

ANDERSON SECONDARY SCHOOL
Preliminary Examination 2021
Secondary Four Express



CANDIDATE NAME:

CLASS:

INDEX NUMBER:

ADDITIONAL MATHEMATICS

4049/02

Paper 2

26 August 2021

2 hours 15 minutes

0800 – 1015h

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

Setter: Ms Lee Siew Lin

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \dots (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1 If $y = (1+x)e^{3x}$, find the value of the constant k for which $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + ky = 0$. [6]

- 2 A hot stone with a temperature of 220°C was dropped into a reservoir of water. As the stone cools, its temperature, $T^{\circ}\text{C}$, t minutes after it enters the water is given by $T = P + 190e^{-kt}$, where P and k are constants.

(i) Show that $P = 30$. [2]

(ii) It took 4 seconds for the temperature of the stone to reach 120°C . Find the value of k . [3]

(iii) Sketch the graph of T against t . [2]

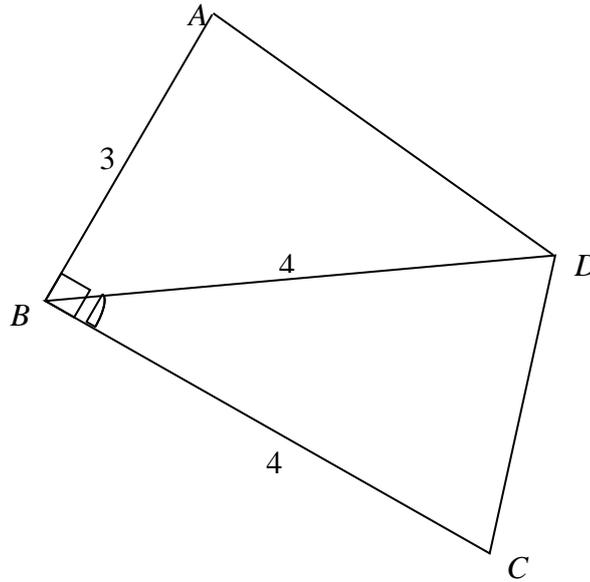
3 (i) Show that $\frac{d}{dx}(\tan^3 x) = 3\sec^4 x - 3\sec^2 x$. [3]

(ii) Hence evaluate $\int_0^{\frac{\pi}{4}} \sec^4 x - 2\sec^2 x \, dx$. [5]

- 4 A curve has the equation $y = \ln \sqrt{1+2x^3}$.
- (i) Show that the curve has only one stationary point and determine the nature of this stationary point. [6]

- (ii) A particle moves along the curve in such a way that the y -coordinate of the particle is decreasing at a constant rate of 0.1 units per second. Find the rate of change of the x -coordinate at the instant when $x = 2$. [3]

- 5 The diagram below shows a quadrilateral $ABCD$ with $AB = 3$ cm, $BC = BD = 4$ cm and $\angle ABC = 90^\circ$. The acute angle DBC is x .



- (i) Show that the area, A cm², of the quadrilateral is given by $A = 6 \cos x + 8 \sin x$. [3]

- (ii) Express A in the form $R \cos(x - \alpha)$, where $R > 0$ and α is acute. [4]

(iii) Hence state the maximum area of the quadrilateral. [1]

(iv) Find x for which the area of $ABCD$ is 7 cm^2 . [3]

6 (i) Show that $\frac{\sin^2 x - 2 \sin x}{1 - \cos 2x} = \frac{1}{2}(1 - 2 \operatorname{cosec} x)$. [3]

(ii) Hence find the equation of the tangent to the curve $y = \frac{\sin^2 x - 2 \sin x}{1 - \cos 2x}$ at the point where $x = \frac{\pi}{4}$. [5]

7 The equation of a curve is $2y = 3x^2 - px - 2q - 1$, where p and q are constants. The line $y = p - q - 2x$ is a tangent to the curve at the point A .

(i) Show that the possible values of p are -2 and -14 . [4]

