

1 A sequence is given by the formula $P_{n+1} = (P_n)^2 + mP_n$, where m is a constant.

(a) Given that $P_1 = 3$, show that $P_2 = 3m + 9$.

Answer

[1]

(b) Given that $P_2 = -\frac{3}{4}$, find the value of m .

Answer $m = \dots\dots\dots$ [2]

(c) (i) By using the answer in (b), find P_3, P_4 and P_5 .

Answer $P_3 = \dots\dots\dots$

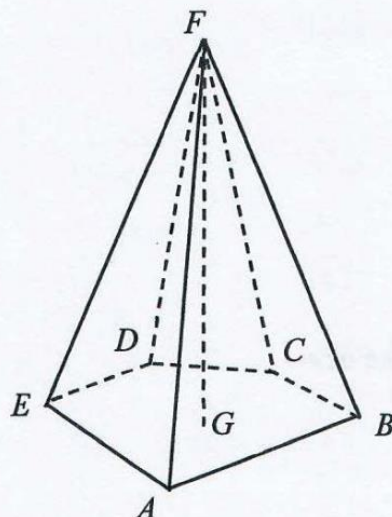
$P_4 = \dots\dots\dots$

$P_5 = \dots\dots\dots$ [3]

(ii) By considering the terms of P_1, P_2, P_3, P_4 and P_5 or otherwise, find the value of P_{2016} .

Answer $\dots\dots\dots$ [1]

- 2 The diagram shows a pyramid $ABCDEF$. The base of the pyramid is a regular pentagon of side 6 cm. The tip F is vertically above the centre of the pentagon, G , and $AF = 14$ cm.



- (a) Calculate the angle AGB .

Answer ° [1]

- (b) Show that $AG = 5.1039$ cm, correct to five significant figures.

Answer

[2]

- (c) Calculate the height of the pyramid, FG .

Answer $FG =$ cm [2]

- (d) The pyramid is the model for a paper weight that is to be gold plated. To reduce costs the pyramid is made smaller such that the smaller pyramid remains geometrically similar to the original pyramid but its height is reduced by 35%.

The surface area of the large pyramid is $S \text{ cm}^2$. Express the surface area of the new pyramid as a percentage of S .

Answer % [3]

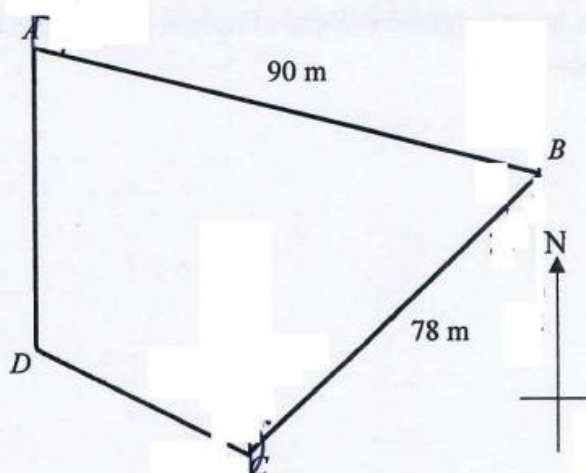
- 3 (i) Given that the points $P(3, k)$, $Q(1, -2)$ and $R(-4, -6k)$ lie on a straight line, find the value of k .

Answer $k =$ [3]

- (ii) Find the length of the line segment PQ .

Answer [2]

- 4 Quadrilateral $ABCD$ is a field with $AB = 90$ m and $BC = 78$ m and A is due north of D . The bearing of B from A is 100° , the bearing of B from C is 025° and the bearing of D from C is 278° .



- (a) (i) Show that angle $ABC = 75^\circ$.

Answer

[2]

- (ii) Calculate the bearing of C from D .

Answer $^\circ$ [2]

- (iii) Calculate the length of AC .

Answer m [3]

- (b) A drone hovers at a height of 70 m above D . A man of height 1.75 m walks along path AC . He stopped at E to take a picture of the drone when the maximum angle of depression from the drone to the top of the man's head was 58° . Calculate the

- (i) length of DE ,

Answer m [2]

- (ii) area of the field $ABCD$.

Answer m^2 [3]

- 5 Adam runs a drink stall franchise in 4 locations. The number of cups for each type of drink sold a day is shown in the table below.

		Types of Drink		
		Coffee	Tea	Fruit Juice
Location	Branch A	60	42	5
	Branch B	24	30	0
	Branch C	30	35	14
	Branch D	30	40	20

- (a) Represent the above information as a 4×3 matrix P .

$$\text{Answer } P = \begin{pmatrix} & & \\ & & \\ & & \\ & & \end{pmatrix} [1]$$

- (b) The price of drinks are shown in the table below.

