Question 1

Since all the coefficients of $\mathrm{f}(z)$ are all real, when $(2\mathrm{i})$ is a root of $\mathrm{f}(z)=0$, then $(-2\mathrm{i})$ is another root.

Thus the product of quadratic factors $=\left[z-(2\mathrm{i})\right]\left[z-(-2\mathrm{i})\right]$

$$=\left[z-(2\mathrm{i})\right]\left[z+(2\mathrm{i})\right]$$

$$=z^2+4$$

$$\begin{aligned}\mathrm{f}(z)&=z^4+2\sqrt{2}z^3+z^2+8\sqrt{2}z-12\\&=\left(z^2+4\right)\left(z^2+Az-3\right)\end{aligned}$$

By comparing coefficients in the terms in z: $4A=8\sqrt{2}\Rightarrow A=2\sqrt{2}$

Thus $\mathrm{f}(z)=\left(z^2+4\right)\left(z^2+2\sqrt{2}z-3\right)$

Consider, $z^2+2\sqrt{2}z-3=0$.

Thus, we have $z=\dfrac{-2\sqrt{2}\pm\sqrt{8+12}}{2}=\dfrac{-2\sqrt{2}\pm\sqrt{20}}{2}=-\sqrt{2}\pm\sqrt{5}$

Hence the other roots are: $-2\mathrm{i}$, $-\sqrt{2}-\sqrt{5}$ and $-\sqrt{2}-\sqrt{5}$.

Question 2

Let a, b and c to be the monthly subscription cost for Amazoon TV, BingeWatch, and Cinematic.

Aaron : $9a+12b+11c=360$

Serene : $12b+12c=300$

Jaycee : $0.9(9a+12b)+11c=337.2$

Solving the SLE $\Rightarrow \begin{cases} 9a+12b+11c=360 \\ 12b+12c=300 \\ 8.1a+10.8b+11c=337.2 \end{cases} \Rightarrow \begin{cases} a=8 \\ b=13 \\ c=12 \end{cases}$

The monthly subscription for Amazoon TV, BingeWatch, and Cinematic are \$8, \$13 and \$12 respectively.

Question 3(i)

$$ar = a + (3-1)d \Rightarrow d = \frac{a(r-1)}{2} \qquad \text{--- (1)}$$

$$ar^3 = a + (5-1)d \Rightarrow d = \frac{a(r^3-1)}{4} \qquad \text{--- (2)}$$

Subst (1) into (2),

$$\frac{a(r^3-1)}{4} = \frac{a(r-1)}{2}$$

$$a(r^3-1) = 2a(r-1)$$

Since $a > 0$,

$$r^3 - 1 = 2(r-1)$$

$$r^3 - 2r + 1 = 0$$

If the common ratio were to be 1, from equation (1), the common difference will be 0. But the question mentioned $d \neq 0$. That is why the common ratio cannot be 1 even though $r = 1$ is a root of the equation.

Question 3(ii)

Let $r^3 - 2r + 1 = (r-1)(Ar^2 + Br + C)$.

Clearly, $A = 1$ and $C = -1$.

Compare coefficient of r: $\quad -2 = A - B$

$$B = A + 2 = 1$$

So, $r^3 - 2r + 1 = (r-1)(r^2 + r - 1) = 0$

Consider $r^2 + r - 1 = 0$,

$$r = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

Since geometric series is convergent, $r \neq \frac{-1-\sqrt{5}}{2}$.

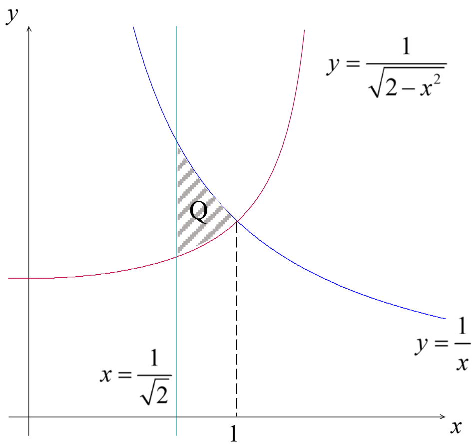
Therefore, the common ratio of the convergent geometric series is $\frac{-1+\sqrt{5}}{2}$.

