# Bukit View Secondary School

Secondary Four Express/Five Normal (Academic)
Preliminary Examination 2021

| CANDIDATE NAME | |
|---|---|
| CLASS | INDEX NUMBER |

# MATHEMATICS
**4048/02**

Paper 2

**1 September 2021**

Candidates answer on the Question Paper.

**2 hours 30 minutes**

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## READ THESE INSTRUCTIONS FIRST

Write your candidate name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142, unless the question requires the answer in terms of $\pi$.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

| For Examiner's use |
|---|
| Marks |
| 100 |

**Setter: Mrs Irni Prasad**        **Parent's Signature:**_____

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This question paper consists of **22** printed pages.

# Mathematical Formulae

*Compound Interest*

$$\text{Total amount} = P(1 + \frac{r}{100})^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

*Statistics*

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\frac{\Sigma fx}{\Sigma f})^2}$$

**1.** **(a)** Express $\dfrac{1}{x+3} - \dfrac{x}{x-4}$ as a single fraction.

*Answer* : …………………………… [2]

**(b)** Simplify $\dfrac{15q^4 r^3}{4s^2} \div \dfrac{3q^5}{8s^3}$ .

*Answer* : …………………………… [2]

**(c)**   $V = \dfrac{4\pi}{3}(p^3 - q)$

**(i)** Evaluate $V$ when $p = 2.5$ and $q = -1.8$.

*Answer* : …………………………… [1]

**(ii)** Rearrange the formula $V = \dfrac{4\pi}{3}(p^3 - q)$ to make $p$ the subject.

*Answer* : …………………………… [2]

**(d) (i)** Find the value of $h$ and $k$ if $x^2 - hx + 1 = (x - 3)^2 + k$.

*Answer* : $h =$ ……………… , $k =$ ……………… [2]

**(ii)** Using your answers in **(d)(i),** write down the coordinates of the minimum point of the graph $y = x^2 - hx + 1$.

*Answer* : (…………… , ……………) [1]

**2.** The diagram shows part of a regular $n$-sided polygon with sides *ABCDE*.
*BCJK* is a square.

It given that angle *CBD* = $x°$, angle *BCD* = $y°$ and angle *JCD* = $7x°$.
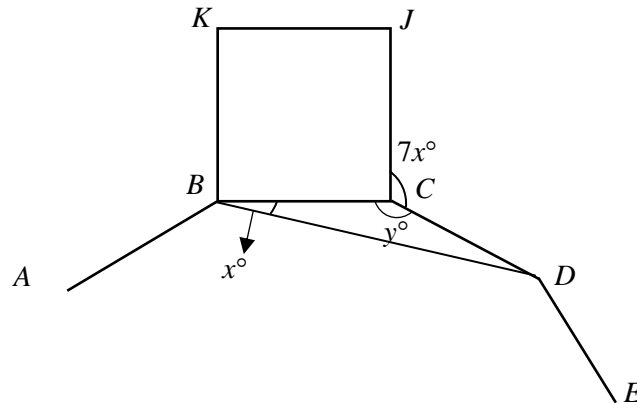


**(a)** By forming two equations and solving them simultaneously, find the value of
$x$ and of $y$.

*Answer :* $x =$ ..................... , $y =$ ........................ [5]

**(b)** Hence, find the value of $n$.

*Answer :* $n =$ ................................... [2]

**3.** The stem and leaf diagram shows the marks obtained by a group of pupils in a Mathematics and English tests.

| Mathematics Marks | | | | | | Stem | English Marks | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 5 | 4 | 2 | 1 | | | | | | |
| | | | | | 8 | 2 | 4 | 4 | 5 | | | |
| | 9 | 4 | 3 | 1 | 0 | 3 | 3 | 5 | 6 | 8 | 8 | 9 |
| | | | | 3 | 2 | 4 | 2 | 2 | 3 | 4 | | |
| | | 9 | 9 | 8 | 6 | 5 | 0 | 1 | | | | |

