1. **(a)** Express $\dfrac{1}{x+3} - \dfrac{x}{x-4}$ as a single fraction.

$\dfrac{1}{x+3} - \dfrac{x}{x-4} = \dfrac{x-4}{(x+3)(x-4)} - \dfrac{x(x+3)}{(x+3)(x-4)}$	M1 (making common denominator and multiplying correct factor)
$= \dfrac{x-4-x^2-3x}{(x+3)(x-4)}$	
$= \dfrac{-x^2-2x-4}{(x+3)(x-4)}$ or $\dfrac{x^2+2x+4}{(x+3)(4-x)}$	A1

Answer : …………………………… [2]

(b) Simplify $\dfrac{15q^4r^3}{4s^2} \div \dfrac{3q^5}{8s^3}$.

$\dfrac{15q^4r^3}{4s^2} \div \dfrac{3q^5}{8s^3} = \dfrac{15q^4r^3}{4s^2} \times \dfrac{8s^3}{3q^5}$	
$= \dfrac{120q^4r^3s^3}{42s^2q^5}$	
$= \dfrac{10r^3s}{q}$	B1 (2 terms correct) B2 (all terms correct)

Answer : …………………………… [2]

(c) $V = \dfrac{4\pi}{3}(p^3 - q)$

    (i) Evaluate V when $p = 2.5$ and $q = -1.8$.

$V = \dfrac{4\pi}{3}[2.5^3 - (-1.8)]$ $= 72.9896$ $= \mathbf{73.0}$	B1

Answer : ……………………………… [1]

    (ii) Rearrange the formula $V = \dfrac{4\pi}{3}(p^3 - q)$ to make p the subject.

		Alternative:	
$V = \dfrac{4\pi}{3}(p^3 - q^3)$ $\dfrac{3V}{4\pi} = p^3 - q$ $p^3 = q + \dfrac{3V}{4\pi}$ $p = \sqrt[3]{q + \dfrac{3V}{4\pi}}$	M1 A1	$V = \dfrac{4\pi}{3}(p^3 - q)$ $3V = 4\pi p^3 - 4\pi q$ $4\pi p^3 = 3V + 4\pi q^3$ $p^3 = \dfrac{3V + 4\pi q}{4\pi}$ $p = \sqrt[3]{\dfrac{3V + 4\pi q}{4\pi}}$	M1 A1

Answer : ……………………………… [2]

(d) (i) Find the value of h and k if $x^2 - hx + 1 = (x-3)^2 + k$.

$x^2 - hx + 1 = (x-3)^2 + k$
              $(x-3)^2 + k = x^2 - 6x + 9 + k$
 	
 $h = 6$
 $9 + k = 1$
    $k = -8$
	
	 B1
	 B1

Answer : $h =$ ……………… , $k =$ ……………… [2]

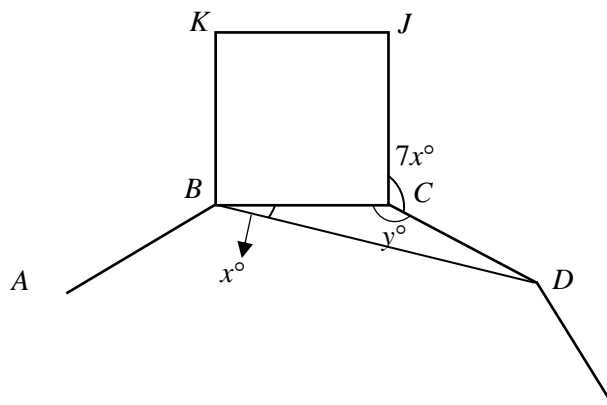   **(ii)** Using your answers in **(d)(i)**, write down the coordinates of the minimum point of the graph $y = x^2 - hx + 1$.

(3, –8)	B1 / (3, ecf)

Answer : (……………, ……………) [1]

2. The diagram shows part of a reagular *n*-sided polygon with sides *ABCDE*.
BCJK is a square.

It given that angle *CBD* = $x°$, angle *BCD* = $y°$ and angle *JCD* = $7x°$.

(a) By forming two equations and solving them simultaneously, find the value of *x* and of *y*.

