Name: __________________________ (   )   Class: ______

CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2021
SECONDARY 5 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS   4047/02
PAPER 2
Duration: 2 hours 30 mins

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Omission of essential working will result in loss of marks.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 The polynomial $f(x)$ has roots of equations $-1,\ 2$ and $-k.$ Given that the coefficient of x^3 is -2 and that $f(x)$ has a remainder of 20 when divided by $x-4$, find the value of *k*. Hence, find the remainder when $f(x)$ is divided by $x - 10$. [6]

2 **(i)** Find the coordinates of the two points A and B on the curve $y = x^2 + 3$ whose tangents pass through the point P (2, 6). [6]

(ii) If the normals at A and B meet at the point Q, find the coordinates of Q. [4]

3 Prove that $\dfrac{\sec^2 x + 2\tan x}{1 - \tan^2 x} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$. [8]

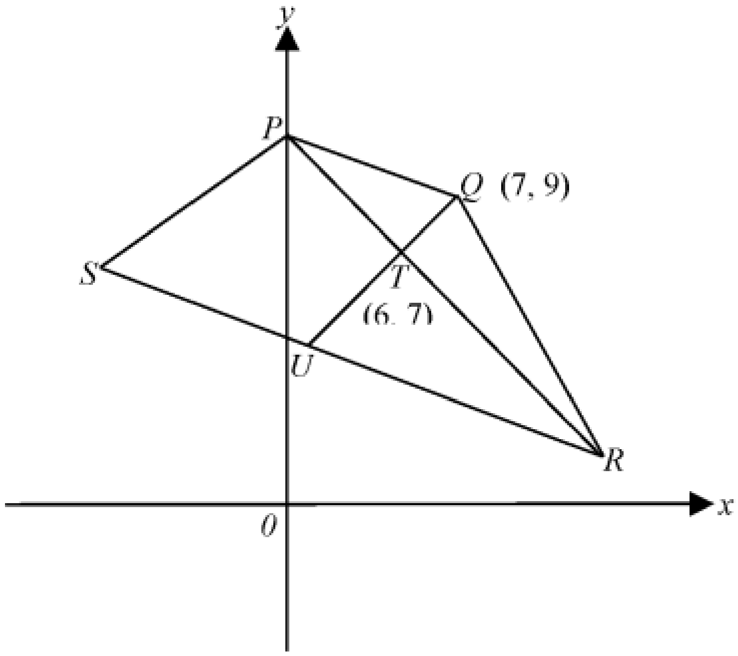
Hence, solve the equation $\dfrac{\cos x + \sin x}{\sin x - \cos x} = \tan x$, giving all solutions between 0 and 2π.

4 **(i)** A cylinder has a diameter of $\left(4+2\sqrt{2}\right)$ cm and a volume of $\left(9+5\sqrt{2}\right)\pi$ cm^3.
Find the height of the cylinder in the form of $\left(a+b\sqrt{2}\right)$ cm. [4]

(ii) Given that $3^{4x+1} \times 7^{x-1} = 27^{2x} \times 49^{x}$, find the value of 63^{x}. [3]

5 Solutions to this question by accurate drawing will not be accepted.

The diagram, which is not drawn to scale, shows a trapezium *PQRS* in which *PQ* is parallel to *SR*. The point *P* lies on the *y*-axis and the point *Q* is (7, 9). The point *T*(6, 7), lies on *PR* such that *QT* is perpendicular to *PR*. *QT* produced meets *SR* at *U* and *QU* = 3*QT*.



Find

(i) the equation of *PR*, [3]

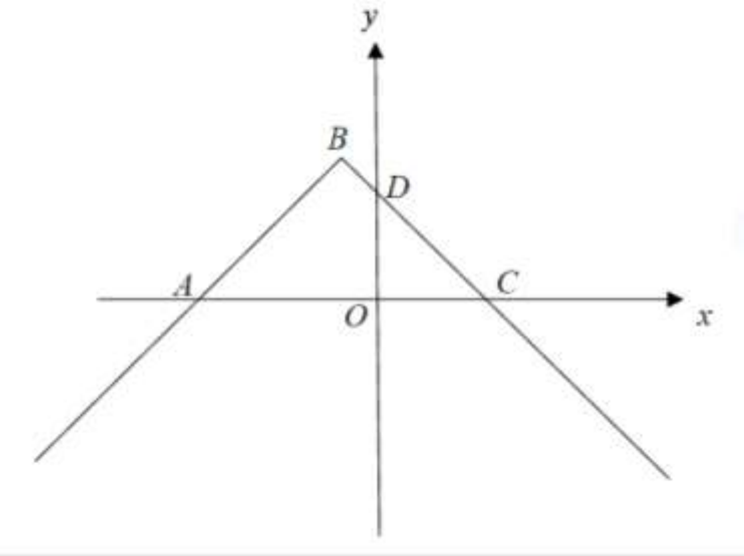
(ii) the coordinates of P, [2]

(iii) the coordinates of V if $PQRV$ is a kite, [2]

(iv) the coordinates of U, [2]

(v) the area of triangle PQU. [2]

6 The diagram shows part of the graph of $y = 4 - |x + 1|$.



(i) Find the coordinates of the points A, B, C and D. [5]