Drink	Price (SGD\$)
Coffee	1.50
Tea	1
Fruit Juice	2

Represent the above information as a 3×1 matrix N .

$$\text{Answer } N = \begin{pmatrix} \\ \\ \end{pmatrix} [1]$$

- (c) Evaluate the matrix $T = PN$.

$$\text{Answer } T = \begin{pmatrix} \\ \\ \end{pmatrix} [1]$$

- (d) State what each of the elements of matrix T represents.

Answer

.....

..... [1]

- (e) (i) The cost of all the ingredients per day for Branch A , B , C and D is shown in the table below.

Branch	All ingredients for drinks	
	In USD	In SGD
A	p	27
B	12	16.20
C	16	21.60
D	23	q

Find the value of p and of q .

Answer $p = \dots\dots\dots$ and $q = \dots\dots\dots$ [2]

- (ii) The rental and operating cost (excluding the cost of ingredients) per day for Branch A , B , C and D is shown in the table below.

Branch	Rental & Operating Cost (SGD)
A	40
B	45
C	50
D	60

Using matrix operations, calculate the total amount of profit in SGD Adam made that day.

Answer \$..... [2]

- 6 44 boys ran the 2.4 km and their timings are shown in the table.

t (minutes)	$8 \leq t < 9$	$9 \leq t < 10$	$10 \leq t < 11$	$11 \leq t < 12$	$12 \leq t < 13$	$13 \leq t < 14$
Frequency	1	h	12	11	k	6

- (a) The estimated mean timing is 11.477 minutes. Estimate to the nearest integer the value of h and the value of k .

Answer $h = \dots\dots\dots$

$k = \dots\dots\dots$ [4]

- (b) Estimate the standard deviation.

Answer $\dots\dots\dots$ minutes [1]

- (c) Explain why in this case, the mean is better than the median as a measure of central tendency.

Answer [1]

- (d) Another group of 35 boys ran the 2.4 km and their mean and standard deviation were 11.7 minutes and 2.10 minutes respectively. Comment on the timings of these two groups of boys.

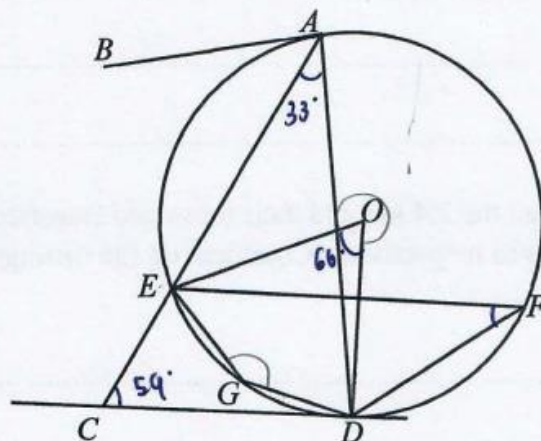
Answer [2]

- 7 It is given that point A lies on the y -axis while point B lies on the x -axis such that $OB = 2OA$, where O is the origin.

Given that the line AB passes through the point $\left(4, \frac{5}{2}\right)$, find a possible equation of the line AB .

Answer [3]

- 8 In the diagram, line AB and line CD are tangents to point A and point D respectively on the circumference of the circle with centre O . Angle $DAE = 33^\circ$, angle $ECD = 59^\circ$ and AEC is a straight line. E , F , and G are points on the circumference of the circle.



- (a) Find angle EOD .

$$\angle EOD = 2(33^\circ) = 66^\circ \quad (\angle \text{at circumference} = 2 \angle \text{at centre}).$$

Answer 66° $^\circ$ [1]

- (b) Find angle EFD .

$$\angle EFD = \angle EAD = 33^\circ \quad (\angle s \text{ in same segment})$$

Answer 33° $^\circ$ [1]

- (c) Find angle EGD .

Answer $^\circ$ [1]

- (d) A circle is drawn with the line AC as its diameter. Explain why point D will not lie on the circumference of the circle.