$2x + y = 180$ ($\angle$ sum of triangle) ----------- [1]	B1 (O.E) – first eq
$7x + y + 90 = 360$ ($\angle$s at a pt) $7x + y = 270$ --------------- [2]	B1 (O.E) – 2nd eq
[2] – [1] : $7x - 2x = 270 - 180$ $5x = 90$ $x = 18$	M1 (solving simultaneous either elimination or substitution using '*their*' 2 equations)
and $y = 180 - 2(18)$ $= 144$	M1 (finding the other variable)
Therefore, *x* = **18** and *y* = **144.**	A1 (both correct)

Answer : .... [5]

(b) Hence, find the value of *n*.

Since $y = 144$, one interior angle $= 144°$ Exterior angle $= 180 - 144$ $= 36°$	M1 (applying sum int angles or ext angle using '*their*' y)
Therefore $n = 360 \div 36$ $= $ **10**	A1

Answer : .... [2]

3. The stem and leaf diagram shows the marks obtained by a group of pupils in a Mathematics and English tests.

Mathematics Marks	Stem	English Marks
5 4 2	1	
8	2	4 4 5
9 4 3 1 0	3	3 5 6 8 8 9
3 2	4	2 2 3 4
9 9 8 6	5	0 1

Key : 2 | 1 means 12   Key : 2 | 4 means 24

(i) Find the median marks for the Mathematic test and English test.

Answer : Median Mathematics mark = ……| 34 ------ B1 |…. [1]

Median English mark = ……| 38 ------ B1 |.. [1]

(ii) Find the interquartile range for the Mathematic test and English test.

	B1
IQR Mathematics Test = 56 – 28 = 28 IQR English Test = 43 – 33 = 10	

Answer : IQR Mathematics mark = …| 28 ------ B1 |… [1]

IQR English mark = …| 10 ------ B1 |... [1]

(iii) Students who scored at least 42 marks were awarded a distinction grade. Find the percentage of students who achieved distinction grade in the English test.

|	
$\dfrac{6}{15} \times 100$ $= 40\%$	B1

Answer : …………….…………….% [1]

(iv) Below are two statements comparing the marks for the two tests.
For each one, write whether you agree or disagree, giving a reason for each
answer. [2]

Statement	Agree/ disagree	Reason
The Mathematics test is easier.	Disagree	Comparing the median marks, English Test is easier as it has a higher median mark. B1 Specify median, Do not accept if they calculate mean.
There is a greater spread of marks for the Mathematics Test.	Agree	The interquartile range/range for Mathematics Test is greater than that of English test. B1 Do not accept if they calculate standard deviation

4. In the diagram, *O* is the centre of the circle passing through points *A*, *B*, *E* and *D*.

The tangent to the circle at *B* meets *AD* produced at *C*.

AOB is a diameter of the circle, *BE* = *ED* and angle *BAD* = 46°.
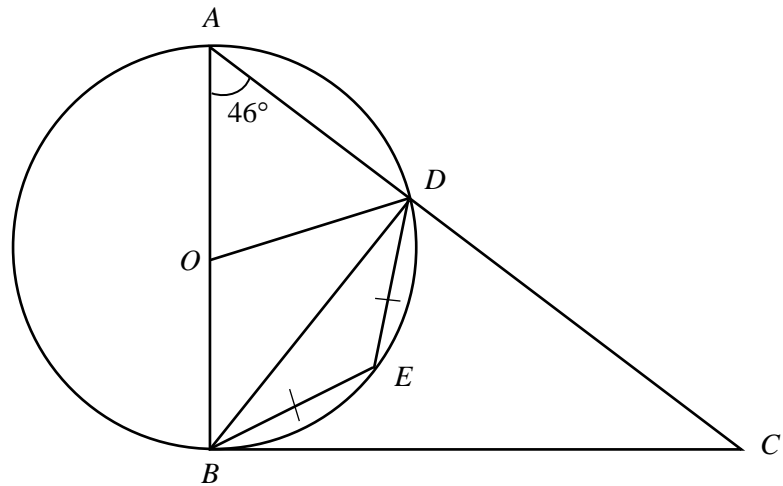
(a) Prove that triangle *ADB* is similar to triangle *ABC*.

