

NAME: _____ ()

CLASS: 4 ()



**ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2021**

S4

MATHEMATICS

4048

Paper 1

26 August 2021

Candidates answer on the Question Paper.

2 hours

READ THESE INSTRUCTIONS FIRST

MARKING SCHEME

Write your name, index number and class in the space at the top of this page.

Write in dark blue or black pen.

You may use a HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiners' Use

Question	1	2	3	4	5	6	7	8	9
Marks									
Question	10	11	12	13	14	15	16	17	18
Marks									
Question	19	20	21	22	23	24	25		
Marks									
Table of Penalties			Units		Clarity / Logic		Precision / Accuracy		
Parent's Name and Signature									
			Total		80				
Date:									

This document consists of 19 printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

1(a) Given that $3^{27} \div 27^3 = 3^k$, find k .

$$\begin{aligned} 3^{27} \div 3^9 &= 3^k \\ 3^{27-9} &= 3^k \\ k &= 18 \end{aligned}$$

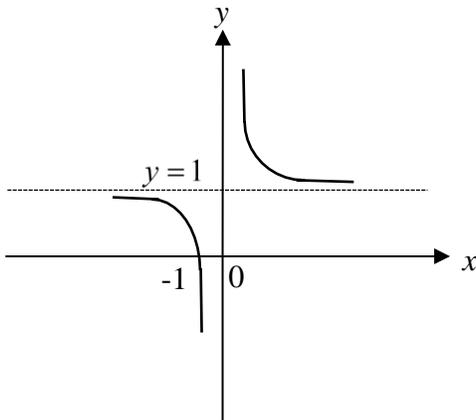
Answer $k =$ _____ [1]

(b) Simplify $\frac{4x^{-4}}{y^3} \times \frac{y^{\frac{2}{3}}}{18} \div \frac{1}{27}$, leaving your answers in positive indices.

$$\begin{aligned} \frac{4y^3}{x^4} \times \frac{y^{\frac{2}{3}}}{18} \times 27 \\ = \frac{6y^{\frac{6}{3}}}{x^4} \\ = \frac{6y^2}{x^4} \end{aligned}$$

Answer _____ [2]

2 The curve below has an equation $y = x^n + c$. State a possible value of n and the value of c .



$$\begin{aligned} y &= x^n + c \\ n &= -1 \\ c &= 1 \end{aligned}$$

Answer $n =$ _____

$c =$ _____ [2]

- 3 Jasmin has 240 two-centimetre cubes. She arranges all of the cubes into a cuboid. The perimeter of the base of the cuboid is 40 cm. Each side of the cuboid has a length greater than 4 cm. Find the height of the cuboid.

$$\begin{aligned} \text{Dimension of cuboid is } 2l \times 2b \times 2h &= 2^3 \times lbh \\ &= 2^3 \times 240 \end{aligned}$$

$$240 = 2^4 \times 3 \times 5$$

$$40 = 4l + 4b$$

$$10 = l + b$$

$$10 = 2 + 2^3 \text{ (rej), } 10 = 2^2 + (2 \times 3)$$

$$2^4 \times 3 \times 5 = (2^2)(2 \times 3)h$$

$$h = 10$$

Height is 20 cm

Answer _____ cm [2]

4. Violet intends to arrange n regular pentagons in a ring. The diagram shows the partially completed ring. Find n .

$$\begin{aligned} \text{Interior angle of the regular pentagon} &= \frac{180(5-2)}{5} \\ &= 108 \end{aligned}$$

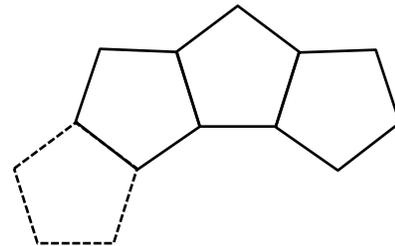
$$\begin{aligned} \text{Interior angle of the regular } n\text{-side polygon form} \\ \text{in the centre of the ring} &= 360 - 2(108) \\ &= 144 \end{aligned}$$

$$144 = \frac{180(n-2)}{n}$$

$$144n = 180n - 360$$

$$36n = 360$$

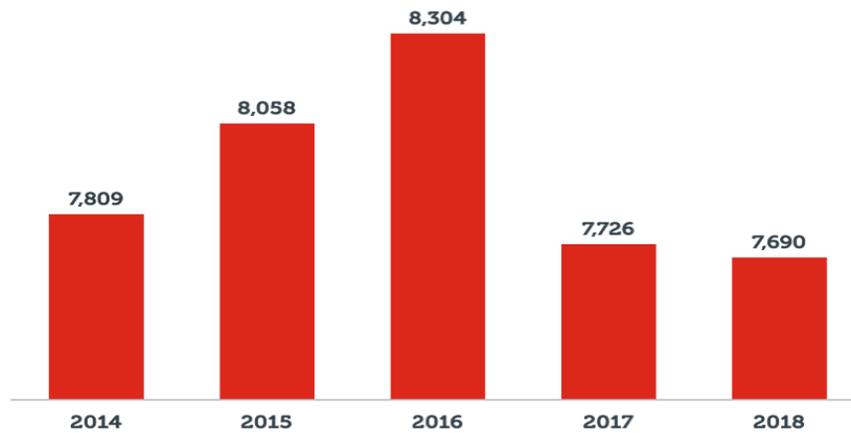
$$n = 10$$



Answer $n =$ _____ [3]

