MARKING SCHEME

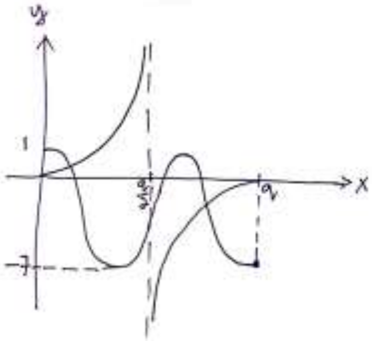
Secondary Four Express

2021 Preliminary Examination Additional Mathematics Paper 1

1	LHS = $\operatorname{cosec} 2\theta + \cot 2\theta$ $= \dfrac{1}{\sin 2\theta} + \dfrac{\cos 2\theta}{\sin 2\theta}$	**4**	**B1**	cosec and cot both correct
	$= \dfrac{1+\cos 2\theta}{2\sin\theta\cos\theta}$		**M1**	Uses $\sin 2\theta = 2\sin\theta\cos\theta$
	$= \dfrac{1+\left(2\cos^2\theta - 1\right)}{2\sin\theta\cos\theta}$		**M1**	Uses $\cos 2\theta = 2\cos^2\theta - 1$
	$= \dfrac{2\cos^2\theta}{2\sin\theta\cos\theta} \quad = \dfrac{\cos\theta}{\sin\theta} \quad = \cot\theta = RHS$		**A1**	All correct
2	$x^2 + (2-3k)x + k^2 + 1 = 0$	**4**	**M1**	Eliminates y
	$D = (2-3k)^2 - 4\left(k^2+1\right)$ $= 5k^2 - 12k$		**M1**	Uses the discriminant
	$k(5k-12) < 0$		**M1**	Correct inequality
	$0 < k < \dfrac{12}{5}$		**A1**	
3	$\dfrac{dy}{dx} = 2x - \dfrac{4}{(7-3x)} + c \quad or \quad 2\left(x - \dfrac{2}{(7-3x)}\right) + c$	**6**	**B2, 1**	Minus 1 for each error (minus 1 mark if arbitrary constant c is missing; minus once only)
	$0 = 4 - \dfrac{4}{(7-6)} + c \quad or \quad 2\left(2 - \dfrac{2}{(7-6)}\right) + c$ $\rightarrow \quad c = 0$		**M1**	Uses $\dfrac{dy}{dx} = 0$ when $x = 2$ (must show steps clearly)
	Integrates again $\rightarrow \quad y = x^2 + \dfrac{4}{3}\ln(7-3x) + d$		**FT B2, 1**	Minus 1 for each error FT from $\dfrac{dy}{dx}$ (No FT mark if nature of the qn is altered or the level of difficulty of the qn is reduced/changed)
	$5 = 2^2 + \dfrac{4}{3}\ln(7-6) + d \quad \rightarrow \quad d = 1$ $\therefore \quad y = x^2 + \dfrac{4}{3}\ln(7-3x) + 1$		**A1**	Uses $S(2,\ 5)$ (and get the answer correctly)

4a	Since $\cos\left(A+B\right)=-\frac{36}{85}<0$ $\Rightarrow$ angle $\left(A+B\right)$ is obtuse	**1**	**A1**	(with correct reasoning)
4bi	$\cos\left(A+B\right)=\cos A\cos B-\sin A\sin B$ $-\frac{36}{85}=\frac{24}{85}-\sin A\sin B \quad \rightarrow \quad \sin A\sin B=\frac{60}{85}=\frac{12}{17}$	**2**	**M1**	Using addition formula for cos
	$\tan A\tan B=\frac{\frac{60}{85}}{\frac{24}{85}}=\frac{5}{2}$		**A1**	
4bii	Since $\cos\left(A+B\right)=-\frac{36}{85} \quad \rightarrow \quad \tan\left(A+B\right)=-\frac{77}{36}$	**3**	**M1**	
	$\tan\left(A+B\right)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ $-\frac{77}{36}=\frac{\tan A+\tan B}{1-\frac{5}{2}} \quad \Rightarrow \quad \tan A+\tan B=\frac{77}{24}$		**M1** **A1**	Using addition formula for tan

5	Integrating curve: $\int_0^{\frac{1}{2}}\sin\pi x-\frac{3}{2}\ \mathrm{d}x=\left[\frac{-\cos\pi x}{\pi}-\frac{3}{2}x\right]_0^{\frac{1}{2}}$	**6**	**M1** **A1**	Knowing to integrate A1 can be implied
	Integrating line: $\int_0^{\frac{1}{2}}\frac{-3x-1}{5}\,\mathrm{d}x=\left[-\frac{3x^2}{10}-\frac{x}{5}\right]_0^{\frac{1}{2}}$		**M1** **A1**	Knowing to integrate A1 can be implied
	Shaded Area $=\left(-\left[\frac{-\cos\pi x}{\pi}-\frac{3}{2}x\right]_0^{\frac{1}{2}}\right)-\left(-\left[-\frac{3x^2}{10}-\frac{x}{5}\right]_0^{\frac{1}{2}}\right)$ *or* $=-\left(\left[\frac{-\cos\pi x}{\pi}-\frac{3}{2}x\right]_0^{\frac{1}{2}}-\left[-\frac{3x^2}{10}-\frac{x}{5}\right]_0^{\frac{1}{2}}\right)$		**M1**	Overall Method: Area of curve – Area of line; Need to be aware that area under curve is negative (must include correct limits)
	$=\frac{3}{4}-\frac{1}{\pi}-\frac{7}{40} \quad =\frac{23}{40}-\frac{1}{\pi}$		**A1**	Correct evaluation of limits