∠*BAD* = ∠*BAC* (Common Angle)	B1 (with correct reason)
∠*ADB* = 90 ° (rt ∠ in a semicircle) ∠*ABC* = 90 ° (tan ⊥ rad) ∠*ADB* = ∠*ABC*	B1 (with correct reasons)
∴ triangle *ADB* is similar to triangle *ABC* (AA Similarity Test)	B1 (full statement, accept AAA/AA)

[3]

(b) Find the following angles. Show your reason(s) clearly.
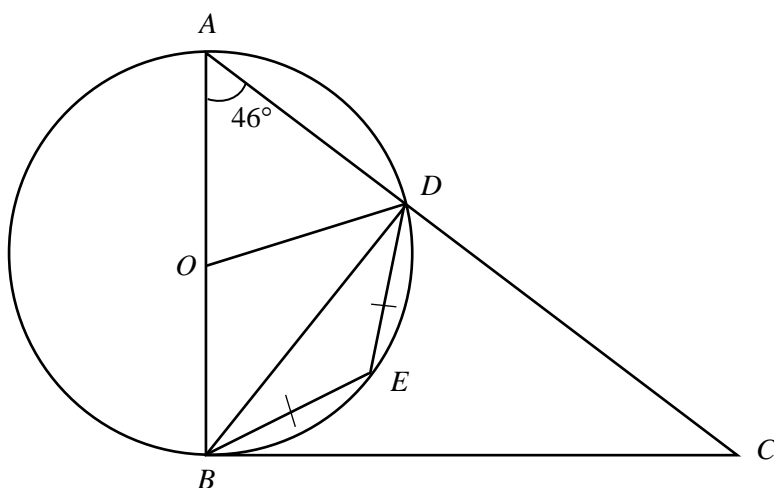
(i) Angle *BOD*

| Angle *BOD* $= 2 \times 46°$ ($\angle$ at centre $= 2 \angle$ at circumference)
$= 92°$ | M1/B1 (with correct reason)
A1/B1 |

Answer : ………………………….. [2]

(ii) Angle *BED*

| Angle *BED* $= 180° - 46°$ ($\angle$s in opp segment)
$134°$ | M1/B1 (with correct reason)
A1/B1 |

Answer : ………………………….. [2]

(iii) Angle *EBC*

Angle *OBD* $= 180° - 90° - 46°$ ($\angle$ sum of triangle) $= 44°$	B1
Angle *DBE* $= \dfrac{180° - 134°}{2}$ ($\angle$ sum of triangle) $= 23°$	M1 ('*their*' $\angle$BED) (with or without reason)
Angle *EBC* $= 90° - 44° - 23°$ $= 23°$	A1 (accept without reasons)

Answer : ………………………….. [3]

----

5. (a) A bag contains 18 red balls and 7 green balls.
Two balls are taken from the bag without replacement.

(i) Complete the tree diagram to show the probabilities of the possible outcomes.

$$\left(\frac{17}{24}\right) \text{ Red}$$

Red

$$\left(\frac{18}{25}\right)$$

$$\left(\frac{7}{24}\right) \text{ Green}$$

$$\left(\frac{18}{24}\right) \text{ Red}$$

$$\left(\frac{7}{25}\right) \text{ Green}$$

$$\left(\frac{6}{24}\right) \text{ Green}$$
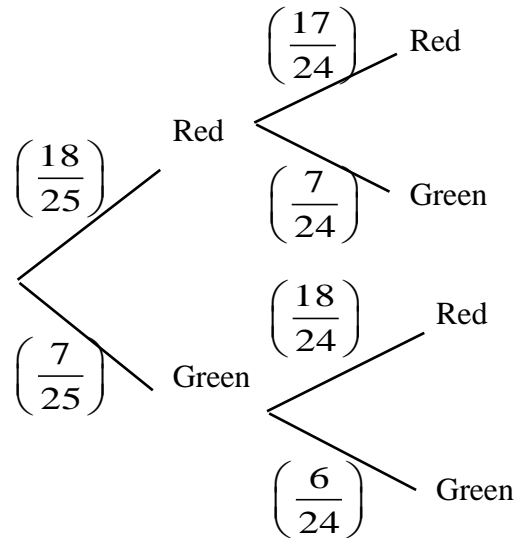
B1 first level
B1 2nd level
[2]

(ii) Find, as a fraction in its simplest form, the probability that
(a) the two balls are the same colour,

P (RR or GG) = $\left(\frac{18}{25}\times\frac{17}{24}\right)+\left(\frac{7}{25}\times\frac{6}{24}\right)$ $=\frac{29}{50}$	B1

Answer : …………….……………. [1]

(b) at least one of the balls is green.