- 5 The bar chart shows the number of traffic accidents resulting in injury from 2014 to 2018. (<https://www.budgetdirect.com.sg/car-insurance/research/road-accident-statistics-in-singapore>)

Number of Accidents Resulting in Injuries (2014 – 2018)



State how this bar chart can be misleading to the reader.

[1]

<p>The bars in the bar chart do not start from zero. The relative heights of the bars can mislead the reader into thinking the differences are larger than what is actually given. For example, The frequency for 2016 is 8304 and the frequency for 2017 is 7726, so the difference is 578, but the height of the 2016 bar is twice that for the 2017 bar.</p>	
---	--

6 (a) Given that

$$\zeta = \{\text{all triangles}\}$$

$$R = \{\text{right-angled triangles}\}$$

$$S = \{\text{triangles with three unequal sides}\}$$

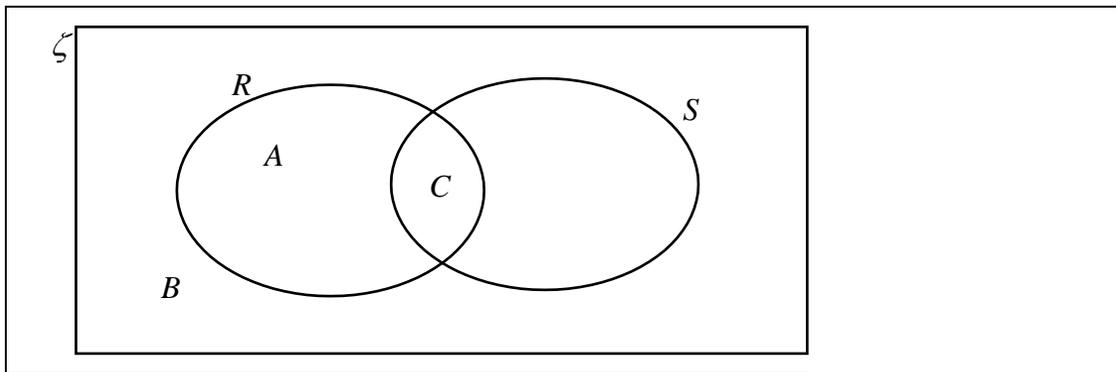
A is a triangle with 45° , 45° and 90° .

B is a triangle with 7 cm, 7 cm and 3 cm.

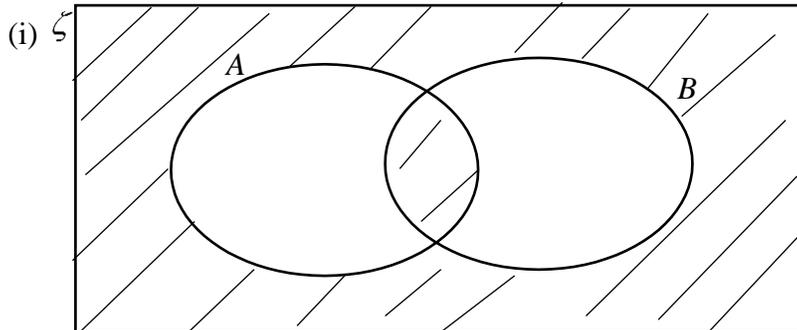
C is a triangle with sides 9 cm, 12 cm and 15 cm.

Represent the above information on a Venn Diagram in the space below.

[2]



(b) Write down the sets represented by the following shaded region



Answer _____ [1]

$(A \cup B)' \cup (A \cap B)$	
-------------------------------	--

7 The speed of light is 3×10^8 m/s. Earth is 150 million km from the sun. How long does light take to travel from the sun to the earth. Round your answer to the nearest minute.

$\frac{150000000}{3 \times 10^5} = 500 \text{ s}$ $= 8.33 \text{ minutes}$ $= 8 \text{ minutes (nearest minute)}$

Answer _____ minutes [2]

- 8 A maximum quadratic curve with the equation $y = -x^2 + bx + c$ has a turning point at $(3, 7)$, find the value of b and of c .

$$y = -(x-3)^2 + 7$$

$$y = -(x^2 - 6x + 9) + 7$$

$$y = -x^2 + 6x - 9 + 7$$

$$y = -x^2 + 6x - 2$$

Therefore, $b = 6$ and $c = -2$.

