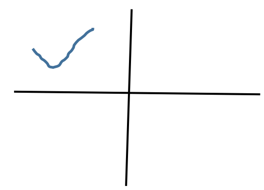
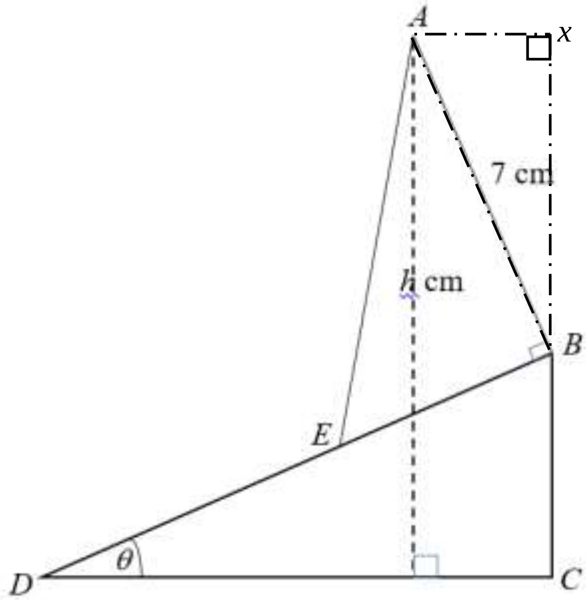
5NA AMath Prelim Paper 1 2021

1(i)	$\sin A = \frac{3}{5}$ $2nd\ Quadrant$ $\sin(90^0 - A) = \cos A$ $= -\frac{4}{5}$ 	
1(ii)	$\frac{\tan A}{\tan B} = \frac{\left(-\frac{3}{4}\right)}{\left(-\frac{1}{2}\right)}$ $= \frac{3}{2}$	
2a	$\lg\left(\frac{4^y - 4}{2^y}\right) = \lg 3$ $\frac{4^y - 4}{2^y} = 3$ $2^{2y} - 3(2^y) - 4 = 0$ $\text{let } 2^y = u$ $u^2 - 3u - 4 = 0$ $(u-4)(u+1) = 0$ $2^y = 4$ $y = 2$ $or\ 2^y = -1(rej)$	

2(b)	$\frac{\log_3 x^3}{\log_3 3^2}=\frac{\log_3 u}{\log_3 3^3}$ $\frac{3\log_3 x}{2\log_3 3}=\frac{\log_3 u}{3\log_3 3}$ $\frac{3}{2}\log_3 x=\frac{1}{3}\log_3 u$ $x^{\frac{3}{2}}=u^{\frac{1}{3}}$ $x=u^{\frac{2}{9}}$ $n=9$	
3(i)	$\frac{dy}{dx}=-\frac{2}{(x-3)^2}$ $=-\frac{1}{2}$ $4=(x-3)^2$ $x-3=\pm 2$ $x=5$ or $x=1(rej)$	
3(ii)	$4-\frac{x}{2}=\frac{2}{x-3}$ $-x+11x+24=0$ $(x-7)(x-4)=0$ $x=4$ or 7 $\int_4^7\left[\left(4-\frac{x}{2}\right)-\frac{2}{x-3}\right]dx$ $=\left[4x-\frac{x^2}{4}-2(\ln(x-3))\right]_4^7$ $\left[4x-\frac{x^2}{4}-2(\ln(x-3))\right]_4^7$ $=\left[\left(4(7)-\frac{49}{4}-2\ln 4\right)-(16-4-2\ln 1)\right]$ $=[12.97741128-12]$ $=0.977$ units^2	