P (no red) = $1-\left(\frac{18}{25}\times\frac{17}{24}\right)$ $= 1-\frac{51}{100}$ $=\frac{49}{100}$	M1 ('*their*' tree diagram) (accept RR+RG+GR) B1

Answer : …………….………………. [2]

(iii) If a third ball is picked, find the probability that none of the balls is red.

P (no red) = P (GGG) $=\frac{7}{25}\times\frac{6}{24}\times\frac{5}{23}$ $=\frac{7}{460}$	M1 A1

Answer : …………….………………. [2]

(b) A gaming app allows a second chance at the game if a player resulted in a draw.

The probability of a win is $\dfrac{5}{6}$.

The probability of a draw is p.
The probability of a loss is q.

Given that the probability of a win at the second try is $\dfrac{5}{144}$, find the value of q.

P (draw and loss) $= \dfrac{5}{144}$ $\dfrac{5}{6}p = \dfrac{5}{144}$ $p = \dfrac{1}{24}$ $q = 1 - \dfrac{5}{6} - \dfrac{1}{24}$ $q = \dfrac{1}{8}$	M1 / B1 (forming eqn o.e) OR $q = 1 - \dfrac{5}{6} - \,'their'\,p$ A1 / B1

Answer : $y =$ …………………………….. [2]

----

6. **(a)** The nth term of a number sequence T_n, is given by $T_n = \dfrac{1+n}{12}$;

Fill in the table for the terms T_2, T_3 and T_4 in the simplest form.
T_1 has been filled in for you.

n	1	2	3	4
T_n	$\dfrac{1}{6}$	$\dfrac{1}{4}$	$\dfrac{1}{3}$	$\dfrac{5}{12}$.

B2/1/0

[2]

(b) The first four terms in another number sequence is as shown below.

$$P_1 = 3^2 - 5 = 4$$
$$P_2 = 4^2 - 6 = 10$$
$$P_3 = 5^2 - 7 = 18$$
$$P_4 = 6^2 - 8 = 28$$

(i) Write down P_5.

$P_5 = 7^2 - 9 = 40$	B1 (40)

Answer : ……………..……………. [1]

(ii) Show that the nth term P_n of this sequence is given by $P_n = n(n+3)$.

P_n $= (n+2)^2 - (n+4)$ $= n^2 + 4n + 4 - n - 4$ $= n^2 + 3n$	M1 A1

Answer : ……………..……………. [2]

(iii) Given that $\dfrac{P_n}{T_n} = 299$, form an equation in n and show that it reduces to

$$12n^2 - 263n - 299 = 0$$

$\dfrac{P_n}{T_n} = 299$ $n^2 + 3n \div \dfrac{1+n}{12} = 299$ $\dfrac{n^2 + 3n}{1} \times \dfrac{12}{1+n} = 299$ $\dfrac{12n^2 + 36n}{1+n} = 299$ $12n^2 + 36n = 299(1+n)$ $12n^2 + 36n = 299n + 299$ $12n^2 + 263n - 299 = 0$	M1 (forming the first equation) B1 single fraction ($\dfrac{12n^2 + 36n}{1+n} = 299$) A1

Answer : ……………..………………. [3]

(iv) Hence, find the value of n.

$12n^2 + 263n - 299 = 0$ $(12n + 13)(n - 23) = 0$ $n = -\dfrac{13}{12}\,(rejected)$ or **$n = 23$** -- *Alternative :* $12n^2 + 263n - 299 = 0$ $n = \dfrac{-263 \pm \sqrt{263^2 - 4(12)(-299)}}{2(12)}$ $n = -\dfrac{13}{12}\,(rejected)$ or **$n = 23$**	M1 (factorization) A1 (final answer n) ---------------------------- M1 (apply quad formula) A1 (final answer n in answer space)

Answer : $n =$ ……………..………………. [2]

----

7. A load *F* hangs vertically down from *B* where *BF* = 3 m.
   G is directly below the load *F*.
   The angle of elevation of *B* from *A* is 45°.