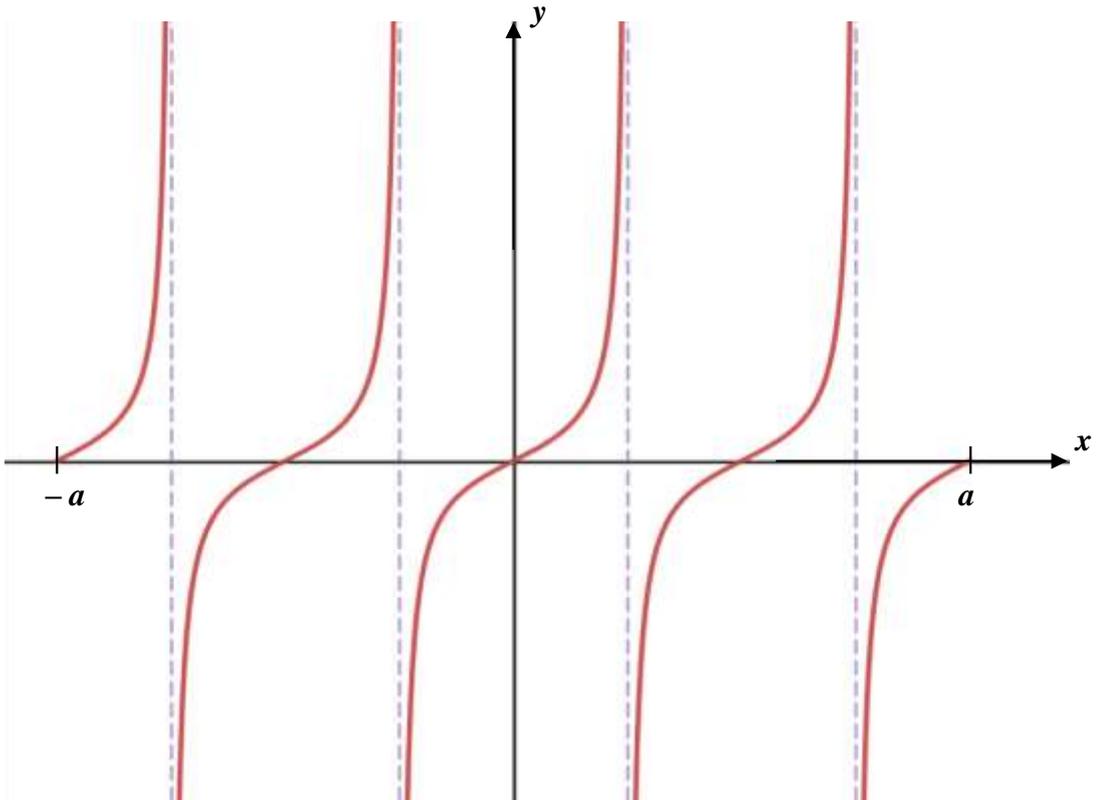
(ii) It is given that the curve passes through the point $(0, -5.5)$.
For the case where $p = -2$, find the coordinates of A . [4]

- 8 (a) Solve the equation $\log_4 x^2 - 3 \log_x 4 = 1$. [6]

(b) (i) Solve the equation $2\log_2(1-x) - \log_2 x - \log_2 2x - 3 = 0$. [5]

(ii) Hence solve $2\log_2(-y) - \log_2(y+1) - \log_2(2y+2) - 3 = 0$. [2]

- 9 (a) The figure shows the graph of $y = \tan 2x$ for $-a \leq x \leq a$, where a is in radians.



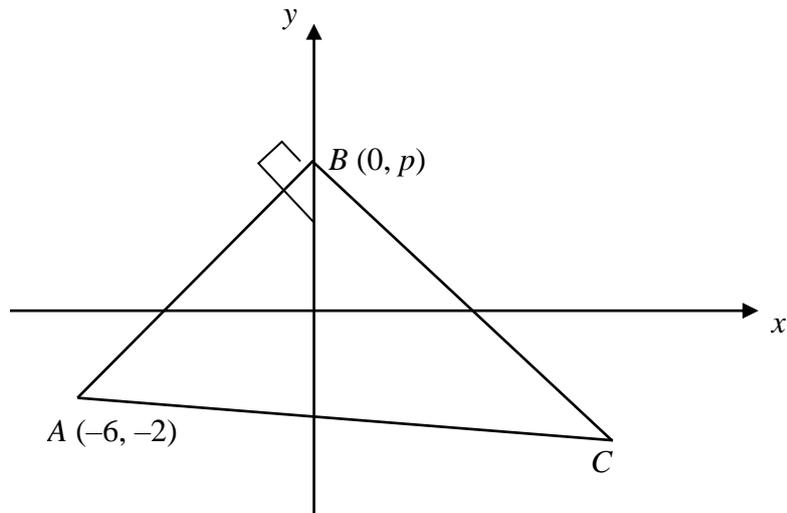
- (i) State the value of a in terms of π . [1]

- (ii) State the amplitude and the period of $y = -4 \cos 3x$. [2]

- (iii) Sketch, on the same axes above, the graph of $y = -4 \cos 3x$ for $-a \leq x \leq a$. [4]

- (b) Eric claims that $x^2 + mx - 2m + \frac{17}{4} > 2$ for all real values of x if $-5 < m < 0$.
Justify whether Eric's claim is true. [5]

- 10 The diagram shows a triangle with vertices $A(-6, -2)$, $B(0, p)$ and C . BC has a gradient of $-\frac{3}{2}$. AB is perpendicular to BC and $3AB = 2BC$.



- (i) Show that $p = 2$. [3]

(ii) Explain why AC is not parallel to the x -axis.

[3]

(iii) Find the area of the triangle ABC .