Answer

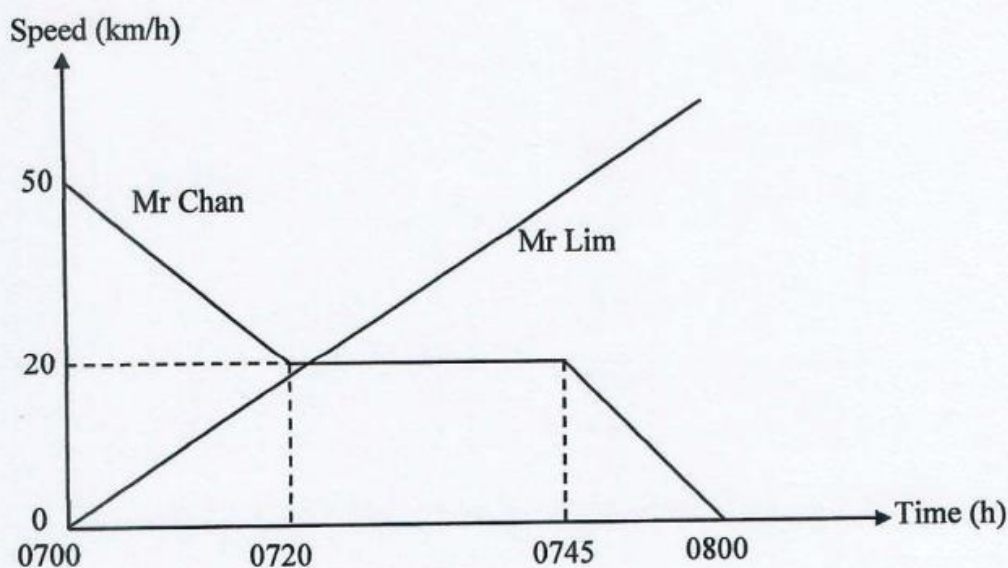
[2]

- (e) Line AB and CD are extended and meet at T . Show that angle $ATD = 4^\circ$.

Answer

[3]

- 9 Mr Chan driving a car at 50 km/h passes a lamppost A and stops at lamppost B , one hour later. When Mr Chan passes the lamppost A , Mr Lim, on a motorcycle, starts from A and overtakes Mr Chan. The motorcycle has uniform acceleration of 80 km/h^2 . The speed-time graphs of Mr Chan and Mr Lim are shown in the diagram.



- (a) Find the
(i) speed of the car at 0715 h,

Answer km/h [2]

- (ii) speed of the motorcycle at 0745 h,

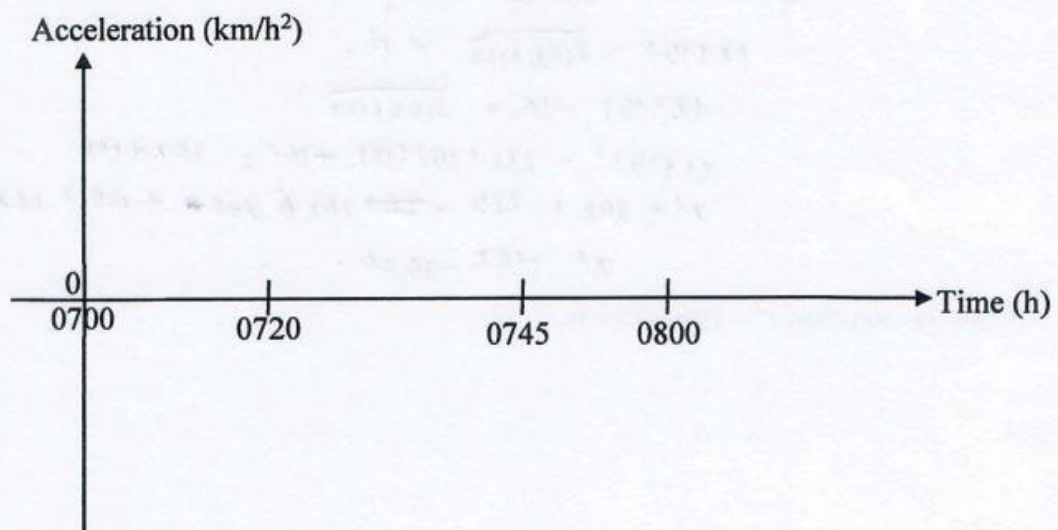
Answer km/h [1]

- (iii) time, to the nearest minute, the motorcycle overtakes the car, given that it was between 0720 h and 0745 h.



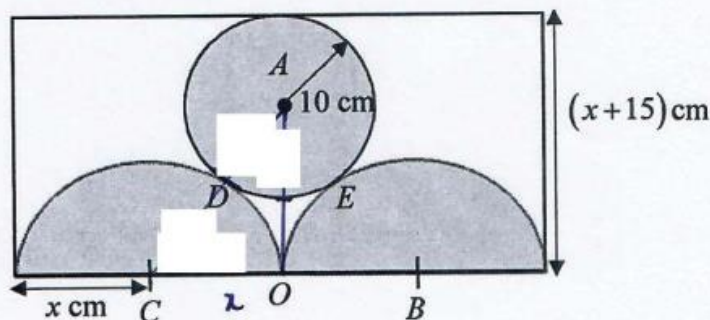
Answer [4]

- (b) Sketch the acceleration time graph for Mr. Chan.



