CHIJ ST. THERESA'S CONVENT
PRELIMINARY EXAMINATION 2021
SECONDARY 4 EXPRESS

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS

4049/01

Paper 1

31 August 2021

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black ink.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions.

1 Prove the identity $\operatorname{cosec} 2\theta + \cot 2\theta = \cot\theta$. [4]

2 Find the range of values of the constant k for which the curve $y = x^2 + (2-3k)x + k^2$ lies entirely above the line $y = -1$. [4]

3 A curve has a stationary point at $S(2,\ 5)$, and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\left(1 - \dfrac{6}{(7-3x)^2}\right)$.

Find the equation of the curve. [6]

4 Angles A and B are both acute, $\cos(A+B) = -\frac{36}{85}$ and $\cos A \cos B = \frac{24}{85}$.

Without using a calculator,

(a) explain why angle $(A+B)$ is acute, obtuse or reflex, giving reason(s) for your answer. [1]

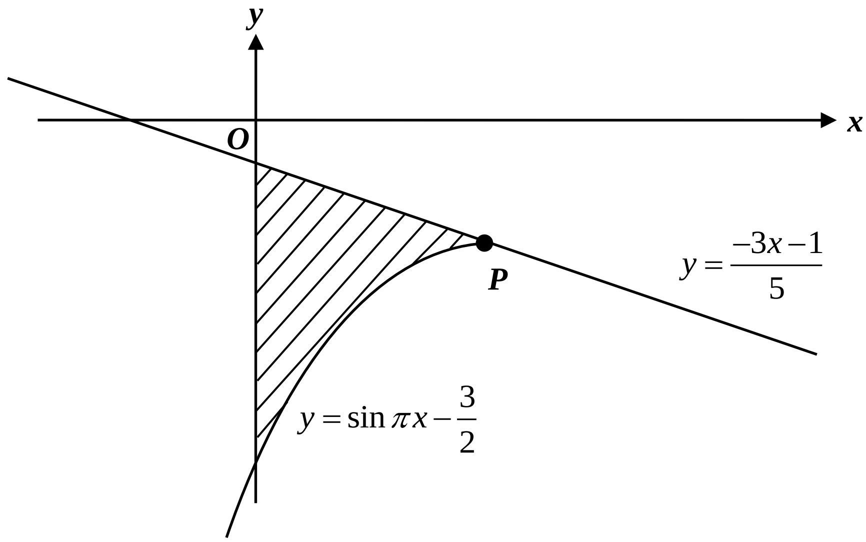
(b) find

(i) $\tan A \tan B$, [2]

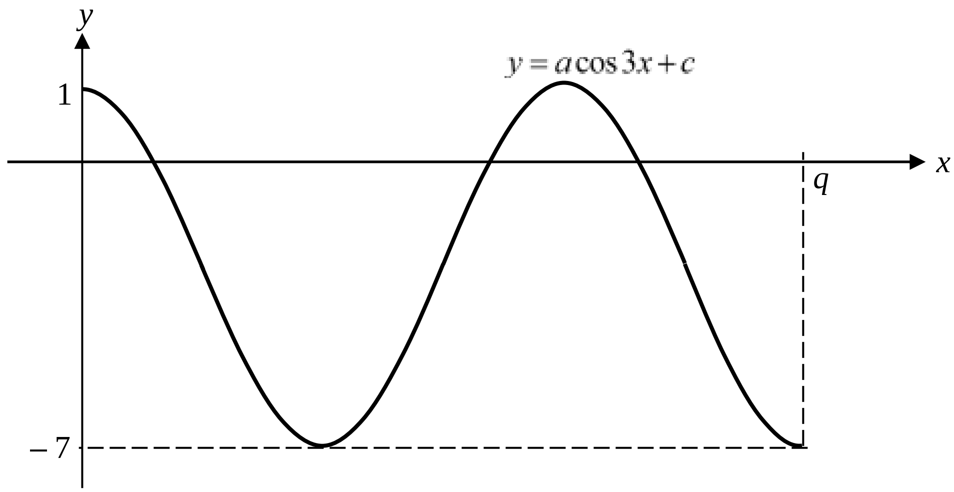
(ii) $\tan A + \tan B$. [3]

5 The diagram shows part of the curve $y = \sin \pi x - \frac{3}{2}$. The line $y = \frac{-3x-1}{5}$ cuts the curve at point P.