- 9 Solve the equation $\frac{1}{x+1} - \frac{6x^2 - 10}{1 - x^2} = 4$.

$$\frac{1}{x+1} - \frac{6x^2 - 10}{1 - x^2} = 4$$

$$\frac{1}{x+1} - \frac{6x^2 - 10}{(1-x)(x+1)} = 4$$

$$\frac{1-x}{(1-x)(x+1)} - \frac{6x^2 - 10}{(1-x)(x+1)} = 4$$

$$\frac{1-x-6x^2+10}{(1-x)(x+1)} = 4$$

$$\frac{-6x^2 - x + 11}{(1-x)(x+1)} = 4$$

$$-6x^2 - x + 11 = 4 - 4x^2$$

$$-6x^2 - x + 11 + 4x^2 - 4 = 0$$

$$-2x^2 - x + 7 = 0$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(-2)(7)}}{2(-2)}$$

$$x \approx 1.6375 \text{ or } x \approx -2.1375$$

$$x \approx 1.64 \text{ or } x \approx -2.14$$

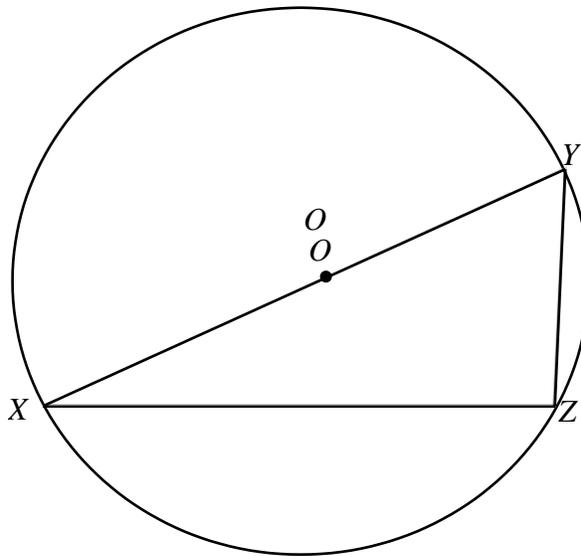
Answer $x =$ _____ or $x =$ _____ [4]

10 Simplify $\frac{27 - 12x^2}{-3 - 2x^2 + 5x} \times \frac{1 - x}{-2x - 3}$.

$$\begin{aligned} & \frac{27 - 12x^2}{-3 - 2x^2 + 5x} \times \frac{1 - x}{-2x - 3} \\ &= \frac{3(9 - 4x^2)}{(-2x + 3)(x - 1)} \times \frac{1 - x}{-2x - 3} \\ &= \frac{3(3 - 2x)(3 + 2x)}{(-2x + 3)(x - 1)} \times \frac{-(x - 1)}{-(2x + 3)} \\ &= 3 \end{aligned}$$

Answer _____ [3]

- 11 In the diagram below, XZ is the chord of a circle. XY is the diameter of the circle, centre O . Given that $XZ = 9$ cm and $YZ = \sqrt{63}$ cm, calculate



- (a) the length of XY ,

(a) By using Pythagoras Theorem,

$$\begin{aligned} XY &= \sqrt{9^2 + (\sqrt{63})^2} \\ &= \sqrt{44} \\ &= 12 \end{aligned}$$

Answer $XY =$ _____ [1]

- (b) $\angle YXZ$,

$$\begin{aligned} \text{(b) } \angle YXZ &= \tan^{-1} \left(\frac{\sqrt{63}}{9} \right) \\ &= 41.4^\circ \text{ (1d.p)} \end{aligned}$$

Answer $\angle YXZ =$ _____ $^\circ$ [1]

(c) $\angle YOZ$ in radian, ΔOXZ is an isosceles triangle.

(c) $\angle YOZ = 2 \times \angle YXZ$

$$\begin{aligned} & \text{(1 ext angle = sum of int. opp. angle)} \\ & = 82.8192^\circ \\ & = 1.45 \text{ rad (3sf)} \end{aligned}$$

OR

 ΔOXZ is an isosceles triangle.

$$\begin{aligned} \angle XOZ &= 180^\circ - (2 \times 41.4096)^\circ \\ &= 97.1807^\circ \text{ (}\angle \text{ on a str. line)} \end{aligned}$$

$$\begin{aligned} \angle YOZ &= \pi - \left(\frac{97.1807}{180} \right) \pi \\ &= 1.45 \text{ rad (3s.f)} \end{aligned}$$

Answer $\angle YOZ =$ _____ rad [2](d) the area of the major segment YZ .