Key : 2 | 1 means 12                  Key : 2 | 4 means 24

**(i)** Find the median marks for the Mathematic test and English test.

*Answer* : Median Mathematics mark = ……………………. [1]

Median English mark = …...……………….. [1]

**(ii)** Find the interquartile range for the Mathematic test and English test.

*Answer* : IQR Mathematics mark = ……………………. [1]

IQR English mark = …...……………….. [1]

**(iii)** Students who scored at least 42 marks were awarded a distinction grade.

Find the percentage of students who achieved distinction grade in the English test.
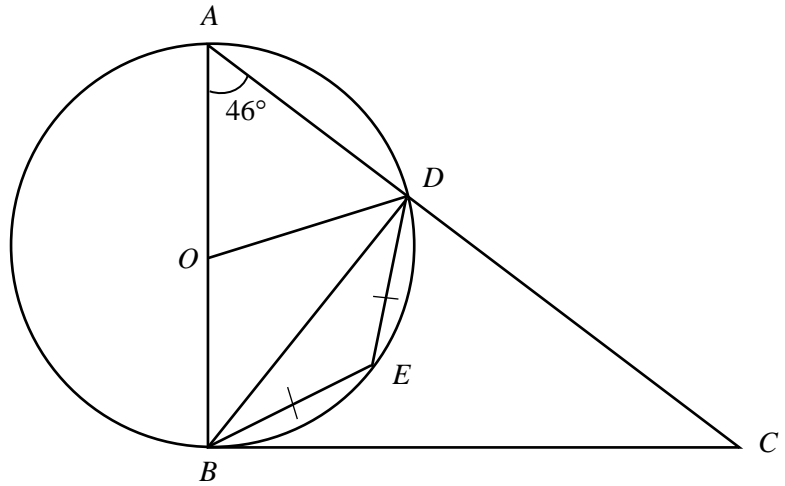
*Answer* : …………….…………….% [1]

**(iv)** Below are two statements comparing the marks for the two tests.
For each one, write whether you agree or disagree, giving a reason for each answer. [2]

| Statement | Agree/ disagree | Reason |
|---|---|---|
| The Mathematics test is easier. | | |
| There is a greater spread of marks for the Mathematics Test. | | |

**4.** In the diagram, $O$ is the centre of the circle passing through points $A$, $B$, $E$ and $D$.
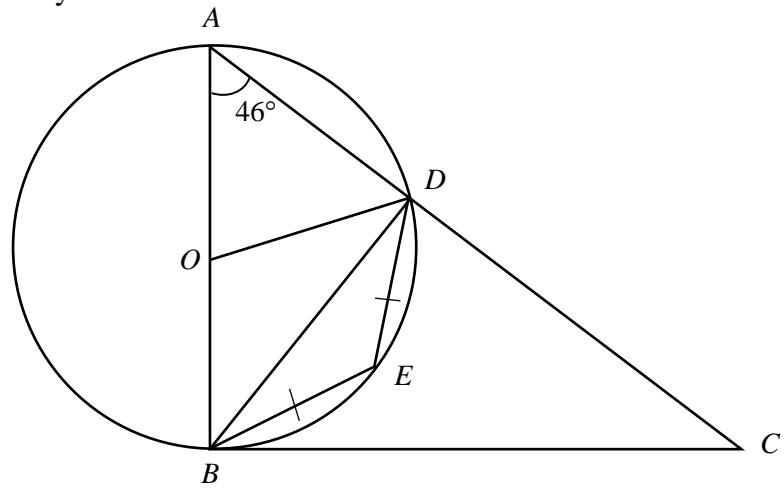
The tangent to the circle at $B$ meets $AD$ produced at $C$.

$AOB$ is a diameter of the circle, $BE = ED$ and angle $BAD = 46°$.