6a	$a=4,\quad c=-3,\quad q=\pi$	**3**	**B3, 2, 1**	Minus 1 for each error
6bi		**3**	**B1**	Correct Shape
			B1	Correct Period
	Number of solutions = 2		**A1**	

7a	$-2x^2-2x-3= \quad -2\left(x+\frac{1}{2}\right)^2-\frac{5}{2}$	**2**	**B2, 1**	Minus 1 for each error
7b	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(2x+3)e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right) \;-\; 2e^{\frac{1}{x}}}{(2x+3)^2}$	**5**	**B1**	Correct derivative, unsimplified $\frac{\mathrm{d}}{\mathrm{d}x}e^{\frac{1}{x}}=e^{\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right)$
			M1	Quotient rule used correctly
	$=\frac{e^{\frac{1}{x}}\left(-\frac{2x+3}{x^2}-2\right)}{(2x+3)^2}=\frac{e^{\frac{1}{x}}\left(-2x^2-2x-3\right)}{x^2(2x+3)^2}$		**A1**	
	Since $e^{\frac{1}{x}},\ x^2,\ (2x+3)^2>0,$ and $-2x^2-2x-3\ =-2\left(x+\frac{1}{2}\right)^2-\frac{5}{2}\ <0$		**M1**	Argues correctly (with link to part (a) because the word “Hence” is used.
	$\frac{\mathrm{d}y}{\mathrm{d}x}<0 \quad\Rightarrow\quad y$ is a decreasing function		**A1**	

8a	$P(1,\ 0)$	**1**	**B1**	
8b	Midpoint of PR = $\left(\frac{1+3}{2},\ \frac{0-1}{2}\right)=\left(2,-\frac{1}{2}\right)$	**4**	**M1**	Midpoint of PR
	Let $S(x,\ y)$ $\left(\frac{x+2}{2},\ \frac{y+2}{2}\right)=\left(2,-\frac{1}{2}\right)$		**M1**	Uses the property of parallelogram
	$x=2,\quad y=-3\quad\rightarrow\quad S(2,-3)$		**A1**	(accept the use of translation)
	Distance between S to centre of circle $=\sqrt{(2-1)^2+(-3)^2}\quad=\sqrt{10}>\sqrt{5}$ Hence, S lies outside the circle		**A1**	Argues correctly
8c	Area of parallelogram $=\frac{1}{2}\begin{vmatrix}1 & 2 & 3 & 2 & 1\\ 0 & -3 & -1 & 2 & 0\end{vmatrix}$ $=\frac{1}{2}\{1(-3)+2(-1)+3(2)+2(0)-1(2)-2(-1)-3(-3)-2(0)\}$	**2**	**M1**	e.c.f. Correct use of shoelace method
	$=\frac{1}{2}(10)\quad=5$ square units		**A1**	

9a	When $t=0,\quad T=-10$ $-10=30-Ae^{0}\quad\rightarrow\quad A=40$	**1**	**B1**	Uses initial temperature
9b	$23.4=30-40e^{-60k}$ $e^{-60k}=0.165$ $-60k=\ln 0.165$	**2**	**M1**	Takes logarithm
	$k=0.03003\quad=0.0300$		**A1**	
9c	$30-40e^{-0.03003t}\geq 20$ $e^{-0.03003t}\leq\frac{1}{4}$	**3**	**M1**	Makes $e^{-0.03003t}$ the subject (ignore the inequality sign if student uses equation but must link the idea back to inequality at the end of the solution; **"at least")**
	$-0.03003t\leq\ln\frac{1}{4}$		**M1**	Takes logarithm correctly
	$t\geq 46.16\quad\rightarrow\quad n=47$		**A1**	
9d	As $t\rightarrow$ a large number, $\quad e^{-0.003003t}\rightarrow 0$	**2**	**B1**	Describes the exponential behaviour for large numbers
	$\therefore T\rightarrow 30$		**B1**	

