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ANDERSON SECONDARY SCHOOL
Preliminary Examination 2021
Secondary Four Express

CANDIDATE NAME:

CLASS: /

INDEX NUMBER:

4049/01

ADDITIONAL MATHEMATICS

Paper 1

23 August 2021

2 hours 15 minutes

1100 – 1315h

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** the questions.
Omission of essential working will result in loss of marks.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Setter: Mr Low Kok Ming

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ∆ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 A curve is such that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2(1-2x)$. The equation of the normal to the curve at the point $(-1, 7)$ is $9y = x + 64$. Find the equation of the curve. [6]

2 **(i)** Show that $p+q$ is a factor of the expression $p^3-3p^2q+4q^3$. [2]

(ii) Factorise $p^3-3p^2q+4q^3$ completely. [2]

(iii) Hence solve the equation $(m+2)^3-3(m+2)^2(m-3)+4(m-3)^3=0$. [2]

3 **(a)** Solve $7\cos\left(\frac{\theta}{2}\right)=10\sin\theta$, for $0\le\theta\le 2\pi$. [5]

(b) Prove that $\frac{\sec x\tan x+\sec^2 x}{(\tan x+\sec x)^2+1}=\frac{1}{2}$. [3]

4 **(i)** Write down and simplify, in descending powers of x, the first three terms in the expansion of $\left(x^5 + \frac{2}{x^6}\right)^n$, where $n > 0$. [2]

(ii) When the third term of the expansion is divided by the second term, $\frac{8}{x^{11}}$ is obtained. Show that $n - 9$. [2]

(iii) Using the value of n found in **(ii)**, without expanding $\left(x^5 + \dfrac{2}{x^6}\right)^n$, show that there is no constant term in the expansion. [3]

5 **(a)** Solve the equation $3\left(9^k\right)+2\left(4^k\right)=5\left(6^k\right)$. [5]

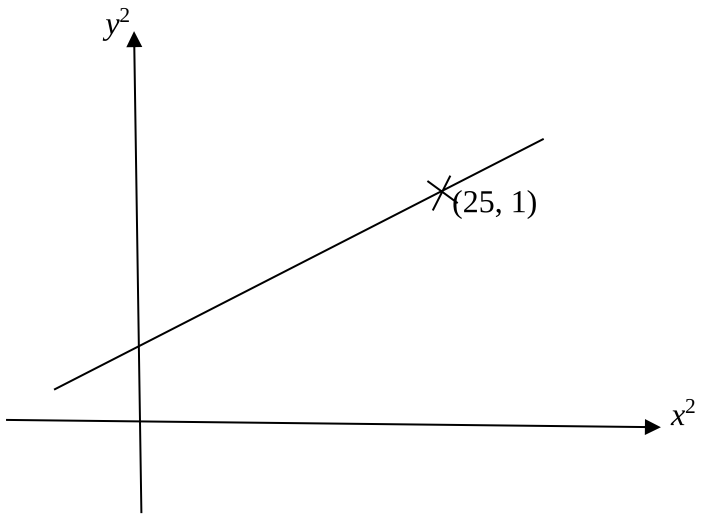
(b) Show that $\dfrac{\sqrt{\left(a-b^2\right)^3\left(a+b^2\right)}}{\left(\sqrt{a}+b\right)\sqrt{a^2-b^4}}$ can be expressed in the form $m\sqrt{a}+nb$ where m and n are constants to be determined. [4]

6 **(i)** Express $\dfrac{4x^3+9x^2-17x-5}{(2x-3)(x^2+1)}$ in partial fractions. [6]

(ii) Differentiate $\ln\left(x^2+1\right)$ with respect to x. [1]

(iii) Hence find $\int \frac{4x^3+9x^2-17x-5}{(2x-3)\left(x^2+1\right)}\ \mathrm{d}x$. [3]

7 **(a)**



Two variables x and y are related by the equation $\frac{x^2}{p^2} = 1 + \frac{3y^2}{q^2}$, where p and q are constants. When the graph of y^2 against x^2 is drawn, a straight line is obtained.

