hours 15 minutes

1000h – 1215h

Name: \_\_\_\_\_ (     )     Class: \_\_\_\_\_

#### READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number on all work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR MARKER'S USE		
	Marks Awarded	Max Marks
Total		90

---

This question paper consists of 17 printed pages including the cover page.

**Mathematical Formulae****1. ALGEBRA***Quadratic Equation*

For the equation  $ax^2 + bx + c = 0$ ,  $a \neq 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

*Binomial expansion*

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$ .

**2. TRIGONOMETRY***Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for  $\triangle ABC$* 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1 Given that  $y = p \sin 2x + q$ , where  $p$  and  $q$  are positive integers. The maximum and minimum values of  $y$  are 8 and -2 respectively.
- (a) Find the value of  $p$  and of  $q$ . [2]

- (b) Hence, sketch the graph of  $y$  for  $0^\circ \leq x \leq 360^\circ$ . [3]

- 2 (a) Express each of  $4x^2 - 6x - 1$  and  $-x^2 - 10x + 3$  in the form  $a(x+b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are constants. [4]

- (b) Using your answers in (a), show that the curves  $y = 4x^2 - 6x - 1$  and  $y = -x^2 - 10x + 3$  will intersect. [3]

3 The equation of a curve is given by  $y = 5xe^{2x+1}$ .

(a) Find  $\frac{dy}{dx}$ . [2]

(b) Hence, find  $\int xe^{2x+1} dx$ . [3]

(c) Determine the range of values of  $x$  for which  $y$  is a decreasing function. [2]

- 4 (a) A curve has equation  $y = x^2 + x + 2k - 4$  and a line has equation  $y - 2x = k$ , where  $k$  is a constant. Find the set of values of  $k$  for which the line and curve do not intersect and represent this set on a number line. [5]

- (b) Given that the curve with equation  $y = ax^2 + 8x + c$ , where  $a$  and  $c$  are constants, lies completely below the  $x$ -axis. Write down the conditions which must apply to  $a$  and  $c$ . [3]

5 Given that a point P(1, 2) lies on the curve  $y = \frac{5}{2-x} - 3$ , find

(a) (i) the value of  $\frac{dy}{dx}$  at point P, [2]

(ii) the equation of the normal to the curve at point P. [3]

(b) Explain whether a stationary point will exist on the curve. [1]

- (c) If  $x$  is increasing at a rate of 0.2 units per second, find the rate of change of  $y$  at the instant when  $y = -4$ . [3]

6 (a) Given that  $\frac{\cos \frac{\pi}{6} - \tan \frac{\pi}{4}}{\sin \frac{\pi}{3} + \cos \frac{\pi}{3}} = \frac{\sqrt{3} + a}{\sqrt{3} + b}$ , where  $a$  and  $b$  are integers.

Find the values of  $a$  and  $b$ .

[3]

(b) Given that  $\sin A = -\frac{5}{13}$  and  $\cos B = \frac{4}{5}$ , where  $A$  and  $B$  are in the same quadrant. Find the exact value of the following:

(i)  $\sin(A+B)$  [3]

(ii)  $\sec 2B$  [3]

- 7 A dot on a computer screen moves in a straight line and passes through a fixed point  $A$ . The distance,  $d$  metres, that it runs in  $t$  s after it passes through  $A$  is given by

$$d = t^3 - 6t^2 + 9t \text{ for } t \geq 0.$$

- (i) Find the dot's speed and acceleration 10 seconds after it passes  $A$ . [6]

- (ii) Find the values of  $t$  at which the dot is instantaneously at rest. [3]

- (iii) Find the distance travelled by the dot in the first 4 seconds. [3]

**8** The area of a quadrilateral is  $(13 - \sqrt{48}) \text{ cm}^2$ .

- (i)** In the case where the quadrilateral is a rectangle with a width of  $(2 - \sqrt{3}) \text{ cm}$ , find, without the use of a calculator, the length of the rectangle in the form of  $(a + b\sqrt{3}) \text{ cm}$ .

[4]

- (ii)** In the case where the quadrilateral is a square with side  $(c + 2\sqrt{3}) \text{ cm}$ , find, without the use of a calculator, the value of the constant  $c$ . [3]

- 9** The price of a new car on 1 January 2021 is \$120 888.00. Given that the value of the car depreciates in such a way that,  $n$  months after the purchase, the sale price,  $\$P$ , is determined by the formula