Given that the curve and line intersect at $P\left(\frac{1}{2}, -\frac{1}{2}\right)$, find the exact area of the shaded region bounded by the curve, line and y- axis. Show all your working clearly. [6]

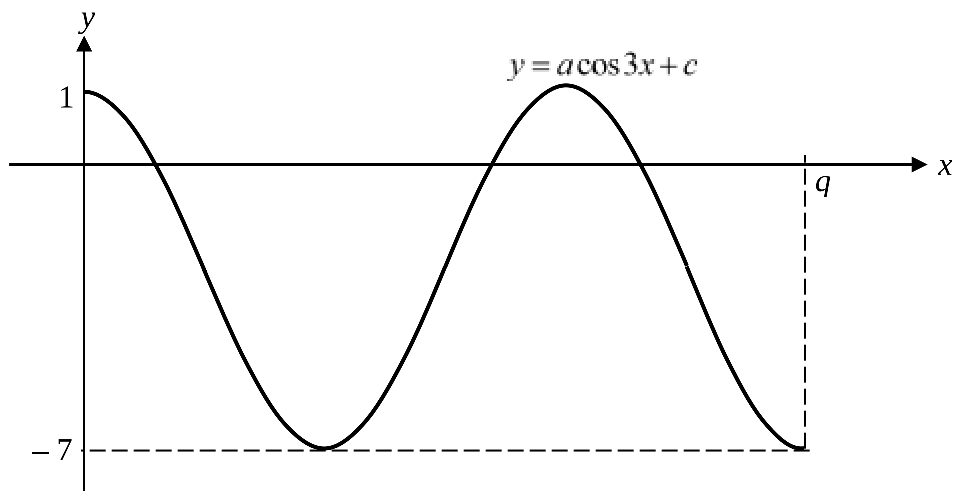


6 The diagram below shows the graph of $y = a\cos 3x + c$, for $0 \le x \le q$, where q is in radians.



(a) Find the value of each of the constants a, c and q. [3]

(b) By drawing an additional curve on the axis below, state the number of solutions to the equation $a\cos 3x = \tan x - c$. [3]



7 **(a)** Express $-2x^2 - 2x - 3$ in the form $a(x+h)^2 + k$, where a, h and k are constants. [2]

(b) Hence, find the derivative of $y = \dfrac{e^{\frac{1}{x}}}{2x+3}$, and determine whether y is an increasing or decreasing function. [5]

8 The equation of a circle, with centre P, is $(x-1)^2 + y^2 = 5$. The circle passes through the points $Q(2,\ 2)$ and $R(3,\ -1)$.

(a) State the coordinates of P. [1]

(b) A point S is such that $PQRS$ is a parallelogram. Find the coordinates of S, and explain whether S is inside the circle, on the circle, or outside the circle. [4]

(c) Find the area of the parallelogram *PQRS*. [2]

9 A deep freezer keeps food frozen at $-10°C$. A piece of frozen fish is taken out to thaw before cooking. The temperature $T°C$ of the fish after *t* minutes of thawing is given by $T = 30 - Ae^{-kt}$, where *A* and *k* are constants.

(a) Explain why $A = 40$. [1]

(b) Given that the temperature of the fish rose to $23.4°C$ after an hour, find the value of *k*. [2]

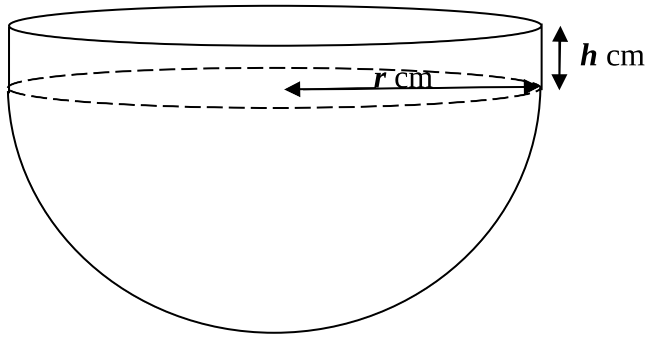
(c) The fish should only be cooked when its temperature is at least $20°C$. Given that the fish takes n minutes to reach $20°C$, where n is an integer, find the smallest value of n. [3]