	Method 2 $\int_4^7 \frac{2}{x-3}dx = 2\left[\ln(x-3)\right]_4^7$ $= \ln 16$ $x = 4, y = 2$ $x = 7, y = \frac{1}{2}$ area of trapezium $= \frac{1}{2} \times 3 \times \left(2 + \frac{1}{2}\right)$ $= \frac{15}{4}$ required area $= \frac{15}{4} - \ln 16$ $= 0.977 \text{ units}^2$	
4(i)	$\cos 2t + 3\sin 2t = 0$ $3\sin 2t = -\cos 2t$ $\tan 2t = -\frac{1}{3}$ $\alpha = 0.32175$ $t = 1.40992$ $t = 1.41$	
4(ii)	$s = \frac{\sin 2t}{2} - \frac{3\cos 2t}{2} + c = 3$ $0 - \frac{3}{2} + c = 3$ $c = 4\frac{1}{2}$ $s = \frac{\sin 2t}{2} - \frac{3\cos 2t}{2} + 4\frac{1}{2}$	
4(iii)	s(0) = 3 m s(1.40992) = 6.081138 m s(2.5) = 3.595044 m total distance = (6.081138 – 3) + (6.081138 – 3.595044) = 5.573832 m = 5.57 m	

5(i)	 $\cos\theta = \frac{BX}{7}$ $BX = 7\cos\theta$ $\sin\theta = \frac{BC}{9}$ $BC = 9\sin\theta$ $h = XB + BC$ Therefore $= 7\cos\theta + 9\sin\theta.$	
5(ii) **Ok!**	$h = 7\cos\theta + 9\sin\theta$ $R = \sqrt{49+81} = \sqrt{130}$ $\tan\alpha = \frac{9}{7}$ $\alpha = 52.125^\circ$ $\sqrt{130}\cos(\theta - 52.1^\circ)$	
5(iii)	Greatest h = $\sqrt{130}$	

	$\cos(\theta - 52.1°) = 1$ $\theta - 52.1° = 0$ $\theta = 52.1°$	
5(iv)	$h = 7\cos\theta + 9\sin\theta$ $\sqrt{130}\cos(\theta - 52.1°) = 10$ $\sqrt{130}\cos(\theta - 52.125°) = 10$ $\cos(\theta - 52.125°) = \frac{10}{\sqrt{130}}$ $\theta - 52.125° = 28.7105148°, \quad -28.7105148°$ $\theta = 80.8°, \quad 23.4°$	
6(a)	$-hx - h + 1 = (h+1)x^2 + hx$ $(h+1)x^2 + 2hx + h - 1 = 0$ $b^2 - 4ac = (2h)^2 - 4(h+1)(h-1)$ $= 4h^2 - 4h^2 + 4$ $= 4 > 0$ since $b^2 - 4ac > 0$, therefore the str line always intersects the curve at 2 distinct points for all real values of h where $h \neq -1$.	

6(b)	$$\begin{array}{r|l} & x-1 \\ \hline x^2-4 & x^3-x^2-4x+1 \\ & x^3 \quad -4x \\ \hline & -x^2 \quad +1 \\ & -x^2 \quad +4 \\ \hline & -3 \end{array}$$ $$\frac{x^3-x^2-4x+1}{x^2-4}=x-1-\frac{3}{(x-2)(x+2)}$$ $$-\frac{3}{(x-2)(x+2)}=\frac{A}{(x+2)}+\frac{B}{(x-2)}$$ let x = 2 -3 = - 4A A = $\frac{3}{4}$ let x = 2 -3 = 4B B = - $\frac{3}{4}$ $$\frac{x^3-x^2-4x+1}{x^2-4}=x-1+\frac{3}{4(x+2)}-\frac{3}{4(x-2)}$$	
6(c)	$$\frac{13\sin\frac{1}{2}x+9-4\cos x}{\cos x}=0$$ $$13\sin\frac{1}{2}x+9-4(1-2\sin^2\frac{1}{2}x)=0$$ $$8\sin^2\frac{1}{2}x+13\sin\frac{1}{2}x+5=0$$ $$\left(8\sin\frac{1}{2}x+5\right)\left(\sin\frac{1}{2}x+1\right)=0$$ $$\sin\frac{1}{2}x=-\frac{5}{8} \quad \text{or} \quad \sin\frac{1}{2}x=-1(NA)$$ $$\alpha=0.67513$$ $$x=7.63$$	

7(i)
Ok!