Given that the line passes through the point (25, 1) and has a gradient $\frac{1}{15}$, find

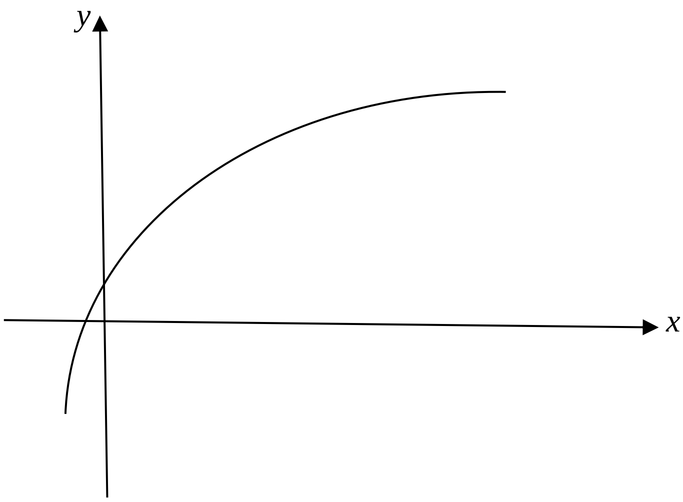
(i) the exact values of p and q. [4]

(ii) the values of x when $y = \sqrt{\frac{2}{5}}$. [2]

(b) Determine, showing your working, if it is possible for any line that passes through the point (2, 1) to be a tangent to the circle with equation $x^2 + y^2 - 6x + 8y - 11 = 0$. [5]

8 **(a)** A curve has the equation $y = \dfrac{4x+15}{x-9}$, where $x \neq 9$. Find the gradient function of the curve and determine, with explanation, whether *y* is an increasing or decreasing function. [4]

(b) The diagram shows part of the curve $y = \ln \sqrt[3]{x}$.



Calculate the area of the region bounded by the curve, the line $x = 3$ and the x-axis. [5]

9 **(a)** Write down the principal value, in radians as a multiple of π, of $\cos^{-1}\left(-\sin\dfrac{\pi}{3}\right)$. [1]

(b) x, y and z are three angles of a triangle. Given that x and y are acute angles such that $\sin x = \dfrac{8}{17}$ and $\sin y = \dfrac{3}{5}$, find the exact value of $\tan z$ without the use of a calculator. [4]

10 A factory is tasked to design an open cylindrical container with a surface area of 243π cm^2. The radius and height of the cylinder is r cm and h cm respectively.

(i) Show that the volume, V cm^3, of the cylinder is $V = \frac{\pi}{2}\left(243r - r^3\right)$. [4]

(ii) Find, in terms of π, the maximum volume of the cylinder. [6]

11 A particle moves in a straight line so that, *t* seconds after passing a fixed point *O*, its velocity, *v* m/s, is given by $v = 2t^2 - 16t + 30$.

(i) Find an expression, in terms of *t*, for the displacement of the particle. [2]

(ii) Sketch the velocity-time graph and hence find the range of times when the particle is travelling towards *O*. [3]

(iii) Calculate the total distance travelled by the particle in the first 7 seconds. [4]

End of Paper

Answers:

1. $y=-\frac{2}{3}x^3+x^2-5x+\frac{1}{3}$

2(ii) $(p+q)(p-2q)^2$ (iii) $m=\frac{1}{2}$ or 8

3(a) $\theta=\pi$, 0.715, 5.57

4(i) $x^{5n}+2nx^{5n-11}+2n(n-1)x^{5n-22}+\ldots$

5(a) $k=-1$ or 0 (b) $\sqrt{a}-b$

6(i) $2+\frac{1}{(2x-3)}+\frac{7x}{(x^2+1)}$ (ii) $\frac{2x}{x^2+1}$

(iii) $2x+\frac{1}{2}\ln(2x-3)+\frac{7}{2}\ln(x^2+1)+C$

7(a)(i) $p=\pm\sqrt{10}$, $q=\pm\sqrt{2}$ (ii) $x=+4$ (b) not possible

8(a) $\frac{dy}{dx}=-\frac{51}{(x-9)^2}$; decreasing function (b) 0.432 units^2

9(a) $\frac{5\pi}{6}$ (b) $-\frac{77}{36}$

10(ii) 729π cm^2

11(i) $s=\frac{2t^3}{3}-8t^2+30t$ (ii) $3<t<5$ (iii) 52 m