Question 4(a)

Let $\dfrac{1}{x}=\dfrac{1}{\sqrt{2-x^2}}$

$\Rightarrow x^2 = 2 - x^2$

$\Rightarrow x = 1$ (since $x > 0$)



$\therefore$ Area of Region Q

$$= \int_{\frac{1}{\sqrt{2}}}^{1} \frac{1}{x}\,dx - \int_{\frac{1}{\sqrt{2}}}^{1} \frac{1}{\sqrt{2-x^2}}\,dx$$

$$= \left[\ln x - \sin^{-1}\frac{x}{\sqrt{2}}\right]_{\frac{1}{\sqrt{2}}}^{1}$$

$$= \left(\ln 1 - \sin^{-1}\frac{1}{\sqrt{2}}\right) - \left(\ln\frac{1}{\sqrt{2}} - \sin^{-1}\frac{1}{2}\right)$$

$$= 0 - \frac{\pi}{4} - \ln\frac{1}{\sqrt{2}} + \frac{\pi}{6}$$

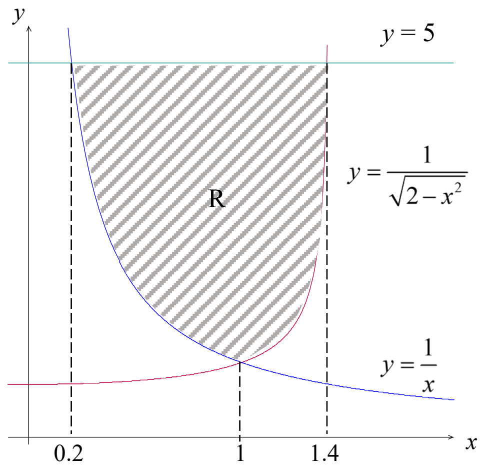
$$= \frac{1}{2}\ln 2 - \frac{\pi}{12} \text{ units}^2$$

*Accept $\ln\sqrt{2} - \dfrac{\pi}{12}$ and $-\dfrac{1}{2}\ln\dfrac{1}{2} - \dfrac{\pi}{12}$

Question 4(b)

Let $5 = \dfrac{1}{x}$ and $5 = \dfrac{1}{\sqrt{2-x^2}}$

From GC, $x = 0.2$ and $x = 1.4$ (since $x > 0$)



Required volume

$$= \text{Vol of cylinder} - \pi\int_{0.2}^{1}\left(\frac{1}{x}\right)^2 dx - \pi\int_{1}^{1.4}\left(\frac{1}{\sqrt{2-x^2}}\right)^2 dx$$

$$= \pi(5)^2(1.4-0.2) - 12.566 - 3.9158$$

$$= 77.8 \text{ units}^3 \text{ (3 s.f.)}$$

Question 5(i)

$y = e^{\sqrt{1+x}}$

$\ln y = \sqrt{1+x}$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}(1+x)^{-\frac{1}{2}}$

$2\ln y\frac{dy}{dx} = y$

$2\ln y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) = \frac{dy}{dx}$

$2\ln y\frac{d^2y}{dx^2} + \frac{2}{y}\left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx}$

Question 5(ii)

$2\ln y\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2}\left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) + \frac{4}{y}\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2\left(-\frac{1}{y^2}\right)\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$

When $x = 0$,
$y = e$
$\frac{dy}{dx} = \frac{e}{2}$
$\frac{d^2y}{dx^2} = 0$
$\frac{d^3y}{dx^3} = \frac{e}{8}$

Maclaurin's series of y is $y = e + \left(\frac{e}{2}\right)(x) + (0)\frac{x^2}{2!} + \left(\frac{e}{8}\right)\frac{x^3}{3!} + \ldots$

$y = e + \frac{e}{2}x + \frac{e}{48}x^3 + \ldots$

Question 5(iii)

$e^{\sqrt{1+x}} = e + \frac{e}{2}x + \frac{e}{48}x^3 + \ldots$

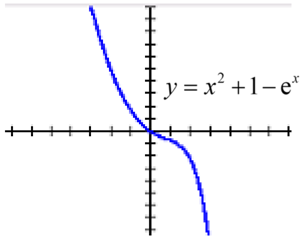
Differentiating both sides with respect to x,

$\frac{1}{2}\frac{e^{\sqrt{1+x}}}{\sqrt{1+x}} = \frac{e}{2} + \frac{e}{48}\left(3x^2\right) + \ldots$