$$\begin{aligned} \text{(d) Area of the major segment } YZ \\ &= \text{area of major sector } YZ + \text{area of triangle } YOZ \end{aligned}$$

$$= \frac{1}{2}(6)^2(2\pi - 1.445469) + \frac{1}{2}(6)^2 \sin(1.445469)$$

$$= 105 \text{ cm}^2 \text{ (3s.f)}$$

OR

$$\begin{aligned} \text{Area of major sector } YOZ &= \frac{360 - 82.8192}{360} \times \pi(6)^2 \\ &= 87.0789 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of triangle } YOZ &= \frac{1}{2} \times (6)^2 \times \sin(82.8192) \\ &= 17.8168 \text{ cm}^2 \end{aligned}$$

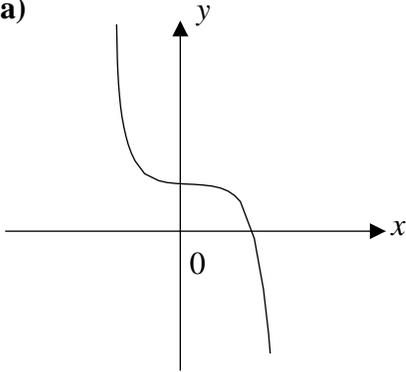
$$\text{Area of major segment } YZ = 105 \text{ cm}^2 \text{ (3s.f)}$$

Answer = _____ cm^2 [3]

$y = x^3 - 4$	$y = -3(4)^x$	$y = 4 - x^2$	$y = 4x^{-2}$
$y = -2x^{-4}$	$y = 4 - x^3$	$y = -3(-4)^x$	$y = x^2 + 4$

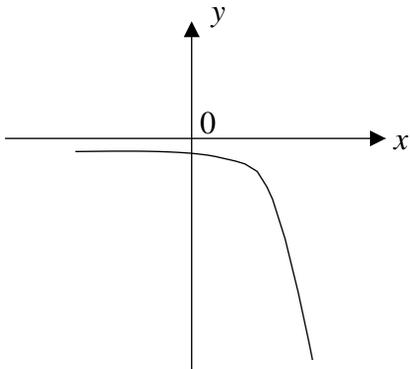
Write down a possible equation for each of the sketch graphs below.
In each case select one of the equations from the box above.

(a)



(a) $y = 4 - x^3$

(b)



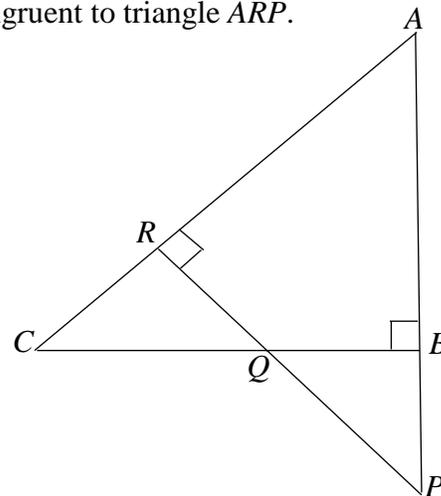
(b) $y = -3(4)^x$

- 13 Make x the subject in the equation $y = \sqrt{x^2 - 8x + 16} - y^2$.

$$\begin{aligned}
 y &= \sqrt{x^2 - 8x + 16} - y^2 \\
 y^2 &= x^2 - 8x + 16 - y^2 \\
 2y^2 &= x^2 - 8x + 16 \\
 2y^2 &= (x - 4)^2 \\
 x - 4 &= \pm\sqrt{2}y \\
 x &= 4 \pm \sqrt{2}y
 \end{aligned}$$

Answer _____ [3]

- 14 In the diagram shown below, it is given that $AP = 5$ cm, $BP = 2$ cm and $AR = 3$ cm. ARC , ABP and RQP are straight lines. Show, with clear reasons, that triangle ABC is congruent to triangle ARP .



[2]

In triangle ABC and triangle ARP ,

Angle A is common.
 Angle $ABC = \text{Angle } ARP = 90^\circ$ (given)
 $AB = 5 - 2 = 3$ cm = AR
 Therefore triangle $ABC \cong$ triangle ARP (AAS)

- 15 A lake has an actual area of 2.5 km^2 . The area of the lake on the map is 40 cm^2 . The distance between two towns on the map is 45 cm. Find the actual distance, in kilometres, between the two towns.