[2]

End of Paper

ANDERSON SECONDARY SCHOOL
Sec 4 Express Additional Mathematics
PRELIM EXAM 2021 PAPER 2

Answer Key

| Qn | Answer |
|----|---|
| 1 | $k - 9$ |
| 2 | (ii) $k = 11.2$ |
| 3 | (ii) $-\frac{2}{3}$ |
| 4 | (ii) $-\frac{17}{120}$ units/s |
| 5 | (ii) $A = 10 \cos(x - 53.1^\circ)$ |
| | (iii) 10 cm^2 |
| | (iv) $x = 7.6^\circ$ |
| 6 | (ii) $y - \left(\frac{1}{2} - \sqrt{2}\right) = \sqrt{2}\left(x - \frac{1}{4}\right)$ |
| 7 | (ii) $(-1, -5)$ |
| 8 | (a) $x = 8$ or $x = \frac{1}{4}$ |
| | (b) (i) $x = \frac{1}{5}$ |
| | (b) (ii) $y = -\frac{4}{5}$ |
| 9 | (a) (i) $a = \pi$ |
| | (a) (ii) Amplitude = 4, Period = $\frac{2\pi}{3}$ |
| 9 | (b) Eric's claim is true. |
| 10 | (iii) 39 units^2 |

ANDERSON SECONDARY SCHOOL
Sec 4 Express Additional Mathematics
PRELIM EXAM 2021 PAPER 2
MARKING SCHEME

- 1 If $y = (1+x)e^{3x}$, find the value of the constant k for which $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + ky = 0$. [6]

[Marking Scheme]

$$\begin{aligned}\frac{dy}{dx} &= 1(e^{3x}) + (1+x)(3)e^{3x} && \text{[M1]} \\ &= e^{3x}(3x+4) && \text{[M1]}\end{aligned}$$

$$\begin{aligned}\frac{d^2y}{dx^2} &= 3e^{3x}(3x+4) + 3e^{3x} \\ &= 3e^{3x}(3x+5) && \text{[M1]}\end{aligned}$$

$$\begin{aligned}\frac{d^2y}{dx^2} - 6\frac{dy}{dx} &= 3e^{3x}(3x+5) - 6e^{3x}(3x+4) && \text{[M1]} \\ &= e^{3x}(9x+15-18x-24) \\ &= -9e^{3x}(x+1) \\ &= -9y && \text{[M1]}\end{aligned}$$

Thus, $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$.

Therefore, $k = 9$. [A1]

- 2 A hot stone with a temperature of 220°C was dropped into a reservoir of water. As the stone cools, its temperature, $T^{\circ}\text{C}$, t minutes after it enters the water is given by $T = P + 190e^{-kt}$, where P and k are constants.

- (i) Show that $P = 30$. [2]

[Marking Scheme]

When $t = 0$, $T = 220$:

$$220 = P + 190e^{-k(0)} \quad \text{[M1]}$$

$$P = 220 - 190(1)$$

$$P = 30 \quad \text{[A1]}$$

- (ii) It took 4 seconds for the temperature of the stone to reach 120°C . Find the value of k . [3]

[Marking Scheme]

When $t = \frac{1}{15}$, $T = 120$:

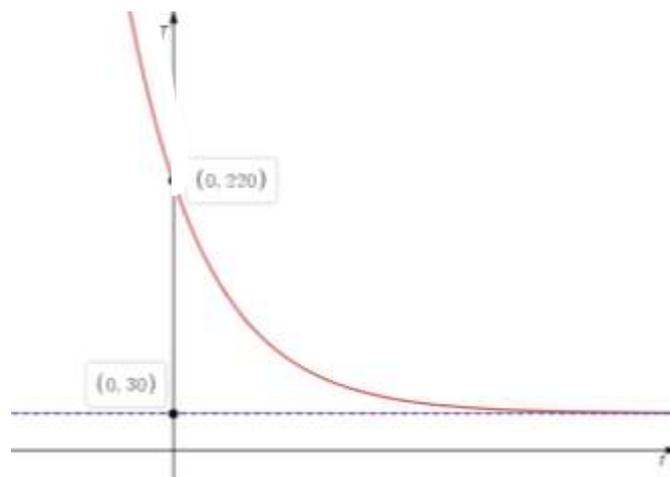
$$120 = 30 + 190e^{-k\left(\frac{1}{15}\right)} \quad \text{[M1]}$$

$$-\frac{k}{15} = \ln\left(\frac{90}{190}\right) \quad \text{[M1]}$$

$$k = 11.2 \text{ (3sf)} \quad \text{[A1]}$$

- (iii) Sketch the graph of T against t . [2]

[Marking Scheme]



[C1] Shape of graph

[C1] Passes through $(0, 220)$, with asymptote $T = 30$.

