5NA AMath Prelim Paper 2 2021

1	$f(x)=-2(x+1)(x-2)(x+k)$ $f(4)=20$ $-2(5)(2)(4+k)=20$ $k=-5$ $f(10)=-2(10+1)(10-2)(10-5)$ $=-880$	
2(i)	$y=x^2+3$ $\frac{dy}{dx}=2x$ $\frac{x^2+3-6}{x-2}=2x$ $x^2-3=2x^2-4x$ $x^2-4x+3=0$ $(x-1)(x-3)=0$ $x=1,\ 3$ ***A* (3, 12) , *B* (1, 4)**	
2(ii)	At A (3, 12), m = 6 Equation of normal at A is $y-12=-\frac{1}{6}(x-3) \Rightarrow y=-\frac{1}{6}x+\frac{25}{2}$ At B (1, 4), m = 2 Equation of normal at B is $y-4=-\frac{1}{2}(x-1) \Rightarrow y=-\frac{1}{2}x+\frac{9}{2}$ A and B meet at the point Q $-\frac{1}{6}x+\frac{25}{2}=-\frac{1}{2}x+\frac{9}{2}$ $-x+75=-3x+27$ $2x=-48 \Rightarrow x=-24$, $y=\frac{33}{2}$	

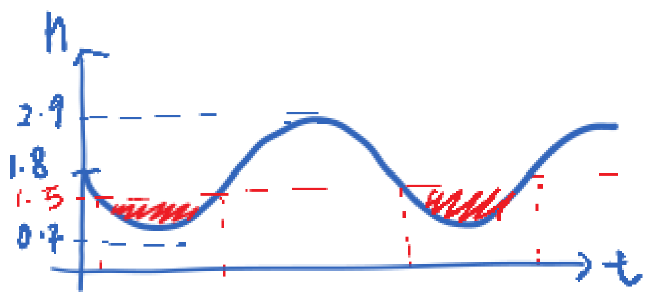
	Coordinates of Q $(-24, \frac{33}{2})$	
3	$\frac{1+\tan^2 x+2\tan x}{1-\tan^2 x}$ $=\frac{(1+\tan x)^2}{(1-\tan x)(1+\tan x)}$ $=\frac{1+\tan x}{(1-\tan x)}$ ------------------ (1) $=\frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}$ $=\frac{\cos x+\sin x}{\cos x-\sin x}$ Proven $\frac{\cos x+\sin x}{\sin x-\cos x}$ $= -\frac{\cos x+\sin x}{\cos x-\sin x}$ $= -\frac{1+\tan^2 x+2\tan x}{1-\tan^2 x}$ $=-\frac{1+\tan x}{(1-\tan x)}$ $-\frac{1+\tan x}{(1-\tan x)} = \tan x$ $\tan^2 x-2\tan x-1=0$ $\tan x=\frac{2\pm\sqrt{8}}{2}$ $\tan x=1\pm\sqrt{2}$ $\alpha=1.1781$ $x=1.18, 2.75, 4.32, 5.89$	

4(i)	$r = 2+\sqrt{2}$ $\pi r^2 h = \left(9+5\sqrt{2}\right)\pi$ $\left(2+\sqrt{2}\right)^2 h = \left(9+5\sqrt{2}\right)$ $\left(6+4\sqrt{2}\right)h = \left(9+5\sqrt{2}\right)$ $h = \frac{\left(9+5\sqrt{2}\right)}{\left(6+4\sqrt{2}\right)} \times \frac{\left(6-4\sqrt{2}\right)}{\left(6-4\sqrt{2}\right)}$ $= \frac{14-6\sqrt{2}}{4} = \frac{7}{2} - \frac{3}{2}\sqrt{2}$	
4(ii)	$\frac{3^{4x+1}}{3^{6x}} = \frac{7^{2x}}{7^{x-1}}$ $3^{1-2x} = 7^{x+1}$ $\frac{3}{7} = 7^x \square 9^x = 63^x$ $63^x = \frac{3}{7}$	
5(i)	$m_{QT} = \frac{2}{1} = 2$ $m_{PR} = -\frac{1}{2}$ $y-7 = -\frac{1}{2}\left(x-6\right)$ $y = -\frac{1}{2}x+10$	
5(ii)	$y = -\frac{1}{2}\left(0\right)+10$ $y = 10$ $P(0,10)$	
5(iii)	$\left(6,7\right) = \left(\frac{7+x}{2}, \frac{9+y}{2}\right)$ $x = 5$ $y = 5$ $V(5,5)$	