Area Scale = $40 \text{ cm}^2 : 2.5 \text{ km}^2$
 $= 40 \text{ cm}^2 : 2.5 \times 100000 \times 100000 \text{ cm}^2$
 $= 1 : 625000000$
 Linear Scale = $1 : 25000$
 $= 1 \text{ cm} : 0.25 \text{ km}$
 $= 45 \text{ cm} : 11.25 \text{ km}$

OR

$$40 \text{ cm}^2 \text{ on the map} = 2.5 \text{ km}^2 \text{ on the ground}$$

$$\sqrt{40} \text{ cm on the map} = \sqrt{2.5} \text{ km on the ground}$$

$$1 \text{ cm} = \frac{\sqrt{2.5}}{\sqrt{40}} \text{ km}$$

$$45 \text{ cm} = 45 \times \frac{\sqrt{2.5}}{\sqrt{40}} = 11.25 \text{ km}$$

OR

Let the distance between the two towns be d km.

$$\left(\frac{45}{d}\right)^2 = \frac{40}{2.5}$$

$$\frac{2025}{d^2} = 16$$

$$d^2 = \frac{2025}{16}$$

$$d = \frac{45}{4} \quad (d > 0)$$

Answer _____ km [3]

- 16 (a) Solve the inequalities $\frac{8x-12}{2} \leq 3x+1 < \frac{17x}{3}$.

$$\frac{8x-12}{2} \leq 3x+1 < \frac{17x}{3}$$

$$\frac{8x-12}{2} \leq 3x+1 \quad \text{and} \quad 3x+1 < \frac{17x}{3}$$

$$8x-12 \leq 6x+2 \quad 3x+1 < \frac{17x}{3}$$

$$8x-6x \leq 2+12 \quad 9x+3 < 17x$$

$$2x \leq 14 \quad \text{and} \quad 9x-17x < -3$$

$$x \leq 7 \quad -8x < -3$$

$$x > \frac{3}{8}$$

Therefore, $\frac{3}{8} < x \leq 7$

Answer _____ [3]

- (b) Hence, write down all the prime numbers that satisfy $\frac{8x-12}{2} \leq 3x+1 < \frac{17x}{3}$.

The prime numbers are 2, 3, 5 and 7

Answer _____ [1]

17 Factorise completely $4x^2 - 12xy + 9y^2 - 1$.

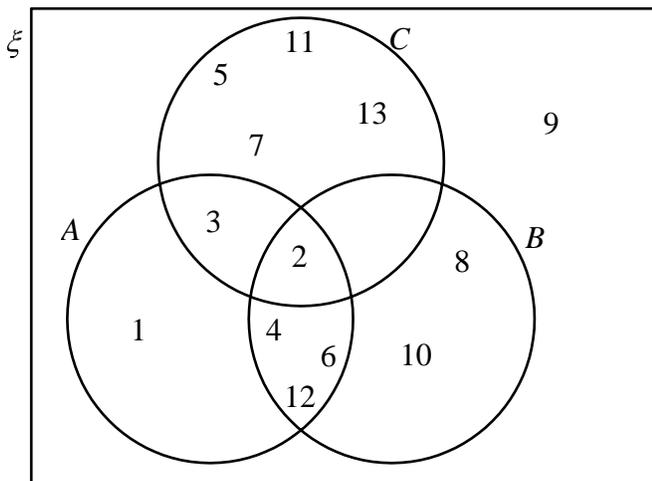
$$4x^2 - 12xy + 9y^2 - 1 = (2x - 3y)^2 - 1^2$$

$$= (2x - 3y - 1)(2x - 3y + 1)$$

Answer _____ [2]

18 The Venn diagram shows the elements of ξ and three sets A , B and C .

$\xi = \{x: x \text{ is a positive integer such that } 0 < x < 14\}$



(a) Describe in words the elements in set C .

(a) The elements in set C are the prime numbers between 0 and 14.
OR The set C is the set of prime numbers.

(b) Use one of the symbols below to complete each statement.

$\emptyset \subset \not\subset \notin \in \xi$

(i) $A' \cap (B \cap C) = \dots\dots\dots$

(ii) $3 \dots\dots\dots A$

(b) (i) \emptyset
(ii) \in

- 19 The time taken to assemble a car is inversely proportional to the number of workers involved. 4 workers can complete the assembly in x days. If 6 more workers are involved, the assembly can be completed 3 days in advance.

(a) Find the value of x .

Let W = number of works, D = number of days required

$$x = \frac{k}{4}$$

$$k = 4x$$

$$x - 3 = \frac{k}{10}$$

$$k = 10x - 30$$

$$10x - 30 = 4x$$

$$x = 5$$

Answer $x =$ _____ [2]

(b) Find the number of workers required if the assembly is to be completed in 2 days.

(b) When $x = 5$,

$$5 = \frac{k}{4}$$

$$k = 20$$

$$D = \frac{20}{W}$$

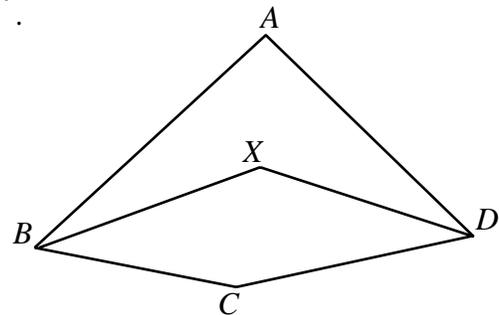
When $D = 2$ days, $W = 10$ workers.