   It is given that *AB* = 15.7 m, *BC* = 12 m, and angle *CDE* = 90°.
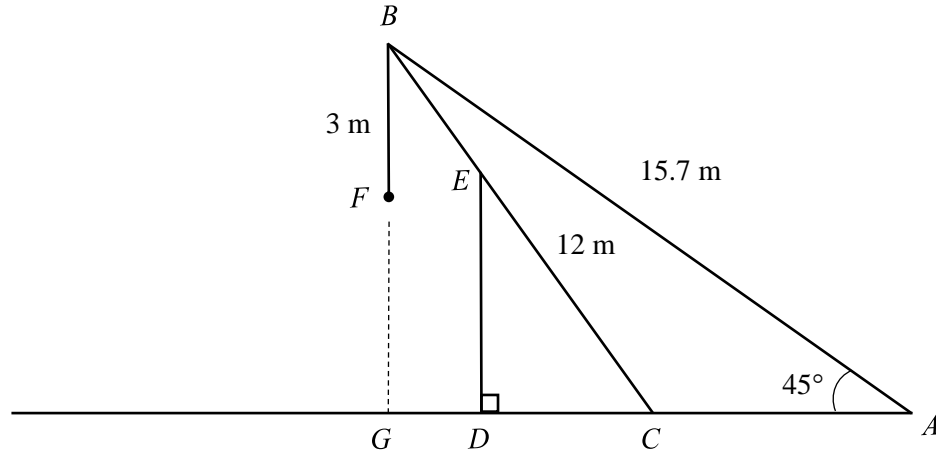
   (i) Calculate angle *BCA*.

Using sine rule: $$\frac{\sin B\hat{C}A}{15.7} = \frac{\sin 45°}{12}$$ $$B\hat{C}A = \sin^{-1}\left(\frac{15.7 \times \sin 45}{12}\right) = 67.6887$$ or $B\hat{C}A = 180 - 67.6887$ $B\hat{C}A = 112.311826 = \mathbf{112.3°}$	M1 (apply sine rule) M1/ B1 (67.7) A1 (1 d.p)

Answer : …………………………° [3]

   (ii) Without finding any lengths, explain why *AG* = *BG*.

Since ∠BAC = 45°, and ΔBAG is a <u>right-angled triangle</u>, ∠GBA = 45° (<u>isosceles triangle</u>). Hence, *AG* = *BG*.	B1

Answer : …………………………………………………………………………….

……………………………………………………………………………………………

..…………………………………………………………………………………… [1]

(iii) Calculate how far the load *F* is above the ground.

$\sin 45° = \dfrac{x}{15.7}$ $x = 11.10157$ m Height of F above ground = 11.10157 – 3 = **8.10 m**	M1 (correct trigo ratio) Or Pyt Th. A1

Answer : ………………………… m [2]

(iv) The load is being lowered from *F* to the ground at *G*.
It took 2 minutes and 42 seconds.

Find the speed in m/s, correct to 3 decimal places.

$\text{Speed} = \dfrac{8.10157 m}{120 + 42 \sec}$ $= \dfrac{8.10157}{162} m/s$ $= 0.050$ m/s	M1 (speed with time convert to seconds) A1

Answer : ……………………… m/s [2]

(v) Given also that $GD : DC = 1 : 2$, find the distance *FE*.

Since $GD : DC = 1 : 2$ Then, $BE : EC = 1 : 2$ (similar triangles) $BE = \dfrac{12}{3} = $ **4 m** $\angle CBF = 180 - 90 - 67.6887 = $ **22.3113°** Using cosine rule, $FE^2 = 3^2 + 4^2 - 2(3)(4)(\cos 22.3113°)$ $FE = 1.67234$ $\quad = $ **1.67 m** (3 s.f)	B1 (BE = 4m) M1 (using *'their'* 67.7 from (i)) M1 (apply cosine rule) A1

Answer : ………………………… m [4]

8. **(a)** A publishing company published a book, which was sold at $32.80 per physical copy.

The company agreed to pay the author:

5% of the selling price for first 3000 copies,

8% of selling price for the next 2000 copies sold, and

10% of the selling price for the remaining copies sold.

(i) If 7945 copies of the books were sold, calculate the amount that the author received.

Amount for first 3000 copies $= \dfrac{5}{100} \times 3000 \times 32.80$ $= \$4920$	M1 (finding first 3000)
Amount for next 2000 copies $= \dfrac{8}{100} \times 2000 \times 32.80$ $= \$5248$	
Amount for remaining 2945 copies $= \dfrac{10}{100} \times 2945 \times 32.80$ $= \$9659.60$	B1 ($9659.60)
Total author received $= \$4920 + \$5248 + \$9659.60$ $= \$ 19\ 827.60$	A1

Answer : $ .... [3]

(ii) Electronic version of the books (e-book) were sold at a discounted price. The company also agrees to pay the author using the same payment arrangement for physical copy.