$PC = \sqrt{PB^2 + BC^2}$
$PB = 1200 - x$
$BC = 1200$
$PC = \sqrt{(1200-x)^2 + 1200^2}$

time to travel from A to P = $\dfrac{x}{2.5}$

Time to travel from P to C = $\dfrac{\sqrt{(1200-x)^2 + 1200^2}}{1.5}$

Therefore $T = \dfrac{x}{2.5} + \dfrac{\sqrt{(1200-x)^2 + 1200^2}}{1.5}$ (proven)

7(ii)

$$\frac{dT}{dx} = \frac{1}{2.5} + \frac{1}{1.5} \times \frac{1}{2}\left((1200-x)^2 + 1200^2\right)^{-\frac{1}{2}}\left(-2(1200-x)\right)$$

$$0 = \frac{1}{2.5} + \frac{-\frac{2}{3}(1200-x)}{\sqrt{(1200-x)^2 + 1200^2}}$$

$$5(1200-x) = 3\sqrt{(1200-x)^2 + 1200^2}$$

$$25(1200-x)^2 = 9\left[(1200-x)^2 + 1200^2\right]$$

$$16(1200-x)^2 = 9(1200)^2$$

$$4(1200-x) = 3(1200), \qquad 4(1200-x) = -3(1200)$$

$$x = 300, \quad x = 2100 \text{ (rejected)}$$

	x = 299	**x = 300**	**x = 301**
$\frac{dT}{dx}$	**-ve**	**zero**	**+ve**
shape	\	—	/

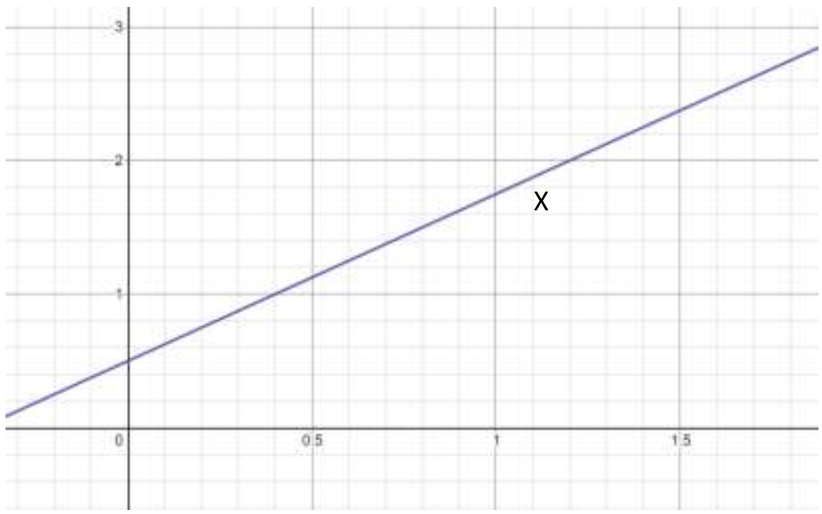
First derivative, minimum T at x = 300

Min $T = \dfrac{300}{2.5} + \dfrac{\sqrt{(1200-300)^2 + 1200^2}}{1.5}$ =1120 seconds

8(i)	Gradient of tangent at B = $-\frac{3}{4}$ Gradient of the perpendicular line to the tangent = $\frac{4}{3}$ Equation of the perpendicular line: $y-5=\frac{4}{3}(x-4)$ $3y=4x-1$ -(1) sub (1) into the given line $3\left(-\frac{3}{4}x+\frac{57}{4}\right)=4x-1$ $x=7$ $y=9$ $B(7,9)$	
8(ii)	Radius of circle = $\sqrt{(4-7)^2+(5-9)^2}=5$ $(x-4)^2+(y-5)^2=25$	
8(iii)	Radius = 5, from the centre vertically down, lowest point *D* on x-axis is *D*(4,0)	
8(iv)	Equation at *D* is $y = 0$ $4(0) + 3y = 57$ $y = 19$ Point is (19,0)	

10(i)

lgx	0.70	1.00	1.18	1.30	1.40
lg(y-12)	1.37	1.75	1.80	2.13	2.25



9(iia)

The correct value is
lg(y-12) = 1.975
y = 106.4

9(iib)

$$\lg(y-12) = n\lg x + \lg a$$

$$\text{gradient} = \frac{1.50-0.50}{0.8-0}$$

$$n = 1.25$$

$$\lg a = 0.5$$

$$a = 3.16$$

9(iic)

lg(80-12) =1.83
lg x = 1.06

x =11.5