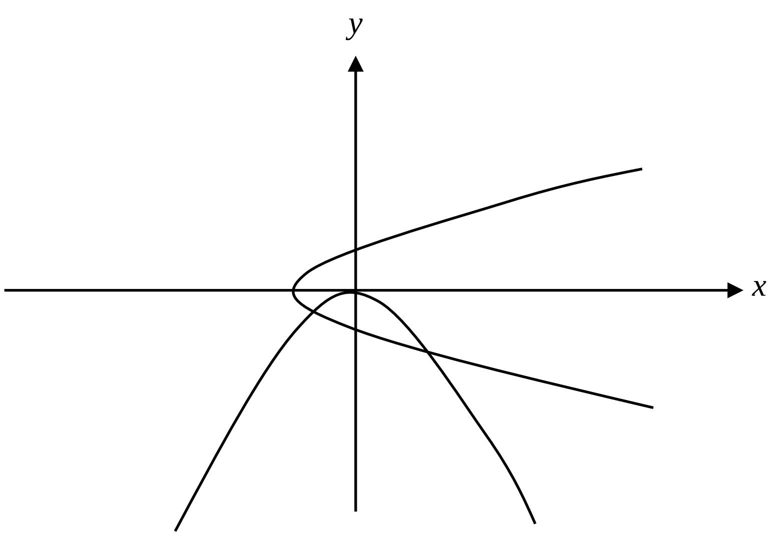
5(iv)	$\overrightarrow{OU}-\overrightarrow{OQ}=3\left(\overrightarrow{OT}-\overrightarrow{OQ}\right)$ $\begin{pmatrix}x\\y\end{pmatrix}=3\begin{pmatrix}6\\7\end{pmatrix}-2\begin{pmatrix}7\\9\end{pmatrix}$ $=\begin{pmatrix}4\\4\end{pmatrix}$ $U(4,3)$	
5(v)	$\text{area}=\frac{1}{2}\left\|\begin{vmatrix}0 & 4 & 7 & 0\\10 & 3 & 9 & 10\end{vmatrix}\right\|$ $=\frac{1}{2}\left[(36+20)-(40+21)\right]$ $=\frac{1}{2}[106-61]$ $=22.5\text{ units}^2$	
6(i)	$y=4-\|x+1\|$ Along the *x*-axis, $y=0$, $4-\|x+1\|=0$ $\|x+1\|=4$ $x+1=4$ or $x+1=-4$ $x=3$ or $x=-5$ For $y=\|x+1\|$, *x*-intercept: $\|x+1\|=0$ $x+1=0$ $x=-1$ Along the *y*-axis, $x-0$, $y=4-\|0+1\|$ $=3$ $A(-5,0)$ $B(-1,4)$ $C(3,0)$ $D(0,3)$	

6(iia)	When $m-2$, $4-\lvert x+1\rvert=2x+3$ Graph $y=2x+3$ will intersect $y=4-\lvert x+1\rvert$ at one point No. of solution = 1	
6(iib)	When $m--1$, $4-\lvert x+1\rvert=-x+3$ Graph $y=-x+3$ is equal to $y=4-(x+1)$ No. of solution = infinite	
6(iii)	When $m--1$ or $m-1$, Graph $y=mx+3$ have infinite number of solutions. Range of values of *m* for two solutions: $-1<m<1$	
7(i)	$\mathrm{f}'(x)=\dfrac{(x-1)\left(-\dfrac{1}{1-x}\right)-\ln(1-x)}{(x-1)^2}$ $=\dfrac{1-\ln(1-x)}{(x-1)^2}$	
7(ii)	$\mathrm{f}(-1)=-\dfrac{1}{2}\ln 2$ $\mathrm{f}'(-1)=\dfrac{1-\ln 2}{4}$ $y-\left(-\dfrac{1}{2}\ln 2\right)=\dfrac{1-\ln 2}{4}(x+1)$ $x=0$ $y=\dfrac{1-\ln 2}{4}-\dfrac{1}{2}\ln 2=\dfrac{1-3\ln 2}{4}$ $A\left(0,\dfrac{1-3\ln 2}{4}\right)$	

7(iii)	$\dfrac{1-\ln\left(1-x\right)}{\left(x-1\right)^{2}}<0$ $x<1$ given $1-\ln\left(1-x\right)<0$ $\ln(1-x)>1$ $1-x>e$ $x<1-e$	-
7(iv)	$\dfrac{1-\ln\left(1-x\right)}{\left(x-1\right)^{2}}>0$ $x>1-e$ $1-e<x<1$	
8(i)	${}^{n}C_{0}\left(1\right)^{n-0}\left(-\dfrac{x}{3}\right)^{0}+{}^{n}C_{1}\left(1\right)^{n-1}\left(-\dfrac{x}{3}\right)^{1}+{}^{n}C_{2}\left(1\right)^{n-2}\left(-\dfrac{x}{3}\right)^{2}+\ldots$ $=1-\dfrac{n}{3}x+\dfrac{n(n-1)}{18}x^{2}+\ldots.$	
8(ii)	$2+\left(p-\dfrac{2n}{3}\right)x+\left[\dfrac{n\left(n-1\right)}{9}-\dfrac{np}{3}+\dfrac{5}{2}\right]x^{2}+\ldots$ $p-\dfrac{2n}{3}=\dfrac{31p}{3}$ ---------(1) $\dfrac{n\left(n-1\right)}{9}-\dfrac{np}{3}+\dfrac{5}{2}=\dfrac{25}{3}$ ------------(2) Simplify (1) $n=-14p$ $n-7$, $p=-\dfrac{1}{2}$	