- 3 (i) Show that $\frac{d}{dx}(\tan^3 x) = 3\sec^4 x - 3\sec^2 x$. [3]

[Marking Scheme]

$$\begin{aligned}\frac{d}{dx}(\tan^3 x) &= 3 \tan^2 x (\sec^2 x) && \text{[M1]} \\ &= 3(\sec^2 x - 1)(\sec^2 x) && \text{[M1]} \\ &= 3\sec^4 x - 3\sec^2 x && \text{[A1]}\end{aligned}$$

- (ii) Hence evaluate $\int_0^{\frac{\pi}{4}} \sec^4 x - 2\sec^2 x \, dx$. [5]

[Marking Scheme]

From (i), $\int 3\sec^4 x - 3\sec^2 x \, dx = \tan^3 x + c$ [M1]

$$\begin{aligned}\int_0^{\frac{\pi}{4}} \sec^4 x - 2\sec^2 x \, dx &= \frac{1}{3} \int_0^{\frac{\pi}{4}} (3\sec^4 x - 3\sec^2 x) - 3\sec^2 x \, dx && \text{[M1]} \\ &= \frac{1}{3} [\tan^3 x]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \sec^2 x \, dx && \text{[M1]} \\ &= \frac{1}{3} [\tan^3 x]_0^{\frac{\pi}{4}} - [\tan x]_0^{\frac{\pi}{4}} && \text{[M1]} \\ &= \frac{1}{3} (1 - 0) - (1 - 0) \\ &= -\frac{2}{3} && \text{[A1]}\end{aligned}$$

- 4 A curve has the equation $y = \ln \sqrt{1+2x^3}$.
- (i) Show that the curve has only one stationary point and determine the nature of this stationary point. [6]

[Marking Scheme]

$$y = \ln \sqrt{1+2x^3}$$

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{1}{2} \ln(1+2x^3) \right) \quad \text{[M1]}$$

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{6x^2}{1+2x^3} \right)$$

$$\frac{dy}{dx} = \frac{3x^2}{1+2x^3} \quad \text{[M1]}$$

At stationary point,

$$\frac{dy}{dx} = 0$$

$$\frac{3x^2}{1+2x^3} = 0 \quad \text{[M1]}$$

$$3x^2 = 0$$

$$x = 0$$

Thus curve has only one stationary point (0, 0). (Shown) [A1]

| | | | |
|-----------------|-------|----|-------|
| x | 0^- | 0 | 0^+ |
| $\frac{dy}{dx}$ | > 0 | 0 | > 0 |
| Slope | / | -- | / |

[M1] (first deriv test)

The stationary point is a point of inflexion. [A1]

- (ii) A particle moves along the curve in such a way that the y -coordinate of the particle is decreasing at a constant rate of 0.1 units per second. Find the rate of change of the x -coordinate at the instant when $x = 2$. [3]

[Marking Scheme]

$$\frac{dy}{dx} = \frac{3x^2}{1+2x^3}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{3x^2}{1+2x^3} \frac{dx}{dt} \quad \text{[M1]}$$

When $x = 2$,

$$-0.1 = \frac{3(2)^2}{1+2(2)^3} \frac{dx}{dt} \quad \text{[M1]}$$

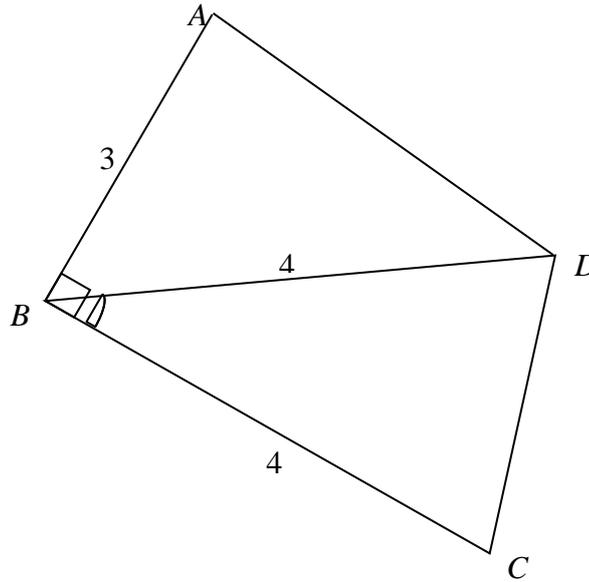
$$\frac{dx}{dt} = \frac{17}{12} \left(-\frac{1}{10} \right)$$

$$\frac{dx}{dt} = -\frac{17}{120}$$

Rate of change of the x -coordinate when $x = 2$ is $-\frac{17}{120}$ units/s.

OR x -coordinate decreases at a rate of $\frac{17}{120}$ units/s when $x = 2$. [A1]

- 5 The diagram below shows a quadrilateral $ABCD$ with $AB = 3$ cm, $BC = BD = 4$ cm and $\angle ABC = 90^\circ$. The acute angle DBC is x .



