CHIJ ST. THERESA'S CONVENT
PRELIMINARY EXAMINATION 2021
SECONDARY 4 EXPRESS

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/2**

Paper 2

27 Aug 2021
2 hours 15 minutes

Candidates answer on the Question Paper as well as on the graph paper provided.

READ THESE INSTRUCTIONS FIRST

Write your index number, and name on all the work you hand in.
Write in dark blue or black ink.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answers in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions.

1(a) Show that the curve $y = -3x^2 + kx + 2$ intersects the line $y = x - 5$ for all real values of k. [3]

(b) Determine the values of k for the line $y = x + 5$ to be a tangent to the curve $y = -3x^2 + kx + 2$. [3]

(c) Solve the equation $x = \sqrt{1-x} - 5$, showing clearly if the answers obtained are correct. [4]

2 When the polynomial $\mathrm{f}(x) = 2x^3 + ax^2 + bx + 4$ is divided by $x - 2$, the quotient and the remainder are $\mathrm{Q}(x)$ and R respectively, where a, b and R are constants.

(i) Form an equation relating $\mathrm{f}(x)$, $\mathrm{Q}(x)$ and R. [1]

It is given that $R = 8$.
It is also given that the gradient of the curve $y = \mathrm{f}(x)$ at the point $x = -1$ is 0.

(ii) Show that $a = 0$ and $b = -6$. [4]

(iii) Factorise $\mathrm{f}(x)$ completely. [3]

3(a) Solve $(\lg x)^2 = 3 - 2\lg x$. [4]

(b) Use a change of base to find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\lg 3x^2\right)$. [3]

(c) **(i)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(e^x \sin x\right)$. [2]

(ii) Hence find $\int_0^{\frac{\pi}{3}} e^x(\sin x + \cos x)\,\mathrm{d}x$. [2]

4(i) Find the coordinates of the stationary point of the curve $y = \dfrac{21 - 20x}{x^2 - 2x + 1}$ where $x \neq 1$. [5]

(ii) Use the **First Derivative Test** to determine the nature of this stationary point. [3]

5(i) Given that $\left(\dfrac{x}{x-1}\right)^2 = A + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$, where A, B and C are constants, find the value of A, of B and of C. [4]

(ii) Hence find $\int_2^3 \left(\dfrac{x}{x-1}\right)^2 \mathrm{d}x$. [3]

6 The variables x and y are related by the equation $y = 10^{-a} b^{x}$, where a and b are constants. The table below shows values of x and y.

x	0	5	10	15	20	25
y	0.16	0.4	0.8	1.6	3.3	6.5

(i) Plot $\lg y$ against x for the given data and draw a straight line graph. [2]

(ii) Use your graph to estimate

(a) the value of a and of b, [3]

(b) the value of x when $y = 4$. [1]

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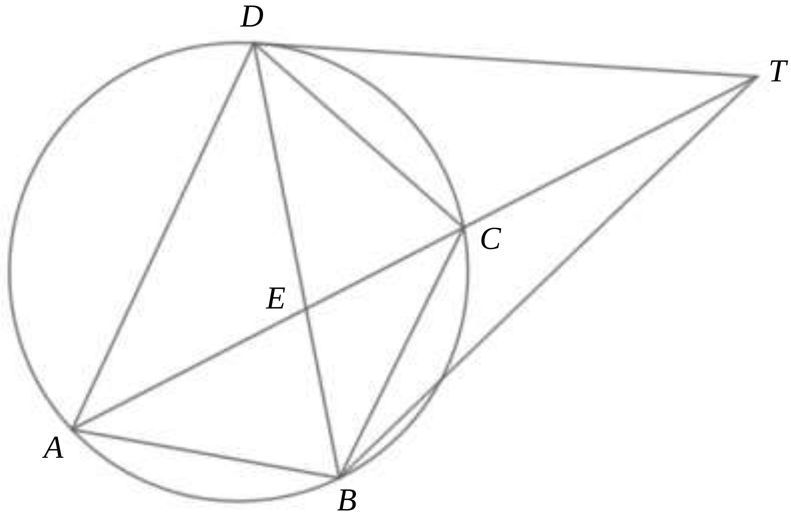

The diagram shows a point D on a circle, and TD is a tangent to the circle. Points A, B, and C lie on the circle such that DC bisects angle TDB and TCA is a straight line. The lines TA and DB intersects at E.

(i) Prove that $CB = CD$. [3]

(ii) Prove that EA bisects angle DAB. [4]

8(i) Express $4\sin 2x + 3\cos 2x + 1$ in the form $R\sin(2x+\alpha) + k$, where R, α and k are constants with $R > 0$ and α an acute angle. [2]

(ii) Solve $4\sin 2x + 3\cos 2x + 1 = 0$ for $0 \le x \le 180°$. [4]

(iii) Sketch, in the same diagram, the graphs of $y = 4\sin 2x + 1$ and $y = -3\cos 2x$ for $0 \le x \le 180°$.
[4]

(iv) Find the coordinates of the points of intersection of the two graphs. [2]

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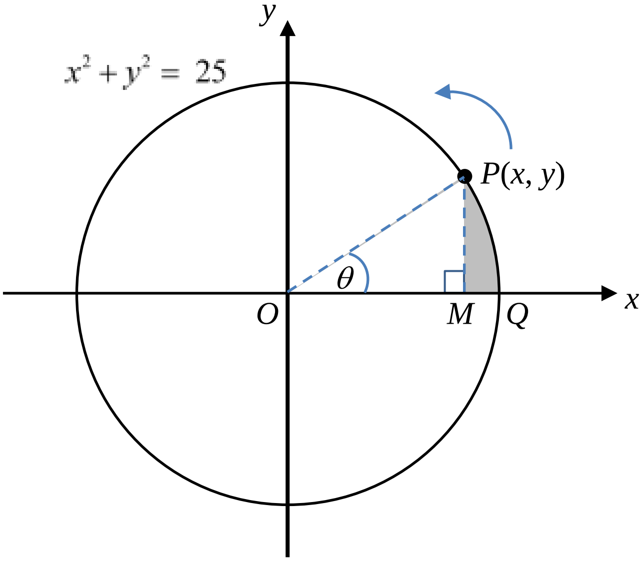

The point $P(x, y)$ starts from the point Q with coordinates (5, 0) and it moves along the circle $x^2 + y^2 = 25$ in an anticlockwise sense. The angle QOP is θ and it increases at a constant rate of 0.04 radians per second.

The point M on the x-axis is such that PM is perpendicular to the x-axis.

(i) Find the area of triangle POM, giving your answer in the form $k \sin 2\theta$ where k is a constant. [4]

(ii) The area of the shaded region PMQ is A unit^2. Obtain an expression for A in terms of θ. [2]

(iii) Find, in terms of θ, the rate at which A is changing. [3]

(iv) Hence find the **exact** value of this rate when P passes through the point (4, 3). [2]

10 A particle moves in a straight line so that t seconds after passing through a fixed point O, its velocity v m/s is given by $v = 7\sin 2t$ where $0 \le t \le \pi$. Find

(i) the initial acceleration of the particle, [2]

(ii) the value of t when the particle first comes to instantaneous rest, [1]

(iii) the maximum displacement of the particle, [3]

(iv) the distance travelled by the particle in the first 2.5 seconds after passing through *O*. [4]

~ ~ End of Paper 2 ~ ~