If the author sold 3450 copies of the e-books and received $5948.28, calculate the percentage discount.

Let the discounted price be x. $\dfrac{5x}{100} \times 3000 + \dfrac{8x}{100} \times 450 = 5948.28$ $150x + 36x = 5948.28$ $186x = 5948.28$ $x = 31.98$ Percentage discount $= \dfrac{32.80 - 31.98}{32.80} \times 100$ $= \mathbf{2.5\ \%}$	M1 (forming an eqution O.E) B1 ($31.98) M1 (using *their* discounted price) A1	*Alternative:* First 3000 =($32.8×3000)×0.05=$4920 Next 450 = ($32.8×450)×0.08 =$1180.80 Total = $4920 + 1180.80 = $6100.80 ------- B1 Discount =$6100.80 – 5948.28 = $152.52 ---- B1 % discount = $\dfrac{152.52}{6100.8} \times 100$ ------ M1 = 2.5% ----- A1

Answer : ....% [4]

(iii) The author decides to save $20 000 in bank at an annual interest rate of 3.6%, per annum, compounded monthly.

How much money will the author have in the bank at the end of 4 years?

Amount of money in bank end of 4 years $$= \$20000\left(1+\frac{0.3}{100}\right)^{48}$$ $= \$23\ 092.70$	M1 (apply compound interest formula with either correct interest '0.3' OR correct $n = 48$) A1

Answer : $ .... [2]

(b) The publishing company's number of books sold and revenue for the year 2020 is shown below.

	2020	
	Number of books sold	*Revenue ($ million)*
Hard Copy Books	793 000	24.5
E-Books	549 000	20.1

(i) Calculate the total number of books (both hard copy and e-books) sold in 2020.
Leave your answer in standard form.

1.342×10^6 -- B1

Answer : .... [1]

(ii) The revenue from sales of books for the year 2019 was $41 735 000.
Calculate the percentage increase in revenue from 2019 to 2020.

Percentage increase $= \dfrac{(24.5+20.1)-41.735}{41.735}\times100$ $= \dfrac{44.6-41.735}{41.735}\times100$ $= \textbf{6.86 \%}$	M1 (% increase) B1 (either '41.735' or 44 600 000) A1

Answer : .... % [3]

9. A number of bacteria are introduced to a culture.

The number of bacteria, *y*, in the culture *t* hours after they are first introduced, is given by the formula

$$y = 40 \times 1.5^{t}$$

The table shows some corresponding values of *t* and *y*, correct to 3 significant figures.

t	0.5	1	2	3	4	5
y	*p*	60	90	135	203	304

(a) Calculate the value of *p*.

49.0   -- B1

Answer : *p* = …………………………… [1]

(b) What was the initial number of bacteria?

40   -- B1

Answer : ……………..………………… [1]

(c) On the grid found in the next page, using a scale of 2 cm to represent 50 units on the *y*–axis and 2 cm to represent 1 unit on the *t*–axis, draw the graph of $y = 40 \times 1.5^{t}$ for $0 \le t \le 5$.

[3]

(d) Use your graph to find how many hours it takes for the number of bacteria to reach 150.

3.25   -- B1
(accept 3.2 to 3.35)

Answer : …………..………………… [1]

(e) (i) By drawing a tangent, find the gradient of the curve at (2, 90).

Tangent line ---- B1
Gradient = **36.7**   ---B1
(accept 32 to 42)