(ii) Find the number of solutions of the equation $4-|x+1|=mx+3$ when

(a) $m - 2$ [1]

(b) $m - -1$ [1]

(iii) State the range of values for *m* for which the equation $4-|x+1|=mx+3$ has two solutions. [1]

7 A curve has the equation $\mathrm{f}(x)=\dfrac{\ln(1-x)}{x-1}$ for $x<1$.

(i) Obtain the expression for $f'(x)$. [2]

(ii) The tangent to the curve at the point where $x = -1$ intersects the y-axis at the point A. Find the **exact** coordinates of the point A. [3]

(iii) Show all necessary workings, find the range of values of x for which $f(x)$ is a decreasing function. [3]

(iv) Hence, deduce the range of values of x for which $f(x)$ is an increasing function. [2]

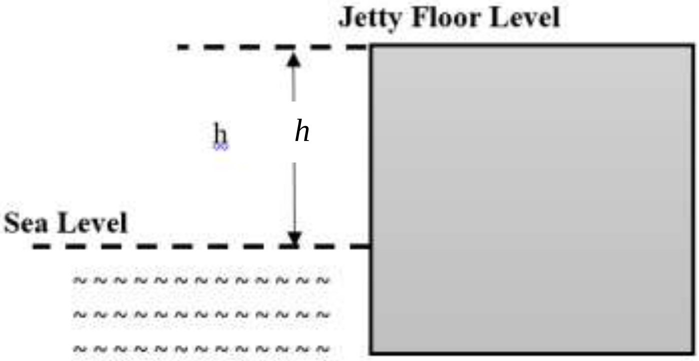
8 **(i)** Write down the first three terms in the expansion, in ascending powers of x, of

[2]

$\left(1-\frac{x}{3}\right)^n$, where n is a positive integer greater than 2.

(ii) In the expansion of $\left(2+px+\frac{5}{2}x^2\right)\left(1-\frac{x}{3}\right)^n$, in ascending powers of x, the first
three terms are $2+\frac{31p}{3}x+\frac{25}{3}x^2$. Find the value of n and p. [6]

9



The height difference, h m, between the jetty floor level and the sea level changes with time due to the tidal effects. It is given that h can be modelled as

$h = 1.8 - 1.1\sin kt$, where k is a constant and t is the time in hours from midnight. The time between two consecutive high tides is 12 hours.

(i) Show that the value of k is $\frac{\pi}{6}$ radian per hour. [2]

(ii) State the maximum value of the height difference h. [1]

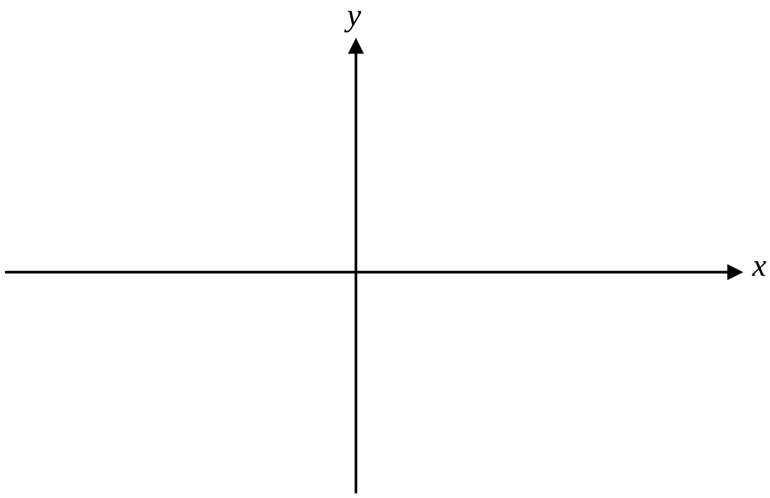
(iii) If the jetty can be used for boats to land only when the height difference, h is within range of values $0.7 \leq h \leq 1.5$, find the total length of time in hours in a day when the boat landings are possible. [4]

10 **(i)** Differentiate $x\cos 2x$ with respect to x. [2]

(ii) Hence, find $\int_0^{\frac{\pi}{4}} \left(x \sin 2x\right) dx.$ [4]

11 **(a)** The roots of the equation $x^2 + 2x + p = 0$, where *p* is a constant, are α and β.
The roots of the equation $x^2 + qx + 27 = 0$, where *q* is a constant, are α^3 and
β^3. Find the value of *p* and of *q*. [6]

(b) On the same axes, sketch the curves $y^2 = 64x$ and $y = -x^2$. [2]



1
2 A jug containing liquid is taken from a refrigerator and placed in a room with a constant temperature of $25°C$. The temperature of the liquid $\theta°C$ after time t minutes is given by $\theta = 25 - Ae^{kt}$, where A and k are real constants. Initially the temperature of the liquid is $9°C$. After 20 minutes, the temperature of the liquid increases to $17°C$.

(i) Find the value of A and show that $k = \frac{1}{20}\ln\frac{1}{2}$. [3]

(ii) Find the **exact duration** it takes for the temperature of the liquid to increase from $17°C$ to $23°C$. [4]

(iii) Explain what will happen to θ for large values of t. [2]

(iv) Sketch a graph of θ against t. [2]

End of Paper