- (i) Show that the area, A cm², of the quadrilateral is given by $A = 6 \cos x + 8 \sin x$. [3]

[Marking Scheme]

$$\begin{aligned} \text{Area} &= \frac{1}{2}(3)(4)\sin(90^\circ - x) + \frac{1}{2}(4)(4)\sin x && \text{[M2]} \\ &= 6 \cos x + 8 \sin x && \text{[A1]} \end{aligned}$$

- (ii) Express A in the form $R \cos(x - \alpha)$, where $R > 0$ and α is acute. [4]

[Marking Scheme]

$$\begin{aligned} \text{Let } 6 \cos x + 8 \sin x &= R \cos(x - \alpha) \\ &= R \cos x \cos \alpha + R \sin x \sin \alpha \end{aligned}$$

Hence

$$R \cos \alpha = 6 \quad (1)$$

$$R \sin \alpha = 8 \quad (2) \quad \text{[M1]}$$

$$\frac{(2)}{(1)}: \quad \tan \alpha = \frac{4}{3}$$

$$\alpha = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ \quad \text{[M1]}$$

$$(1)^2 + (2)^2: \quad R = \sqrt{8^2 + 6^2} = 10 \quad \text{[M1]}$$

Therefore, $A = 10 \cos(x - 53.1^\circ)$ **[A1]**

- (iii) Hence state the maximum area of the quadrilateral. [1]

[Marking Scheme]

Max Area = 10 cm^2 **[B1]**

- (iv) Find x for which the area of $ABCD$ is 7 cm^2 . [3]

[Marking Scheme]

$$10 \cos(x - 53.13^\circ) = 7$$

$$\cos(x - 53.13^\circ) = 0.7 \quad \mathbf{[M1]}$$

$$x - 53.13^\circ = -45.57^\circ, 45.57^\circ \quad \mathbf{[M1]}$$

$$x = 7.6^\circ \text{ or } x = 98.7^\circ \text{ (rejected since } x \text{ is acute)}$$

Thus $x = 7.6^\circ$ **[A1]**

6 (i) Show that $\frac{\sin^2 x - 2 \sin x}{1 - \cos 2x} = \frac{1}{2}(1 - 2 \operatorname{cosec} x)$ [3]

[Marking Scheme]

$$\begin{aligned} \frac{\sin^2 x - 2 \sin x}{1 - \cos 2x} &= \frac{\sin^2 x - 2 \sin x}{1 - (1 - 2 \sin^2 x)} && \text{[M1] (double angle formula)} \\ &= \frac{\sin^2 x - 2 \sin x}{2 \sin^2 x} && \text{[M1] (simplify denominator)} \\ &= \frac{\sin x - 2}{2 \sin x} \\ &= \frac{1}{2} - \operatorname{cosec} x && \text{[A1]} \\ &= \frac{1}{2}(1 - 2 \operatorname{cosec} x) \end{aligned}$$

(ii) Hence find the equation of the tangent to the curve $y = \frac{\sin^2 x - 2 \sin x}{1 - \cos 2x}$ at the point where $x = \frac{\pi}{4}$. [5]

[Marking Scheme]

$$\begin{aligned} y &= \frac{\sin^2 x - 2 \sin x}{1 - \cos 2x} \\ \text{From (i), } y &= \frac{1}{2}(1 - 2 \operatorname{cosec} x) \\ \frac{dy}{dx} &= \frac{1}{2}(0 + 2 \operatorname{cosec}^2 x \cos x) && \text{[M2]} \\ &= \frac{\cos x}{\sin^2 x} \end{aligned}$$

When $x = \frac{\pi}{4}$,

$$y = \frac{1}{2} \left(1 - \frac{2}{\sin \frac{\pi}{4}} \right) = \frac{1}{2} - \sqrt{2} \quad \text{[M1]}$$

$$\frac{dy}{dx} = \frac{\cos \left(\frac{\pi}{4} \right)}{\sin^2 \left(\frac{\pi}{4} \right)} = \frac{1}{\left(\frac{1}{\sqrt{2}} \right)^2} = \sqrt{2} \quad \text{[M1]}$$

Equation of tangent line:

$$y - \left(\frac{1}{2} - \sqrt{2} \right) = \sqrt{2} \left(x - \frac{\pi}{4} \right) \quad \text{[A1]}$$

$$\text{or } y = \sqrt{2}x + \frac{1}{2} - \sqrt{2}\left(\frac{1}{4} + 1\right) \quad \text{or } y = \sqrt{2}x - 2.02$$

- 7 The equation of a curve is $2y = 3x^2 - px - 2q - 1$, where p and q are constants. The line $y = p - q - 2x$ is a tangent to the curve at the point A.

- (i) Show that the possible values of p are -2 and -14 . [4]

[Marking Scheme]

At point of intersection between the line and the curve,

$$2(p - q - 2x) = 3x^2 - px - 2q - 1$$

$$3x^2 + (4 - p)x - 2p - 1 = 0 \quad \text{----(1) [M1]}$$

Since line is tangent to the curve, equation (1) has real and equal roots.

Discriminant = 0

$$(4 - p)^2 - 4(3)(-2p - 1) = 0 \quad \text{[M1]}$$

$$p^2 - 8p + 16 + 24p + 12 = 0$$

$$p^2 + 16p + 28 = 0$$

$$(p + 2)(p + 14) = 0 \quad \text{[M1]}$$

$$p = -2 \quad \text{or} \quad p = -14 \quad \text{[A1]}$$

- (ii) It is given that the curve passes through the point $(0, -5.5)$.