Answer : ……………..………………… [2]
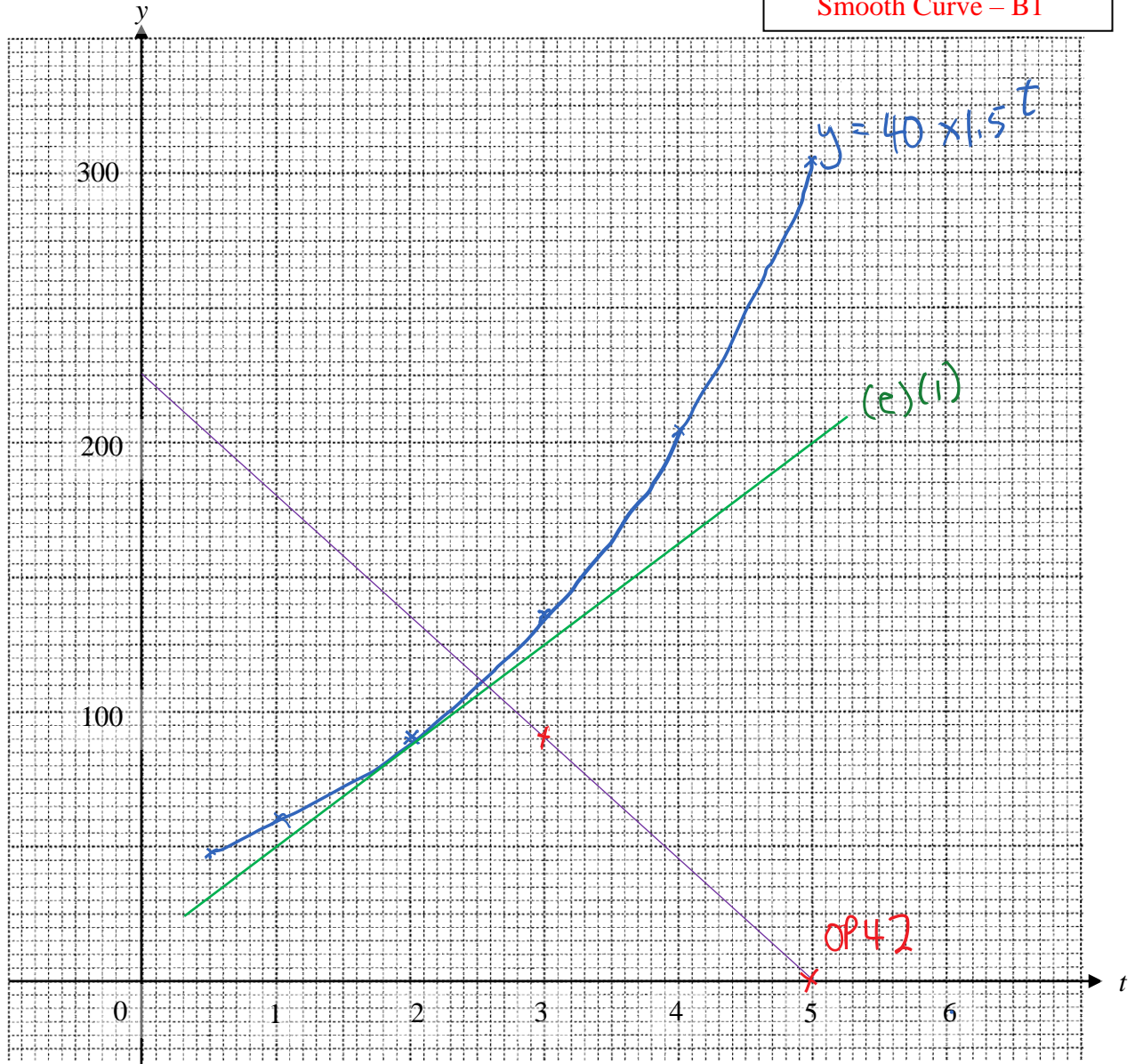
(ii) What does this gradient represent?

The rate at which the bacteria grow per hour, when $t = 2$ hr. ---- B1

Answer : …………………………………………………………………………………… [1]

Answer (c):

$y = 40 \times 1.5^t$

(e)(i)

OP42

(f) A species of organism 'OP42' was introduced into the same culture.

The population of organism 'OP42' declined at a steady rate.
At $t = 3$, its population reduced to 90.
At $t = 5$, the population reached zero.

Find the equation of the line that represents the 'OP42' population.

(3, 90) and (5, 0) Gradient $= \dfrac{90-0}{3-5} = -45$ Using (5, 0) : $0 = -45(5) + c$ $\qquad\qquad c = 225$ Equation is $y = -45x + 225$	*drawing of line not required* M1 (gradient) A1 (accept if c was found graphically)

Answer : …………………………….. [2]

10. The diagram shows a kids' pool in a refurbished hotel.
Its surface is made up two sections; a mat area in the shape of a segment and the remaining circle, with centre O, makes up the actual pool.