**(a)** Prove that triangle $ADB$ is similar to triangle $ABC$.

[3]

**(b)** Find the following angles.
Show your reason(s) clearly.



**(i)** Angle *BOD*

*Answer* : …………….……………° [2]

**(ii)** Angle *BED*

*Answer* : …………….……………° [2]

**(iii)** Angle *EBC*

*Answer* : …………….……………° [3]

**5.** **(a)** A bag contains 18 red balls and 7 green balls.
Two balls are taken from the bag without replacement.

**(i)** Complete the tree diagram to show the probabilities of the possible outcomes.

$$\left(-\right) \quad \text{Red}$$

Red

$$\left(-\right)$$

$$\left(-\right) \quad \text{Green}$$

$$\left(-\right) \quad \text{Red}$$

$$\left(-\right)$$
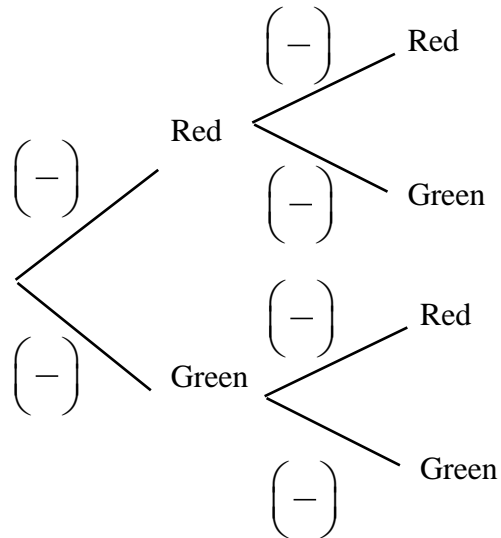
Green

$$\left(-\right) \quad \text{Green}$$

[2]

**(ii)** Find, as a fraction in its simplest form, the probability that

**(a)** the two balls are the same colour,

*Answer* : …………….……………. [1]

**(b)** at least one of the balls is green.

*Answer* : …………….……………. [2]

**(iii)** If a third ball is picked, find the probability that none of the balls is red.

*Answer* : …………….……………. [2]

**(b)** A gaming app allows a second chance at the game if a player resulted in a draw.

The probability of a win is $\dfrac{5}{6}$ .

The probability of a draw is $p$.

The probability of a loss is $q$.

Given that the probability of a win at the second try is $\dfrac{5}{144}$ , find the value of $q$.

*Answer* : $q$ = …………………………….. [2]

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**6.** **(a)** The $n$th term of a number sequence $T_n$, is given by $T_n = \dfrac{1+n}{12}$.

Fill in the table for the terms $T_2$, $T_3$ and $T_4$ in the simplest form.
$T_1$ has been filled in for you.

| $n$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $T_n$ | $\dfrac{1}{6}$ | | | |

[2]

**(b)** The first four terms in another number sequence is as shown below.

$$P_1 = 3^2 - 5 = 4$$
$$P_2 = 4^2 - 6 = 10$$
$$P_3 = 5^2 - 7 = 18$$
$$P_4 = 6^2 - 8 = 28$$

**(i)** Write down $P_5$ .

*Answer* : ……………..……………… [1]

**(ii)** Show that the $n$th term $P_n$ of this sequence is given by $P_n = n(n+3)$ .

*Answer* :

[2]

**(iii)** Given that $\dfrac{P_n}{T_n} = 299$, form an equation in $n$ and show that it reduces to

$12n^2 - 263n - 299 = 0$.

*Answer* :

[3]

**(iv)** Hence, find the value of $n$.

*Answer* : $n =$ ……………………………… [2]