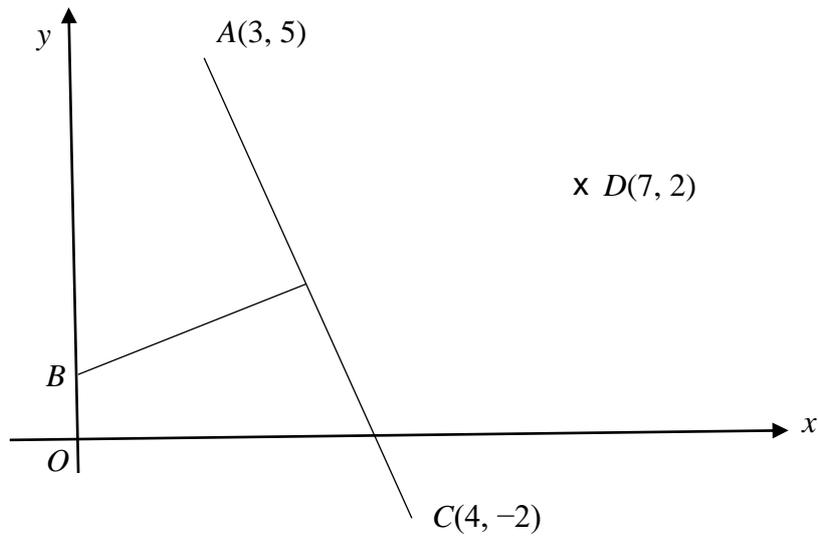
$$P = 120888e^{-0.015n},$$

estimate

- (i) the sale price of the car after 1 year, giving your answer correct to the nearest \$1000, [2]

- (ii) the month and year when the sale price of the car is less than \$50 000. [4]

10



The figure shows three points  $A$ ,  $C$  and  $D$  whose coordinates are  $(3, 5)$ ,  $(4, -2)$  and  $(7, 2)$  respectively.

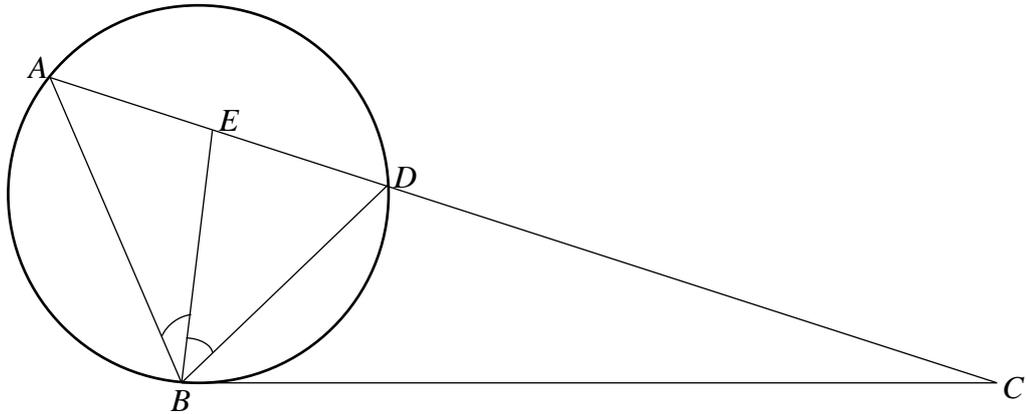
(i) Find the equation of the perpendicular bisector of  $AC$ . [2]

(ii) Find the coordinate of  $B$ , the point where the perpendicular bisector cuts the y-axis. [1]

(iii) Show that  $ABCD$  is a square.

[3]

- 11** In the diagram,  $AC$  is a straight line intersecting a circle at  $A$  and  $D$ . The point  $B$  lies on the circle and  $BC$  is a tangent to the circle. The point  $E$  lies on  $AC$  such that the line  $BE$  bisects angle  $ABD$ .



- (i) State a reason why angle  $DBC$  and angle  $BAD$  are equal. [1]
- (ii) Show that angle  $EBC =$  angle  $BEC$ . [3]
- (iii) Hence name an isosceles triangle and explain why it is so. [2]

- 12** Find, in ascending powers of  $x$ , the first three terms in the expansion of
- (i)  $(1+4x)^7$ , [3]

(ii)  $(2-x)^7$ . [3]

Hence, find the coefficient of  $x^2$  in the expansion of  $(2+7x-4x^2)^7$ . [2]

**- End of paper -**

## Answers

<b>1(a)</b>	$P = 5, q = 3$
<b>2(a)</b>	$-(x+5)^2 + 28$
<b>3(a)</b>	$5e^{2x+1}(2x+1)$
<b>(b)</b>	$\frac{1}{2}xe^{2x+1} - \frac{1}{4}e^{2x+1} + c$
<b>(c)</b>	$x < -\frac{1}{2}$
<b>4(a)</b>	$k > 4\frac{1}{4}$
<b>(b)</b>	$a < 0$ and $ac > 16$
<b>5(a)(i)</b>	When $x = 1, \frac{dy}{dx} = 5$
<b>(a)(ii)</b>	$5y + x - 11 = 0$
<b>(b)</b>	Since $(2-x)^2 \geq 0,$ $\frac{dy}{dx} = \frac{5}{(2-x)^2} \geq 0$
<b>(c)</b>	$\frac{1}{25}$
<b>6(a)</b>	$a = -2, b = 1$
<b>(b)(i)</b>	$-\frac{56}{65}$
<b>(b)(ii)</b>	$\frac{25}{7}$
7i	$48 \text{ m/s}^2$
7ii	$t = 1$ or $t = 3$
7iii	12 metres
8i	$(14 + 5\sqrt{3}) \text{ cm}$
8ii	$c = -1$
9i	\$ 101 000 (to the nearest \$1000)
9ii	The month and year is November 2025
10i	Equation of perpendicular bisector is $y = \frac{1}{7}x + 1$
10ii	$B$ has the coordinates $(0, 1)$
12i	$1 + 28x + 336x^2 + \dots$
12ii	$128 - 448x + 672x^2 + \dots$
12iii	31136