For the case where $p = -2$, find the coordinates of A. [4]

[Marking Scheme]

Sub $p = -2$, and $(0, -5.5)$ into eqn of the curve:

$$2(-5.5) = -2q - 1$$

$$q = 5 \quad \text{[M1]}$$

At point of intersection between the line and the curve,

$$3x^2 + 6x + 4 - 1 = 0 \quad \text{-- From (1) [M1]}$$

$$x^2 + 2x + 1 = 0$$

$$(x + 1)^2 = 0$$

$$x = -1 \quad \text{[M1]}$$

Sub into equation of line:

$$y = p - q - 2x$$

$$= -2 - 5 - 2(-1)$$

$$= -5$$

Coordinates of $A = (-1, -5)$ **[A1]**

- 8 (a) Solve the equation $\log_4 x^2 - 3 \log_x 4 = 1$. [6]

[Marking Scheme]

$$\log_4 x^2 - 3 \log_x 4 = 1$$

$$2 \log_4 x - \frac{\log_4 4^3}{\log_4 x} = 1$$

[M1] (power law) + [M1] (change of base)

$$2(\log_4 x)^2 - 3 = \log_4 x$$

$$2(\log_4 x)^2 - \log_4 x - 3 = 0$$

[M1] (forming quad eqn)

Let $\log_4 x$ be u .

$$2u^2 - u - 3 = 0$$

$$(2u - 3)(u + 1) = 0$$

$$u = \frac{3}{2} \text{ or } -1$$

$$\log_4 x = \frac{3}{2} \text{ or } \log_4 x = -1$$

[M1]

$$x = 4^{\frac{3}{2}} \text{ or } x = 4^{-1}$$

$$x = 8 \text{ or } x = \frac{1}{4}$$

[A2]

- (b) (i) Solve the equation $2 \log_2(1-x) - \log_2 x - \log_2 2x - 3 = 0$. [5]

[Marking Scheme]

$$2 \log_2(1-x) - \log_2 x - \log_2 2x - 3 = 0$$

$$2 \log_2(1-x) - \log_2 x - \log_2 2x - \log_2 2^3 = \log_2 1$$

[M1]

$$\log_2 \frac{(1-x)^2}{8(x)(2x)} = \log_2 1$$

[M1] (product and quotient law)

$$\frac{(1-x)^2}{16x^2} = 1$$

[M1]

$$x^2 - 2x + 1 = 16x^2$$

$$15x^2 + 2x - 1 = 0$$

$$(5x-1)(3x+1) = 0$$

[M1]

$$x = \frac{1}{5} \text{ or } -\frac{1}{3} \text{ (reject as } 0 < x < 1) \quad \mathbf{[A1]}$$

(ii) Hence solve $2\log_2(-y) - \log_2(y+1) - \log_2(2y+2) - 3 = 0$. [2]

[Marking Scheme]

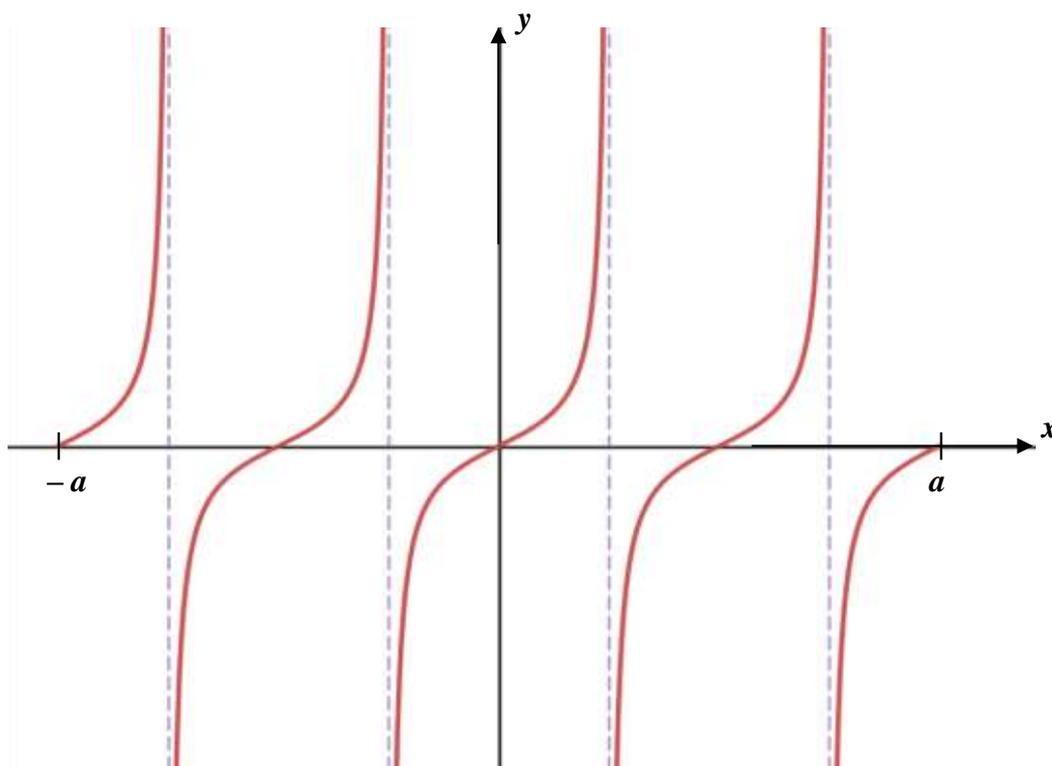
By observation, equation in (ii) can be obtained from (i) by replacing x by $y+1$. **[M1]**

Hence, from (i), solution is

$$y+1 = \frac{1}{5}$$

$$y = -\frac{4}{5} \quad \mathbf{[A1]}$$

- 9 (a) The figure shows the graph of $y = \tan 2x$ for $-a \leq x \leq a$, where a is in radians.