9	-	
9(i)	$12=\frac{2\pi}{k}$ $k=\frac{\pi}{6}$	
9(ii)	max h = 1.8 − 1.1 (− 1) = 2.9	
9(iii)	 h = 0.7 = min h = 1.8 – 1.1(1) = 0.7 $h=1.8-1.1\sin kt\le 1.5$ $\sin\frac{\pi}{6}t\ge 0.2727$ $let\ \ \frac{\pi}{6}t=0.27622, 2.8654, 6.5594, 9.1486$ $t=0.5275, 5.4725, 12.52, 17.4725$ length of time where boat landing is possible $=(5.4725-0.5275)+(17.4725-12.52)$ = 9.90 h	

10(i)	$\frac{d}{dx}(x\cos 2x) = x(-2\sin 2x) + \cos 2x$ $= -2x\sin 2x + \cos 2x$	
10(ii)	$\int(-2x\sin 2x + \cos 2x)dx = x\cos 2x + c$ $\int x\sin 2x\ dx + \frac{-1}{2}\int \cos 2x\ dx = -\frac{1}{2}\ (x\cos 2x + c)$ $\int x\sin 2x\ dx = -\frac{1}{2}\ x\cos 2x - \frac{1}{2}c + \frac{1}{2}\left(\frac{\sin 2x}{2}\right) + c_2$ $\int x\sin 2x\ dx = -\frac{1}{2}\ x\cos 2x + \left(\frac{\sin 2x}{4}\right) + d$ $\int_0^{\frac{\pi}{4}}(x\sin 2x)dx = -\frac{1}{2}\ \left(\frac{\pi}{4}\right)\cos\left(2\times\frac{\pi}{4}\right) + \left(\frac{\sin(2\times\frac{\pi}{4})}{4}\right)$ $= \frac{1}{4}$	
11(a)	$x^2 + 2x + p = 0$ S.O.R: $\alpha + \beta = -2$ P.O.R: $\alpha\beta - p$ $x^2 + qx + 27 = 0$ S.O.R: $\alpha^3 + \beta^3 = -q$ P.O.R: $\alpha^3\beta^3 = 27$ $\alpha^3\beta^3 = 27$ $(\alpha\beta)^3 = 27$ $p^3 = 27$ $p = 3$	

<table>
<tr><td></td><td>$\alpha^3+\beta^3=-q$
$(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)=-q$
$(-2)\left[(\alpha+\beta)^2-3\alpha\beta\right]=-q$
$(-2)\left((-2)^2-3(3)\right)=-q$
$q=-10$</td><td></td></tr>
<tr><td>11(b)</td><td></td><td></td></tr>
<tr><td>12(i)</td><td>$9=25-Ae^0$
$A=16$
$17=25-16e^{20k}$
$e^{20k}=\frac{1}{2}$
$20k=\ln\left(\frac{1}{2}\right)$</td><td></td></tr>
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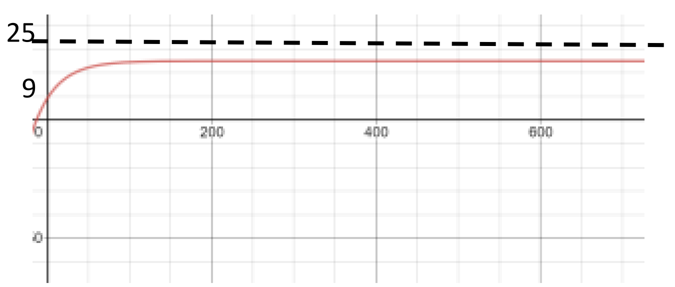
shown

12(ii)	$23 = 25 - 16\left(\frac{1}{2}\right)^{\frac{t}{20}}$ $\left(\frac{1}{2}\right)^{\frac{t}{20}} = \frac{1}{8}$ $\frac{t}{20} = \frac{\log_2 \frac{1}{8}}{\log_2 \frac{1}{2}} = \frac{-3}{-1} = 3$ $t = 60$ Duration = 60 – 20 = 40 minutes	
12(iii)	$t \to \infty, \left(\frac{1}{2}\right)^{\frac{t}{20}} \to 0$ $so\ \theta \to 25$ for large values of t. $\boxed{\text{Approaches / tends towards}}$	
12(iv)	 	-