[2]

- 10 The diagram below shows a rectangle with breadth $(x + 15)$ cm. The circle with centre at A has a radius of 10 cm. The semicircle with centre at B and the semicircle with centre C are congruent and each has a radius of x cm. The small circle with centre A touches the semicircles at point D and E . The line AO bisects the length of the rectangle and is a tangent to both of the semicircles.



- (a) State the length AC , in terms of x .

Answer cm [1]

- (b) State the length OA , in terms of x .

Answer cm [1]

- (c) Hence, write down an equation and show that it simplifies to $x^2 - 10x - 75 = 0$.

Answer

[3]

- (d) Solve the equation $x^2 - 10x - 75 = 0$.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [2]

- (e) Hence, find the shaded area.

Answer cm^2 [2]

- 11 A couple intends to purchase a HDB flat and they intend to take a loan from a financial institution.

The formula to calculate the monthly mortgage payment is given by

$$M = \frac{P \left(\frac{i}{100} \left(1 + \frac{i}{100} \right)^n \right)}{\left(\left(1 + \frac{i}{100} \right)^n - 1 \right)}$$

Where M is the monthly mortgage payment, P is the principal loan amount, i is the monthly interest rate, and n is the number of months required to repay the loan.

(Source: <https://www.businessinsider.com/personal-finance/how-to-calculate-mortgage-payment#:~:text=If%20you%20want%20to%20do,0.04%2F12%20%3D%200.0033>).

- (a) If the couple takes a \$100000 loan to be repaid in 10 years, find the monthly mortgage payment, assuming an interest rate of 2% per annum.

Answer \$..... [3]

The couple intends to take a loan from a financial institution, so they will need to pay 25% of the price as down-payment. For the down-payment, they intend to pay up to \$50000.

An online search yielded information in the tables below.

2021 Property Prices in Singapore

Type	HDB BTO Flats (Non-Mature Estates)	HDB BTO Flats (Mature Estates)	Resale Flats
Two-Room (Flexi)	\$90,000 to \$162,000	\$137,000 to \$277,000	—
Three-Room	\$164,000 to \$248,000	\$205,000 to \$421,000	\$350,000 to \$380,000
Four-Room	\$253,000 to \$381,000	\$311,000 to \$617,000	\$420,000 to \$550,000
Five-Room	\$405,000 to \$516,000	\$423,000 to \$725,000	\$520,000 to \$700,000

Source: <https://www.singsaver.com.sg/blog/costs-of-bto-flat-resale-flat-cc-and-condo-in-singapore>

HDB Flat Types	2-Room Flexi	3-Room	4-Room	5-Room
Approx. floor area (square metres)	36 and 45	60 to 65	90	110
Total no. of bedrooms	1	2	3	3
Total no. of bathrooms	1	2	2	2

(Source: <https://www.hdb.gov.sg/residential/buying-a-flat/resale/getting-started/types-of-flats>)

- (b) Determine all the types of flats that the couple can consider purchasing.

Answer

[3]

- (c) Based on the information given in the tables only, give the type of flat that gives the best value for the money spent. State one assumption that the couple could have made.

Answer

Assumption:

.....

.....

[3]

- 12 The variables of x and y are connected by the equation $y = \frac{x^2}{6} + \frac{2}{x} - 3$.

Some corresponding values of x and y , correct to two decimal places, are given in the table below.

x	0.5	1	1.5	2	2.5	3	4	5	6
y	p	-0.83	-1.29	-1.33	-1.16	-0.83	0.17	1.57	3.33

- (a) Find the value of p .

Answer $p = \dots\dots\dots$ [1]

- (b) Use a scale of 2 cm to represent 1 unit, draw a horizontal x -axis from $0 \leq x \leq 6$. Use a scale of 4 cm to represent 1 unit, draw a vertical y -axis from $-2 \leq y \leq 4$.

On your axes, plot the points given in the table and join them with a smooth curve.

[3]

- (c) Use your graph to find the solutions of $\frac{x^2}{6} + \frac{2}{x} - 2 = 0$.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [2]

- (d) By drawing a tangent, find the gradient of the curve at $(3, -0.83)$.

Answer $\dots\dots\dots$ [2]

- (e) (i) On the same axes, draw the line of with gradient -0.5 that passes through the point with coordinates $(4, -1)$.

[1]

- (ii) Write down the equation of this line.

Answer $\dots\dots\dots$ [1]

- (iii) Write down the x -coordinates of the points where the line intersects the curve.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [2]

- (iv) These values of x are the solutions of the equation $x^3 + Ax^2 - 24x + B = 0$. Find the value of A and of B .

Answer $A = \dots\dots\dots$ and $B = \dots\dots\dots$ [2]