Name: __________________________ (   )   Class: ______

CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2021
SECONDARY 5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS — 4047/01
PAPER 1

Duration: 2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Omission of essential working will result in loss of marks.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Angles A and B lie on the same quadrant such that $\sin A = \frac{3}{5}$ and $\cos B = -\frac{2}{\sqrt{5}}$.
Without using calculators, find the exact value of

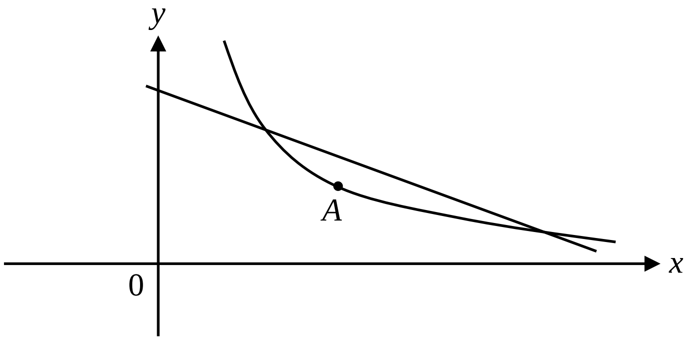
(i) $\sin(90° - A)$, [2]

(ii) $\frac{\tan A}{\tan B}$, [2]

2 **(a)** Solve the equation $\lg(4^y - 4) - y\lg 2 = \lg 3$. [4]

(b) Given that $\log_9 x^3 = \log_{27} u$, where $x = \sqrt[n]{u^2}$, find the integer value of n. [4]

3



The diagram shows part of the curve $y=\frac{2}{x-3}$ intersecting the line $y=4-\frac{x}{2}$.

The point A lies on the curve and the tangent at A is parallel to the line $y=4-\frac{x}{2}$.

(i) Find the x-coordinate of A. [3]

(ii) Find the area of the region bounded by the curve and the line. [5]

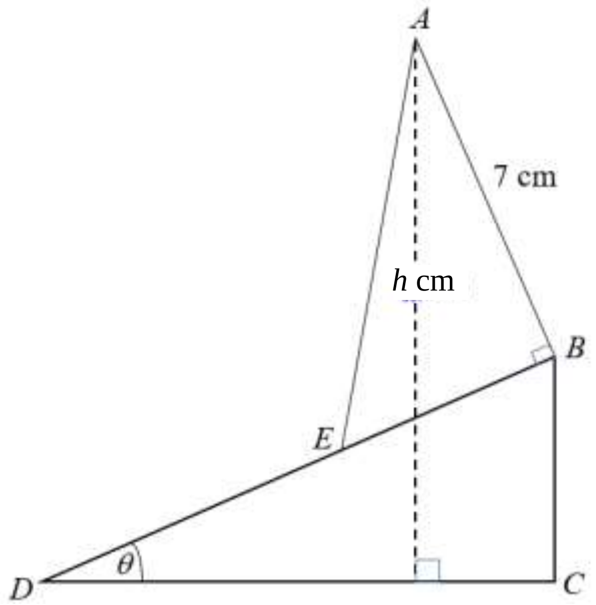
4 Starting from a point that is 3 metres from a fixed point O, a particle moves in a straight line. After t seconds, its velocity, v m/s, is given by $v = \cos 2t + 3\sin 2t$.

(i) Find the value of t when the particle first comes to instantaneous rest. [2]

(ii) Find an expression, in terms of t, for the displacement of the particle. [3]

(iii) Find the total distance travelled by the particle in 2.5 seconds. [3]

5



The diagram shows two right-angled triangles, *ABE* and *BCD*.

It is given that angle *BDC* = θ, *AB* = 7 cm and *BD* = 9 cm.

h cm is the perpendicular distance from *A* to *CD*.

(i) Show that $h = 7\cos\theta + 9\sin\theta$. [3]

(ii) Express *h* in the form $R\cos(\theta - \alpha)$, where *R* > 0 and $0° < \alpha < 90°$. [3]

(iii) Find the greatest possible value of h and the value of θ at which this occurs. [3]

(iv) Find the **values** of θ for which $h = 10$. [3]

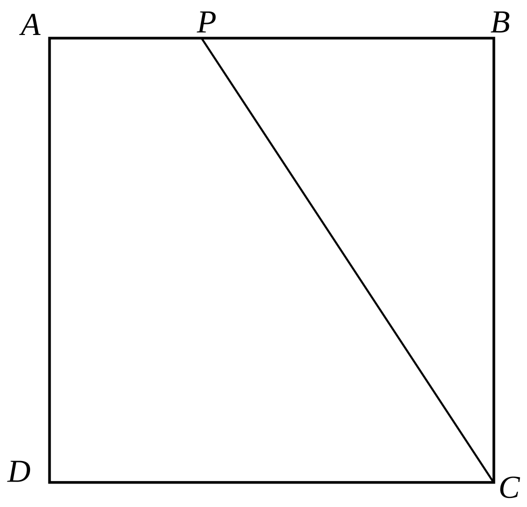
6 **(a)** Show that the straight line $y + hx = -h$ always intersects the curve $y + 1 = (h+1)x^2 + hx$ at two distinct points for all real values of *h* where $h \neq -1$. [4]

(b) Express $\dfrac{x^3 - x^2 - 4x + 1}{x^2 - 4}$ in partial fractions [6]

(c) Find all the angle(s) between 4 and 8 which satisfy the equation

$\sec x\left(13\sin\frac{1}{2}x+9\right)-4=0.$ [4]

7 *ABCD* is a field in the shape of a square with side 1200 m. A man started walking from *A* to *C*. He first walked along *AB* with a speed of 2.5 m/s. At a certain point *P*, he cuts across the field, walking with a speed of 1.5 m/s, in a straight line from *P* to *C*.



(i) Show that the total time taken by the man is given by

$$T = \frac{x}{2.5} + \frac{\sqrt{(1200-x)^2 + 1200^2}}{1.5},$$

where x m is the distance *AP*. [3]

(ii) Find the shortest time that the man would spend and the corresponding value of x. [6]

8 **(i)** Given that the tangent to the circle, C, with centre (4,5) at the point *B* is

$4y+3x-57=0$, find the coordinates of B. [3]

(ii) Find the equation of the circle, C. [2]

(iii) The lowest point on the circle is D, show that D lies on the x-axis. [1]

(iv) Find the coordinates of the point at which the tangents to the circle at B and D intersect. [2]

9 The population y, in millions, of a bacteria colony was recorded in a laboratory experiment for a period of 25 minutes. It is given that $y = ax^n + 12$, where x is the time measured in minutes and a and n are constants. The results are shown in the table below.

x	5	10	15	20	25
y	35.6	68.2	75.1	145.7	188.6

(i) Plot $\lg(y-12)$ against $\lg x$ on the **graph grid on page 17**, using a scale of 4 cm to 0.5 units on the horizontal axis and 4 cm to 1 unit on the vertical axis. [3]

(ii) Use your graph to estimate

(a) the correct reading of y for which an error has been made, [2]

(b) the value of a and of n, [2]

(c) the time in which the population reached 80 millions. [2]

End of Paper