10a	$\frac{8\sqrt{2}-6}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$	**3**	**M1**	For rationalising denominator
	$= 22-14\sqrt{2}$		**A1,** **A1**	Correct p Correct q
10b	$\left(a+\sqrt{3}\right)\left(4-b\sqrt{3}\right)=\left(14-6\sqrt{3}\right)$ $(4a-3b)+(4-ab)\sqrt{3}=14-6\sqrt{3}$ $\Rightarrow \begin{array}{l} 4a-3b=14 \\ 4-ab=-6 \end{array}$	**5**	**M1**	Comparing coefficients
	$\Rightarrow a=\frac{10}{b}$ or $\Rightarrow b=\frac{10}{a}$ $\frac{40}{b}-3b=14$ or $4a-\frac{30}{a}=14$		**M1**	Solving non-linear equation by eliminating a or b
	$3b^2+14b-40=0$ or $4a^2-14a-30=0$ $(b-2)(3b+20)=0$ or $(a-5)(2a+3)=0$		**M1**	Solving Quadratic
	$a=5, b=2$ or $a=-\frac{3}{2}, b=-\frac{20}{3}$		**A1**	
	When $a=-\frac{3}{2}, b=-\frac{20}{3}$ $\rightarrow$ Speed $\approx 0.232 km/h$, Time $\approx 15.5h$ which is not realistic.		**A1**	Argues correctly

11a	$2700\pi=\frac{2}{3}\pi r^3+\pi r^2 h$	**4**	**M1**	Using volume to express h in terms of r
	$h=\frac{2700-\frac{2}{3}r^3}{r^2} \quad \Rightarrow \quad h=\frac{2700}{r^2}-\frac{2}{3}r$		**A1**	Correct expression for h, unsimplified
	$A=2\pi r^2+2\pi r\left(\frac{2700}{r^2}-\frac{2}{3}r\right)$		**B1**	Correct expression for area
	$=\frac{5400\pi}{r}+\frac{2}{3}\pi r^2$ (*shown*)		**A1**	Answer was given, so all workings must be correct
11b	$\frac{\mathrm{d}A}{\mathrm{d}r}=-\frac{5400\pi}{r^2}+\frac{4}{3}\pi r$	**4**	**B1**	Correct differentiation
	$-\frac{5400\pi}{r^2}+\frac{4}{3}\pi r=0$ $r^3=4050$		**M1**	Sets to 0 and solves
	$r=15.93988 \quad \approx 15.94 \quad \approx 15.9\text{cm}$		**A1**	Correct value of r

	$A=\frac{5400\pi}{15.94}+\frac{2}{3}\pi(15.94)^2$ $=1596.43 \quad \approx 1600\text{cm}^2$		**A1**	Finding corresponding value of A

12a	$\frac{QA}{AP}=\frac{1}{2}$ *or* $\frac{QA}{QP}=\frac{1}{3}$	**4**	**M1**	Using proportion or similar triangles or finding equation of line $L_1: Y=\frac{1}{2}X+3$
	$\therefore A(0,3)$		**A1**	Coordinates of A
	At A, $x-y=0 \quad \rightarrow \quad x=y$ $xy=3 \quad \rightarrow \quad x^2=3$		**M1**	Comparing coordinates with axes
	$\Rightarrow \begin{array}{l} x=\sqrt{3}, \quad y=\sqrt{3} \\ x=-\sqrt{3}, \quad y=-\sqrt{3} \end{array}$		**A1**	
12b	Gradient of L_1 = $\frac{4-1}{2-(-4)}=\frac{1}{2}$	**6**	**B1**	
	Midpoint = $\left(\frac{-4+2}{2},\frac{1+4}{2}\right) = \left(-1, \frac{5}{2}\right)$		**B1**	
	Gradient of L_2 = –2		**M1**	Using $m_1m_2=-1$
	$L_2: \ Y-\frac{5}{2}=-2(X+1) \Rightarrow Y=-2X+\frac{1}{2}$		**M1**	
	$xy=-2(x-y)+\frac{1}{2} \Rightarrow 2xy=-4x+4y+1$ $4y-2xy=4x-1$		**M1**	Grouping y terms on one side of the equation
	$y=\frac{4x-1}{4-2x}$		**A1**	

13a	$\left(1-\frac{x}{2}\right)^6 \approx 1-3x+\frac{15}{4}x^2$	**2**	**B2, 1**	Minus 1 for each error
13b	Term independent of x = $9-12k+\frac{15}{4}k^2$	**5**	**B1**	
	$9-12k+\frac{15}{4}k^2>0$ $5k^2-16k+12>0$ $(5k-6)(k-2)>0$		**M1**	Factorisation
	$k<\frac{6}{5}$ or $k>2$		**A1,** **A1**	A1 for correct values A1 for correct inequality
			FTB1	Correct diagram for range of values obtained
13c	$T_{r+1}=\binom{99}{r}x^{99-r}\left(\frac{2}{x}\right)^r$	**3**	**M1**	Applying binomial general term
	$T_{r+1}=\binom{99}{r}2^r x^{99-2r}$		**A1**	Combining the x
	Since 99 is odd and $2r$ is even, 99 – $2r$ will always be odd. Hence, all the powers of x are odd.		**A1**	Argues correctly

~~~ End of Paper 1 ~~~
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