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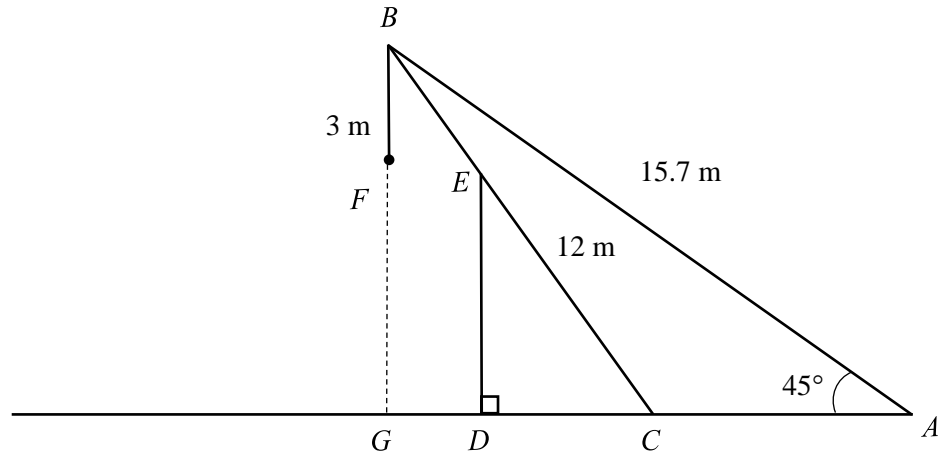
**7.** The diagram shows a metal structure *ABCDE*, where *ACDG* is on the ground.

A load *F* hangs vertically down from *B* where *BF* = 3 m.
*G* is directly below the load *F*.
The angle of elevation of *B* from *A* is 45°.

It is given that *AB* = 15.7 m, *BC* = 12 m, and angle *CDE* = 90°.



**(i)** Calculate angle *BCA*.

*Answer* : ……………………………° [3]

**(ii)** Without finding any lengths, explain why *AG* = *BG*.

*Answer* : …………………………………………………………………………….

……………………………………………………………………………………………

..…………………………………………………………………………………… [1]

**(iii)** Calculate how far the load *F* is above the ground.

*Answer* : ………………………… m [2]

**(iv)** The load is being lowered from *F* to the ground at *G*.
It took 2 minutes and 42 seconds.

Find the speed in m/s, correct to 3 decimal places.

*Answer* : ……………………… m/s [2]

**(v)** Given also that *GD* : *DC* = 1 : 2 , find the distance *FE*.

*Answer* : ………………………… m [4]

**8.** **(a)** A publishing company published a book, which was sold at $32.80 per hard copy.

The company agreed to pay the author:

5% of the selling price for first 3000 copies,

8% of selling price for the next 2000 copies sold, and

10% of the selling price for the remaining copies sold.

**(i)** If 7945 copies of the books were sold, calculate the amount that the author received.

*Answer* : $ ............................................... [3]

**(ii)** Electronic version of the books (e-book) were sold at a discounted price. The company also agrees to pay the author using the same payment arrangement for hard copies.

If the author sold 3450 copies of the e-books and received $5948.28, calculate the percentage discount.

*Answer* : ...............................................% [4]

**(iii)** The author decides to save $20 000 in a bank at an annual interest rate of 3.6% per annum, compounded monthly.

How much money will the author have in the bank at the end of 4 years?

*Answer* : $ …….......................................... [2]

**(b)** The publishing company's number of books sold and revenue for the year 2020 is shown below.

| | **2020** | |
| --- | --- | --- |
| | *Number of books sold* | *Revenue ($ million)* |
| **Hard Copy Books** | 793 000 | 24.5 |
| **E-Books** | 549 000 | 20.1 |

**(i)** Calculate the total number of books (both hard copy and e-books) sold in 2020.
Leave your answer in standard form.

*Answer* : ................................................ [1]

**(ii)** The revenue from sales of books for the year 2019 was $41 735 000.
Calculate the percentage increase in revenue from 2019 to 2020.

*Answer* : .......................................... % [3]

**9.** A number of bacteria are introduced to a culture.

The number of bacteria, $y$, in the culture $t$ hours after they are first introduced, is given by the formula

$$y = 40 \times 1.5^{\,t}$$

The table shows some corresponding values of $t$ and $y$, correct to 3 significant figures.

| $t$ | 0.5 | 1 | 2 | 3 | 4 | 5 |
|-----|-----|---|---|---|---|---|
| $y$ | $p$ | 60.0 | 90.0 | 135 | 203 | 304 |

**(a)** Calculate the value of $p$.

*Answer* : $p =$ ……………………………… [1]

**(b)** What was the initial number of bacteria?

*Answer* : …………..…………………… [1]

**(c)** On the grid found in the next page, using a scale of 2 cm to represent 50 units on the $y$–axis and 2 cm to represent 1 unit on the $t$–axis, draw the graph of $y = 40 \times 1.5^{\,t}$ for $0 \le t \le 5$ .