The radius of the circle is 4 m and the area of the minor sector OPQ is 11.2 m^2.
The depth of the pool is 0.5 m (not including the mat area).
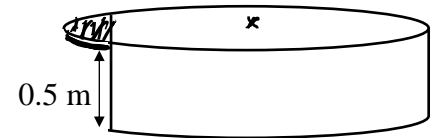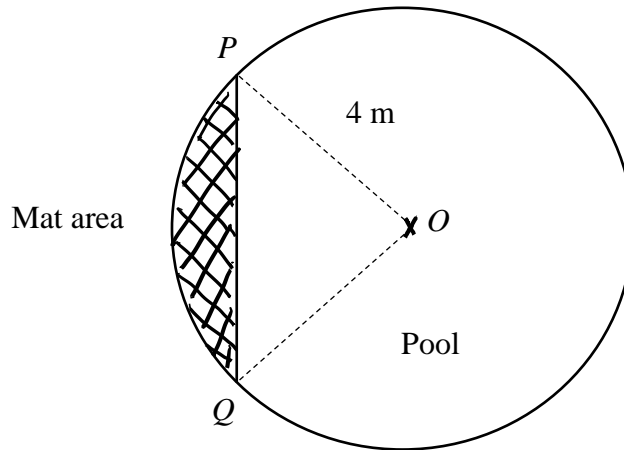
(a) (i) Find angle POQ in radians.

$\frac{1}{2}r^2\theta =$ Area of sector $\frac{1}{2}(4)^2\theta = 11.2$ Angle $POQ = 1.4$ rad	B1 (working shown)

Answer : Angle POQ = ……………………… rad [1]

(ii) By showing all calculations clearly, show that the capacity of the kids' pool is approximately 23.47 m^3.

Area circle $= \pi \times 4^2$ $\quad = 16\pi$ Area $\Delta POQ = \frac{1}{2} \times 4 \times 4 \times \sin 1.4$ $\quad = 7.88359$ m^2 Area segment (mat area) = 11.2 − 7.88539 $\quad = 3.3164$ m^2 Capacity of pool (volume) $= (16\pi - 3.3164) \times 0.5$ $= 46.949 \times 0.5$ $= 23.47454$ $= 23.47$ m^3	B1 (7.88) M1 (Finding area segment using *their* values) M1 (circle – area segment x 0.5) A1	Alternative: Area $\Delta POQ =$ $\frac{1}{2} \times 4 \times 4 \times \sin 1.4 = 7.88359$ m^2 B1 Area major sector POQ $= \frac{1}{2}(4)^2(2\pi - 1.4)$ $= 39.065$ ------- B1 Volume $= (7.88359 + 39.065) \times 0.5$ --- M1 $= 23.47$ m^3 ----- A1

[4]

(b) The kids' pool is to be filled up with water.

- In order to make it safe to enter the pool, a set of stairs made from 1.05 m³ cement was placed at point *P* and point *Q* **inside the pool**.
- The pool is then set to be filled up to **95% capacity**.

Company Alpha provides water-filling service for swimming pools.

Water Pump	**8 gallons per minute**
Cost of water pump	**$17 per 100 gallons of water**
Labour cost	**$30 per hour**

1 gallon = 3.785 litres
1 litre = 0.001 m³

Mr Loh, the manager in charge, thinks that it will take approximately **10 hours** to fill and at a cost of approximately **$1200**.

Is Mr Loh correct? Justify with calculations.

Capacity after stairs = 23.47 – 1.05 = 22.42 m³	
95% capacity = $\dfrac{95}{100} \times 22.42$ = **21.299 m³**	B1 (21.299)
Convert to litres = $\dfrac{21.299}{0.001} = 21299$ litres	
Convert to gallons = $\dfrac{21299}{3.785} = 5627.212682$ gallons	M1 (÷ 3.785 to gallons)
Cost of water pump = $17 \times \dfrac{5627.212682}{100}$ = $956.62	M1 (finding cost water)
Time taken = $\dfrac{5627.212682}{8} = 703$ min $\qquad = 11.72$ h $\qquad = 12$ hr (round off)	M1 (÷8 to find minutes/hr)
Labour cost = $30 × 12 = $360	B1 $360
Total cost = $956.62 + 360 = $1316.62	
Mr Loh is **incorrect** as it will take approximately **12 hours** with an estimated cost of **$1316.62**.	A1 (their logical conclusion)

……………………………………………………………………………………………… [6]
End of Paper