Answer _____ [2]

- 20 In the figure, $ABCD$ is a quadrilateral. The point X is such that XB and XD are the angle bisectors of angle ABC and angle ADC respectively.

Reflex angle $BCD = 200^\circ$ and reflex angle $BXD = 225^\circ$.

Calculate angle BAD .



Obtuse angle $BCD = 360^\circ - 200^\circ = 160^\circ$ (Angles at a point)

Obtuse angle $BXD = 360^\circ - 225^\circ = 135^\circ$ (Angles at a point)

Angle CBX + angle $CDX = 360^\circ - 160^\circ - 135^\circ = 65^\circ$ (angle sum of quadrilateral $BCDX$)

Since XB and XD bisect angle ABC and angle ADC respectively.

Angle ABC + angle $ADC = (\angle CBX + \angle CDX) \times 2$

$$= 65^\circ \times 2 = 130^\circ$$

Angle $BAD = 360^\circ - 130^\circ - 160^\circ = 70^\circ$ (angle sum of quadrilateral $ABCD$)

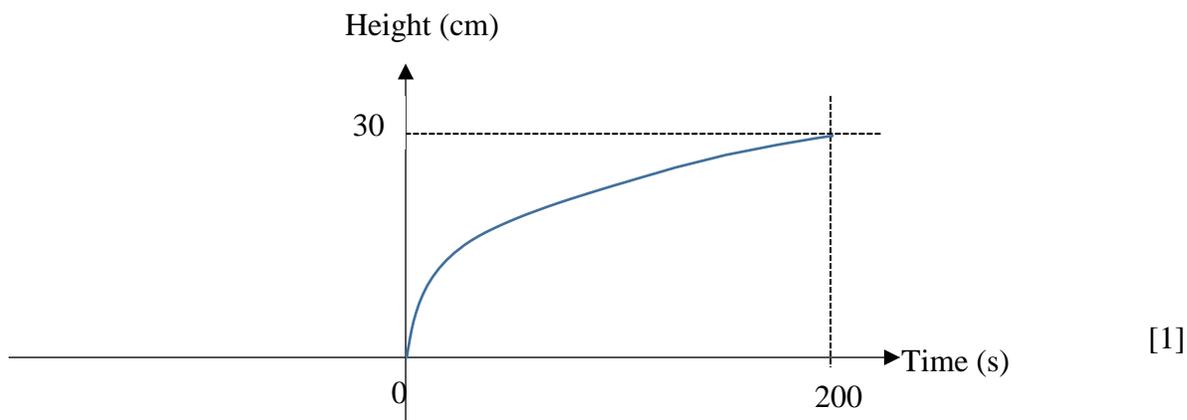
Answer _____ [4]

- 21 (a) An open container in a shape of an inverted cone has radius of 10 cm and height of 30 cm. Water is poured into the container at a constant rate of $5\pi \text{ cm}^3/\text{s}$ until it is completely filled to the brim.
Find the time taken for the container to be completely filled.

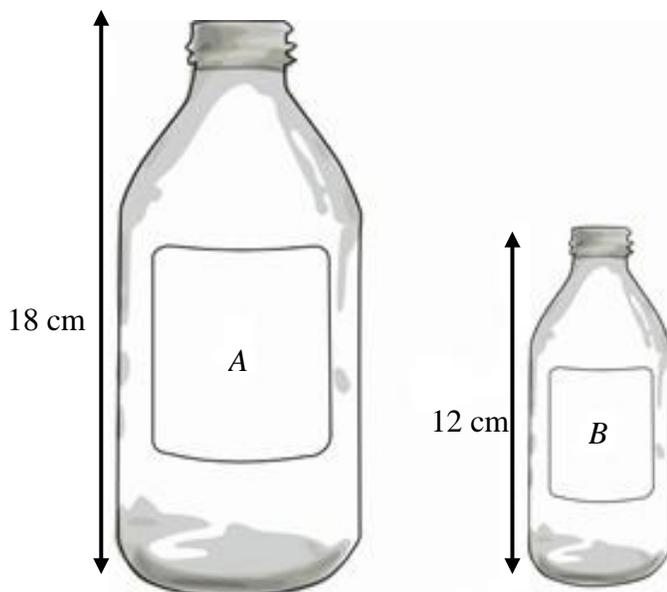
<p>(a) Volume of conical container = $\frac{1}{3} \pi(10)^2(30)$ $= 1000 \pi \text{ cm}^3$</p> <p>Time taken = $\frac{1000\pi}{5\pi}$ $= 200 \text{ s}$</p>	
---	--

Answer _____ s [1]

- (b) Sketch the graph of the water-level against time below.