[3]

**(d)** Use your graph to find how many hours it takes for the number of bacteria to reach 150.
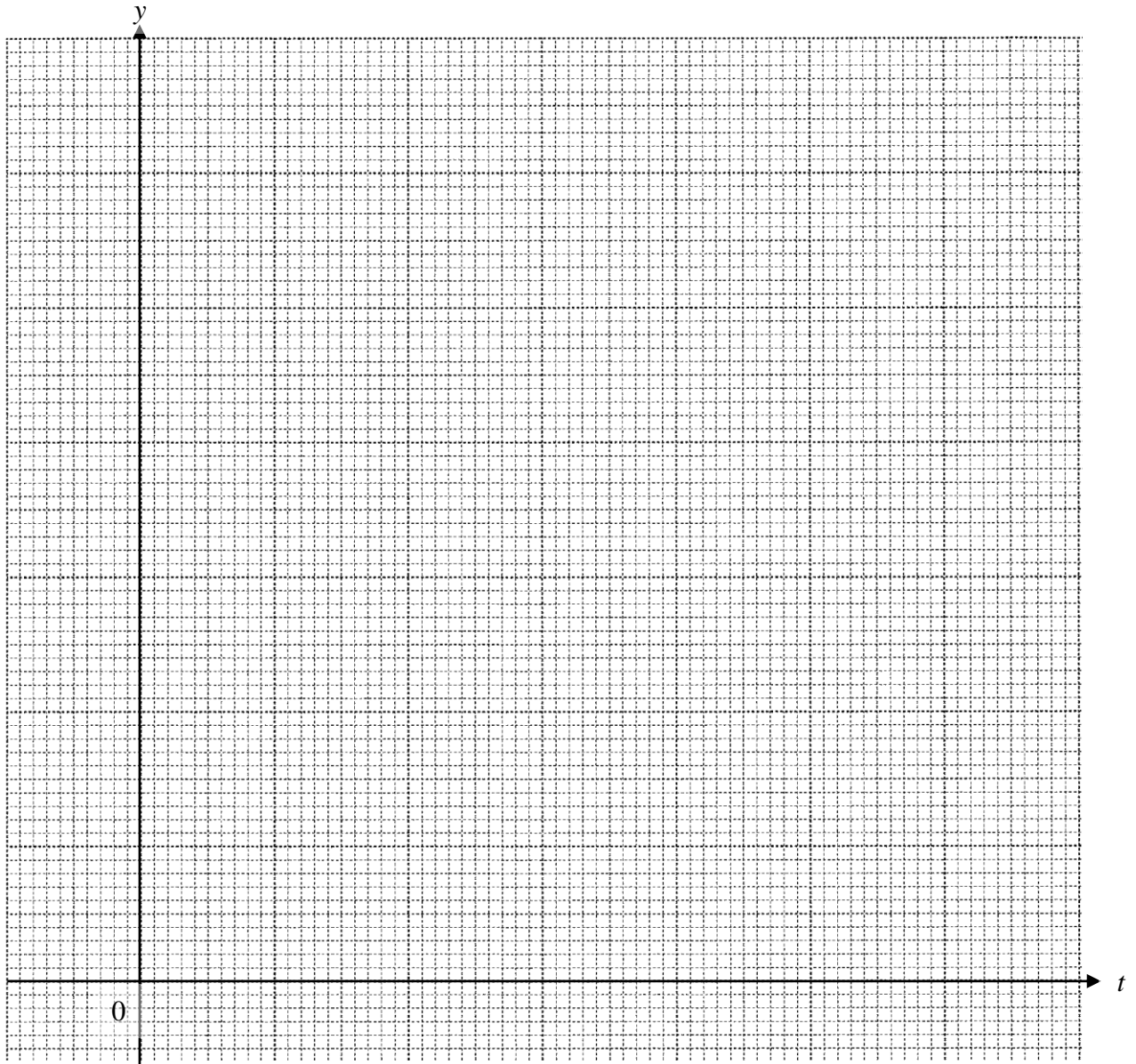
*Answer* : …………..………………… h [1]