- (i) State the value of a in terms of π . [1]

[Marking Scheme]

$$a = \pi \quad \mathbf{[B1]}$$

- (ii) State the amplitude and the period of $y = -4 \cos 3x$. [2]

[Marking Scheme]

$$\text{Amplitude} = 4 \quad \mathbf{[B1]}$$

$$\text{Period} = \frac{\pi}{3} \quad \mathbf{[B1]}$$

- (iii) Sketch, on the same axes above, the graph of $y = -4 \cos 3x$ for $-a \leq x \leq a$. [4]

[Marking Scheme]

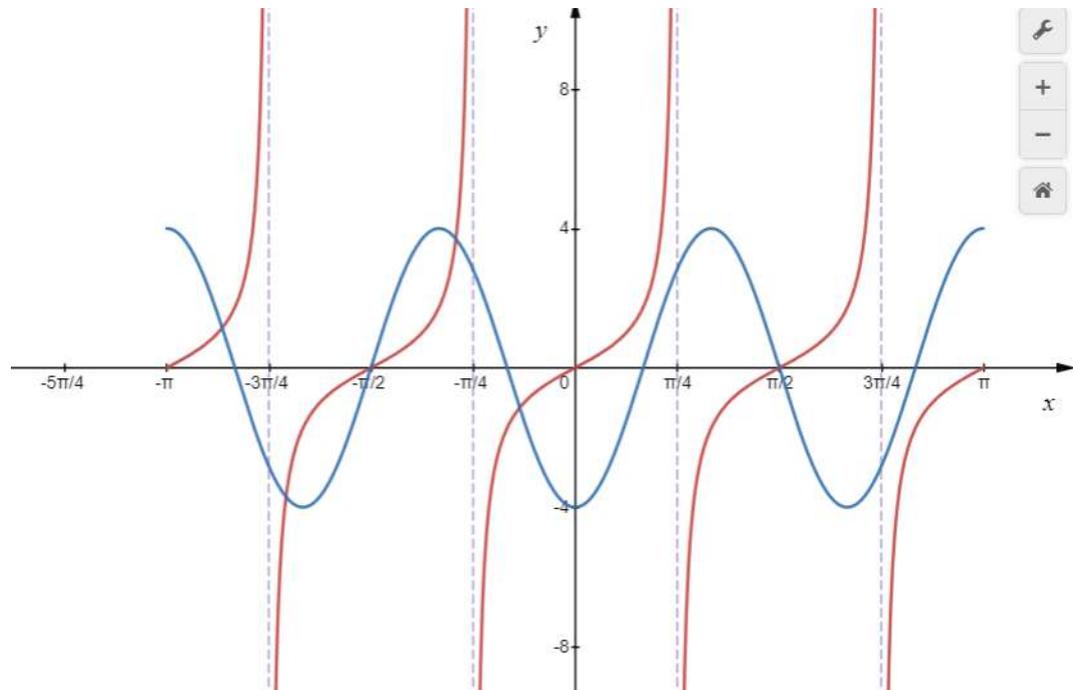
Shape of graph [C1]

3 cycles shown [C1]

Correct amplitude shown and passes through $(0, -4)$ [C1]

Intersects $y = \tan 2x$ at $\left(\frac{\pi}{2}, 0\right)$ and $\left(-\frac{\pi}{2}, 0\right)$

[C1]



- (b) Eric claims that $x^2 + mx - 2m + \frac{17}{4} > 2$ for all real values of x if $-5 < m < 0$.
Justify whether Eric's claim is true. [5]

[Marking Scheme]

If $x^2 + mx - 2m + \frac{17}{4} > 2$ for all real x ,

$$x^2 + mx - 2m + \frac{9}{4} > 0 \text{ for all real } x. \quad \text{[M1]}$$

Then $x^2 + mx - 2m + \frac{9}{4}$ has no real roots.