$\frac{e^{\sqrt{1+x}}}{\sqrt{1+x}} = e + \frac{e}{8}x^2 + \ldots$

<u>Question 6(a)</u>

From graph, $x^2+1-\mathrm{e}^x<0$ for $x>0$.



$$\int_{-a}^{a}\left|x^2+1-\mathrm{e}^x\right|\,\mathrm{d}x$$
$$=\int_{-a}^{0}x^2+1-\mathrm{e}^x\,\mathrm{d}x+\int_{0}^{a}-x^2-1+\mathrm{e}^x\,\mathrm{d}x$$
$$=\left[\frac{x^3}{3}+x-\mathrm{e}^x\right]_{-a}^{0}+\left[-\frac{x^3}{3}-x+\mathrm{e}^x\right]_{0}^{a}$$
$$=-2+\mathrm{e}^{-a}+\mathrm{e}^{a}$$

<u>Question 6(b)</u>

Differentiating $x=\tan\theta$ with respect to θ

$$\frac{\mathrm{d}x}{\mathrm{d}\theta}=\sec^2\theta$$

$$\int\frac{x^2}{\left(1+x^2\right)^2}\,\mathrm{d}x=\int\frac{\tan^2\theta}{\left(1+\tan^2\theta\right)^2}\left(\sec^2\theta\right)\mathrm{d}\theta$$
$$=\int\frac{\tan^2\theta}{\sec^2\theta}\,\mathrm{d}\theta$$
$$=\int\sin^2\theta\,\mathrm{d}\theta$$
$$=\frac{1}{2}\int\left(1-\cos2\theta\right)\mathrm{d}\theta$$
$$=\frac{1}{2}\left(\theta-\frac{\sin2\theta}{2}\right)+c$$, where c is an arbitrary constant

Since $x=\tan\theta$, $\sin\theta=\dfrac{x}{\sqrt{1+x^2}}$ and $\cos\theta=\dfrac{1}{\sqrt{1+x^2}}$

$$\therefore\int\frac{x^2}{\left(1+x^2\right)^2}\,\mathrm{d}x=\frac{1}{2}\left(\theta-\frac{\sin2\theta}{2}\right)+c$$
$$=\frac{1}{2}\left(\theta-\frac{2\sin\theta\cos\theta}{2}\right)+c$$
$$=\frac{1}{2}\left(\tan^{-1}x-\frac{x}{1+x^2}\right)+c$$

Question 7(i)

$$y=\frac{(x+2)^2}{x+1} \xrightarrow{\text{Replace } x \text{ with } x-2} y=\frac{[(x-2)+2]^2}{(x-2)+1}=\frac{x^2}{x-1}$$ (Translation of 2 units in the positive x-direction)

$$\xrightarrow{\text{Replace } x \text{ with } px} y=\frac{(px)^2}{(px)-1}=\frac{p^2x^2}{px-1}$$ (Scaling of scale factor $\frac{1}{p}$ along the x-axis)

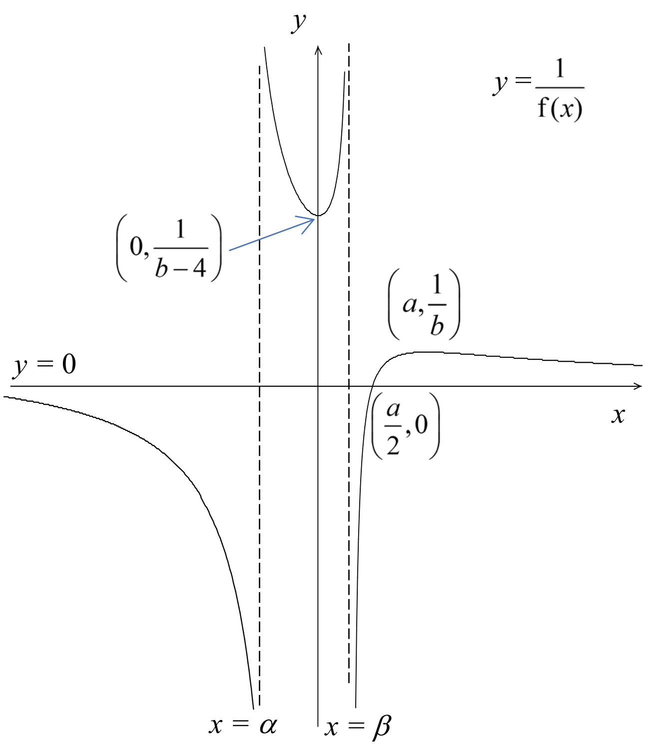
$$\xrightarrow{\text{Replace } y \text{ with } y-q} y=\frac{p^2x^2}{px-1}+q$$ (Translation of q units in the positive y-direction)

The minimum turning point on C_1, (0, 4) corresponds to (a, b) on C_2.