**(e) (i)** By drawing a tangent, find the gradient of the curve at (2, 90).

*Answer* : …………..………………… [2]

**(ii)** What does this gradient represent?

*Answer* : ……………………………………………………….………… [1]

**Answer (c):**



**(f)** A species of organism 'OP42' was introduced into the same culture.

The population of organism 'OP42' declined at a steady rate.
At $t = 3$, its population reduced to 90.
At $t = 5$, the population reached zero.

Find the equation of the line that represents the 'OP42' population.

*Answer* : ……………………………….. [2]

**10.** The diagram shows a kids' pool in a refurbished hotel.

Its surface is made up two sections; a mat area in the shape of a segment and the remaining circle, with centre $O$, makes up the actual pool.

The radius of the circle is 4 m and the area of the minor sector $OPQ$ is 11.2 m$^2$.
The depth of the pool is 0.5 m (not including the mat area).
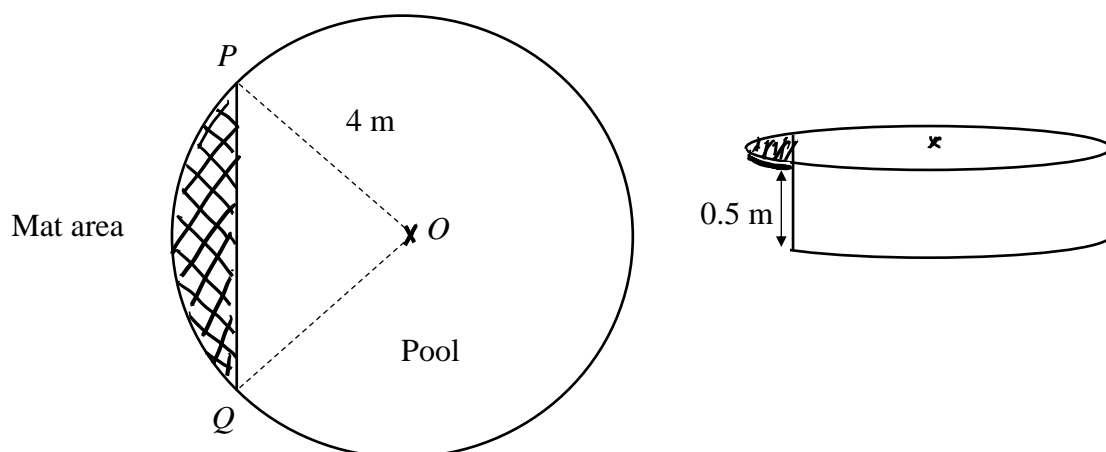


**(a) (i)** Find angle $POQ$ in radians.

Answer : Angle $POQ$ = ………………………… rad [1]

**(ii)** By showing all calculations clearly, show that the capacity of the kids' pool is approximately 23.47 m$^3$.

[4]

**(b)** The kids' pool is to be filled up with water.

- In order to make it safe to enter the pool, a set of stairs made from 1.05 m³ cement was placed at point *P* and point *Q* **inside the pool**.
- The pool is then set to be filled up to **95% capacity**.

Company Alpha provides water-filling service for swimming pools.

| Water pump | **8 gallons per minute** |
|---|---|
| Cost of water pump | **$17 per 100 gallons of water** |
| Labour cost | **$30 per hour** |

*1 gallon = 3.785 litres*
*1 litre = 0.001 m³*

Mr Loh, the manager in charge, thinks that it will take approximately **10 hours** to fill and at a cost of approximately **$1200**.

Is Mr Loh correct? Justify with calculations.

*[Working and answer space to be continued onto the next page.]*

……………………………………………………………………………………………………………..

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…………………………………………………………………………………………………… [6]

**End of Paper**