Discriminant < 0

$$m^2 - 4(1)\left(-2m + \frac{9}{4}\right) < 0 \quad \text{[M1]}$$

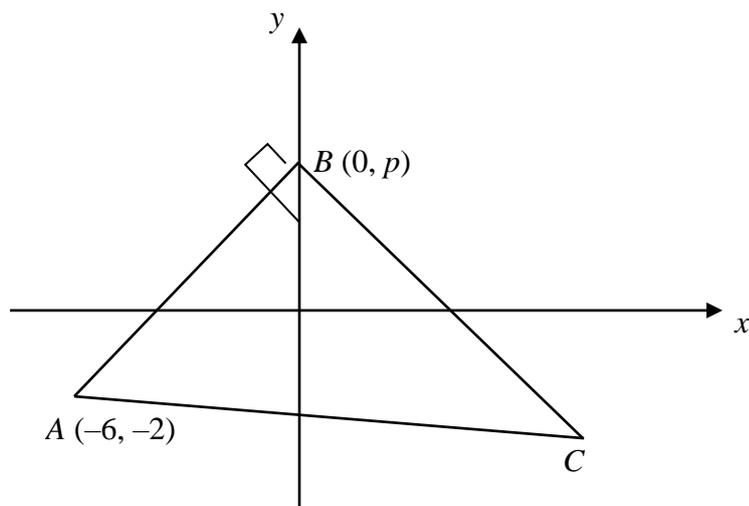
$$m^2 + 8m - 9 < 0$$

$$(m-1)(m+9) < 0 \quad \text{[M1]}$$

$$-9 < m < 1 \quad \text{[M1]}$$

Therefore, if $-5 < m < 0$, $x^2 + mx - 2m + \frac{17}{4} > 2$ for all real x .
Thus Eric's claim is true. **[A1]**

- 10 The diagram shows a triangle with vertices $A(-6, -2)$, $B(0, p)$ and C . BC has a gradient of $-\frac{3}{2}$. AB is perpendicular to BC and $3AB = 2BC$.



- (i) Show that $p = 2$. [3]

[Marking Scheme]

Since BC is perpendicular to AB ,

$$\text{Gradient of } AB = \frac{-1}{-\frac{3}{2}}$$

$$= \frac{2}{3} \quad \text{[M1]}$$

$$\Rightarrow \frac{p - (-2)}{0 - (-6)} = \frac{2}{3} \quad \text{[M1]}$$

$$\frac{p+2}{6} = \frac{2}{3}$$

$$p+2 = 4$$

Thus, $p = 2$ (shown) [A1]

- (ii) Explain why
- AC
- is not parallel to the
- x
- axis.

[3]

[Marking Scheme]

Equation of BC : $y = -\frac{3}{2}x + 2$

Let the coords of C be $\left(x, 2 - \frac{3}{2}x\right)$.

Since $3AB = 2BC$,

$$3\left(\sqrt{4^2 + 6^2}\right) = 2\left(\sqrt{x^2 + \left(\frac{3}{2}x\right)^2}\right) \quad \text{[M1]}$$

$$4^2 + 6^2 = \frac{4}{9}\left(\frac{13}{4}x^2\right)$$

$$x^2 = 36$$

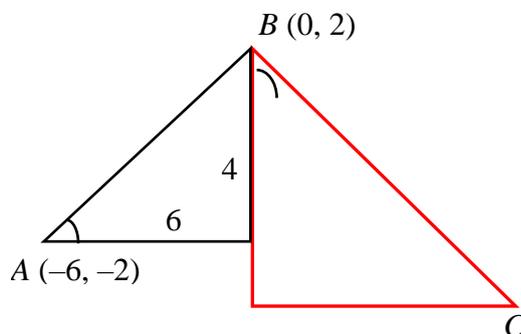
$$x = 6 \quad (\text{since } x > 0)$$

The y -coord of $C = 2 - \frac{3}{2}(6)$
 $= -7$ [M1]

Since the y -coord of $C \neq y$ -coord of A , AC is not parallel to the x -axis. [A1]

[ALTERNATIVE SOLUTION]

Since $3AB = 2BC$, $BC = \frac{3}{2}AB$.
 By similar triangles,



[M1]

The y -coord of $C = 2 - \frac{3}{2}(6)$
 $= -7$ [M1]

Since the y -coord of $C \neq y$ -coord of A , AC is not parallel to the x -axis. [A1]

(iii) Find the area of the triangle ABC .

[2]

[Marking Scheme]

Since AB is perpendicular to BC and $3AB = 2BC$

$$\begin{aligned} \text{Area of triangle } ABC &= \frac{1}{2}(AB)(BC) \\ &= \frac{1}{2}(\sqrt{4^2 + 6^2})\left(\frac{3}{2}\sqrt{4^2 + 6^2}\right) && \mathbf{[M1]} \\ &= \frac{3}{4}(4^2 + 6^2) \\ &= 39 \text{ units}^2 && \mathbf{[A1]} \end{aligned}$$

[ALTERNATIVE SOLUTION]

$$\begin{aligned} \text{Area of triangle } ABC &= \frac{1}{2} \begin{vmatrix} 0 & -6 & 6 & 0 \\ 2 & -2 & -7 & 2 \end{vmatrix} && \mathbf{[M1]} \\ &= \frac{1}{2}(42 + 12) - (-12 - 12) \\ &= 39 \text{ units}^2 && \mathbf{[A1]} \end{aligned}$$