(d) Describe what will happen to the temperature of the fish if it is left to thaw for an extended period of time. Explain your answer clearly. [2]

10 **(a)** Express $\dfrac{8\sqrt{2}-6}{1+\sqrt{2}}$ in the form $p+q\sqrt{2}$ where p and q are rational numbers. [3]

(b) A boy was jogging at a constant speed of $\left(a+\sqrt{3}\right)$ km/h. After jogging for $\left(4-b\sqrt{3}\right)$ h, he saw that he had travelled $\left(14-6\sqrt{3}\right)$ km. Find the values of a and b, and explain why there should be only one set of possible answers. [5]

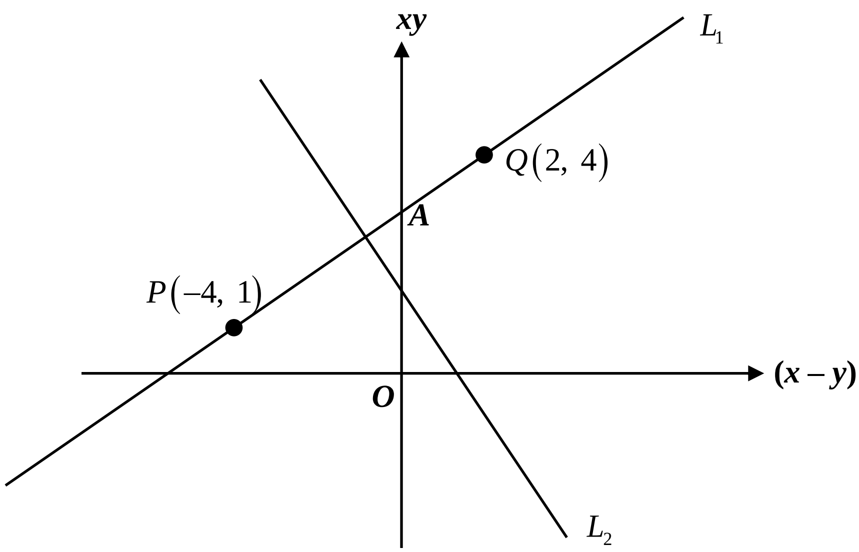
11 A manufacturer wants to manufacture round bottom frying woks made from carbon steel of negligible thickness. It was proposed that the frying wok be made up of a cylinder of height h cm, fixed to the top of a hemisphere of radius r cm.



(a) Given that the wok needs to have a volume of 2700π cm^3, show that the surface area of the wok is given by $A = \dfrac{5400\pi}{r} + \dfrac{2}{3}\pi r^2$. [4]

(b) The manufacturer would like to keep the cost of production at the lowest.
Find the value of *A* that would keep the manufacturing cost minimal. [4]

12 The diagram shows two straight line graphs obtained when the values of *xy* are plotted against $(x-y)$. The line L_1 passes through the points $P(-4,1)$ and $Q(2,4)$.



(a) Find the values of *x* and *y* at the vertical intercept *A* on line L_1. [4]

(b) Given that the line L_2 is the perpendicular bisector of *PQ*, find the equation of L_2, expressing *y* in terms of *x*. [6]

13 **(a)** Find the first three terms in the expansion, in ascending power of x, of $\left(1-\frac{x}{2}\right)^6$. Give the terms in their simplest form. [2]

(b) Hence, find the set of values of k such that the term independent of x in the expansion of $\left(9+\frac{4k}{x}+\frac{k^2}{x^2}\right)\left(1-\frac{x}{2}\right)^6$ is positive, and represent the set on a number line. [5]

(c) Explain why there are no terms with even powers in the expansion of $\left(x+\frac{2}{x}\right)^{99}$. [3]

~~~ End of Paper 1 ~~~
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