- 22 A popular drink is produced in two similar bottle sizes. The height of the large bottle A is 18 cm while the height of the smaller bottle B is 12 cm.



If the selling prices of bottles A and B are \$24.90 and \$6.90 respectively, which bottle provides better value for money? Justify your answer clearly. [3]

$$\frac{\text{Volume of A}}{\text{Volume of B}} = \left(\frac{18}{12}\right)^3 = \frac{5832}{1728} = \frac{27}{8}$$

$$\text{Cost of 1 unit}^3 \text{ of volume for A} = \frac{\$24.90}{27}$$

$$\approx \$0.9222$$

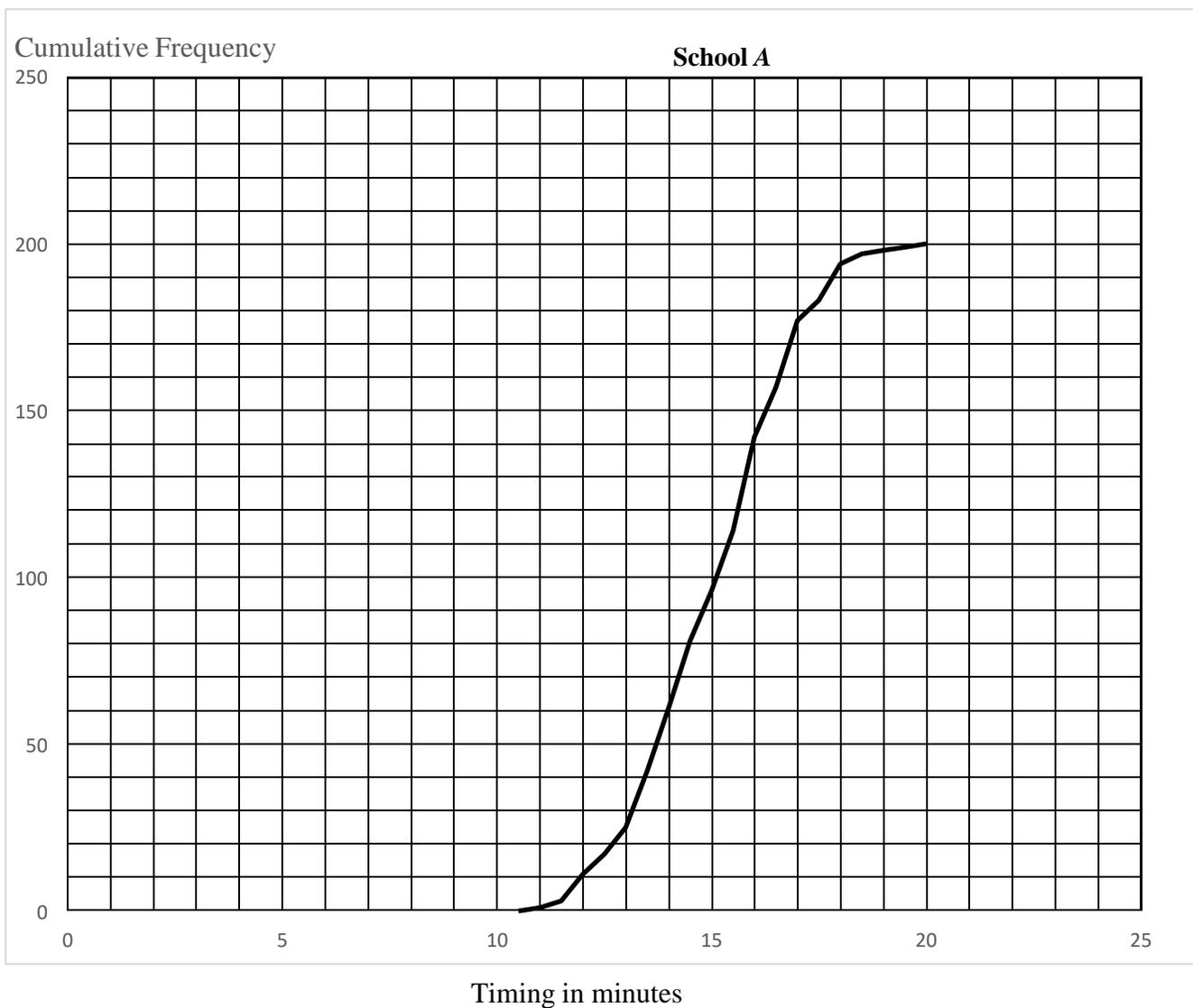
$$\text{Cost of 1 unit}^3 \text{ of volume for B} = \frac{\$6.90}{8}$$

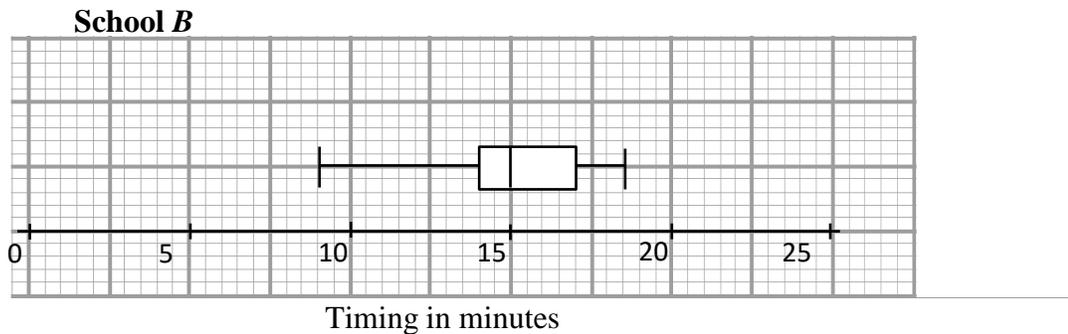
$$\approx \$0.8625$$

Since 1 unit³ of liquid in bottle *B* cost lesser than 1 unit³ of liquid in bottle *A*, bottle *B* is more value for money.

Note: Only award first method mark if student use 1 cm³ instead of 1 unit³

- 23 The cumulative frequency diagram shows the times taken by 200 girls from school *A* running 2.4 km test. The box-and-whisker plot shows the times for another group of girls from school *B*.





- (a) 75% of the girls in school B failed the test. Find the number of girls who passed in school A.

(a) From school B, $Q_1 = 14$ mins, which is the passing time.
Hence, number of girls in school A who passed the test
= 60 (from curve)

Answer..... [1]

- (b) 30% of the girls in school A took longer than t minutes. Find t . [2]

30% of the girls = $\frac{30}{100} \times 200 = 60$
From the curve, 140 girls took 16 minutes or less. So $t = 16$

Answer..... [2]

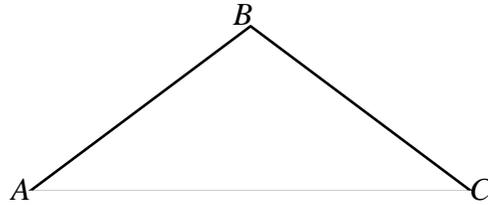
- (c) Find the proportion of girls in school A who took between 14.5 minutes and 17 minutes to complete the run.

$t = 14.5$ Cumulative Frequency = 80
 $t = 17$ Cumulative Frequency = 177
Proportion of girls = $\frac{177 - 80}{200} \times 100 = 48.5\%$

Answer_____ [2]

- 24 (a) Construct kite $ABCD$. $AD = CD = 9$ cm. AB and BC have already been drawn. Measure and state the length of the longest diagonal.

Answer



(a) Construct

Length of longest diagonal = 6.9cm – 7.1cm

- (b) Construct the perpendicular bisector BC . [1]
 (c) Construct the bisector of the angle ABC . [1]

(b) / (c) See diagram

- (d) $ABCD$ represents a plot of land which is to be used for a park. A café is to be built in park, nearer to A than to D and nearer to AD than AB . Shade the region where the café is to be built.

(d) See diagram

- 25 Ahmad and Beng Hai want to rent lockers in school. The lockers are in two levels. Lockers 1A to 1C are on the lower level and Lockers 2A to 2C are on the next level. Lockers are assigned to each student randomly.

- (e) Using a possibility diagram, represent the two lockers that the two boys can be allocated such that they are next to each other on the same level.

Answer

		Ahmad					
Beng Hai		1A	1B	1C	2A	2B	2C
	1A		1	1	1	1	1
	1B	1		1	1	1	1
	1C	1	1		1	1	1
	2A	1	1	1		1	1
	2B	1	1	1	1		1
	2C	1	1	1	1	1	

[2]

- (b) Find the probability that Ahmad and Beng Hai are randomly allocated lockers next to each other on Level 2.

Total number of possible outcomes = 30 P(Ahmad and Beng Hai have lockers on Level 2) $= \frac{4}{30} = \frac{2}{15}$	
--	--

Answer _____ [1]

- (c) Find the probability that Ahmad and Beng Hai are randomly allocated lockers on different levels.

Total number of possible outcomes = 30 P(Ahmad and Beng Hai have lockers on different levels) $= \frac{18}{30} = \frac{3}{5}$	
---	--

Answer _____ [1]

- (d) If the lockers 2C was not available. Find the probability that the friends will be allocated lockers next to each other at any level.

Total number of possible outcomes = 20 P(Ahmad and Beng Hai have lockers next to each other) $= \frac{6}{20} = \frac{3}{10}$
--

Answer _____ [2]

End of paper