Hence $a=\frac{1}{p}(0+2) \Rightarrow p=\frac{2}{a}$

$b = 4 + q \Rightarrow q = b - 4$

Question 7(ii)



Question 8(i)

$$
\begin{aligned}
\sum_{r=1}^{n}\left((r+1)^3-r^3\right) &= \cancel{2^3}-1^3 \\
&\quad +\cancel{3^3}-\cancel{2^3} \\
&\quad +\cancel{4^3}-\cancel{3^3} \\
&\quad\ \ \vdots \\
&\quad +\ \ \cancel{(n)^3}-\cancel{(n-1)^3} \\
&\quad +(n+1)^3-\cancel{(n)^3} \\
&= (n+1)^3-1
\end{aligned}
$$

Question 8(ii)

$$(r+1)^3-r^3 = r^3+3r^2+3r+1-r^3$$

$$=3r^2+3r+1$$

$$\sum_{r=1}^{n}\left((r+1)^3-r^3\right)=\sum_{r=1}^{n}\left(3r^2+3r+1\right)$$

$$(n+1)^3-1=3\sum_{r=1}^{n}r^2+3\sum_{r=1}^{n}r+\sum_{r=1}^{n}1$$

$$n^3+3n^2+3n=3\sum_{r=1}^{n}r^2+3\left(\frac{n}{2}\right)(1+n)+n$$

$$3\sum_{r=1}^{n}r^2=n^3+3n^2+3n-\frac{3}{2}n-\frac{3}{2}n^2-n$$

$$\sum_{r=1}^{n}r^2=\frac{1}{6}\left(2n^3+3n^2+n\right)$$

$$=\frac{n}{6}(n+1)(2n+1)=\frac{n}{6}\left(2n^2+3n+1\right)$$

Question 8(iii)

$$\sum_{r=5}^{N}(r+2)^2=\sum_{r=7}^{N+2}r^2$$

$$=\sum_{r=1}^{N+2}r^2-\sum_{r=1}^{6}r^2$$

$$=\frac{N+2}{6}\big((N+2)+1\big)\big(2(N+2)+1\big)-\frac{6}{6}\big(6+1\big)\big(2(6)+1\big)$$

$$=\frac{1}{6}\big(N+2\big)\big(N+3\big)\big(2N+5\big)-91$$

Alternatively

$$\sum_{r=5}^{N}(r+2)^2=\sum_{r=5}^{N}(r^2+4r+4)$$

$$=\sum_{r=5}^{N}r^2+\sum_{r=5}^{N}(4r+4)$$

$$=\sum_{r=1}^{N}r^2-\sum_{r=1}^{4}r^2+\sum_{r=5}^{N}(4r+4)$$

$$=\frac{N}{6}(N+1)(2N+1)-\frac{4}{6}(5)(9)+\frac{N-4}{2}\big(24+4N+4\big)$$

$$=\frac{N}{6}(N+1)(2N+1)-30+(N-4)(14+2N)$$

$$=\frac{N}{6}(2N^2+3N+1)-30+14N+2N^2-56$$

$$=\frac{N^3}{3}+\frac{5}{3}N^2+\frac{37}{6}N-86$$

Question 9(i)

When $x=0$,

$$\tan 2\theta - 1 = 0 \Rightarrow 2\theta = \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{8}$$

$$y = 1 - 2\sec 2\left(\frac{\pi}{8}\right) = 1 - 2\sec\frac{\pi}{4} = 1 - 2\sqrt{2}$$

The coordinates of the point where C cuts the y-axis is $(0, 1-2\sqrt{2})$

Question 9(ii)

$$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\sec^2 2\theta \,; \ \frac{\mathrm{d}y}{\mathrm{d}\theta} = -4\sec 2\theta \tan 2\theta$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}} = \frac{-4\sec 2\theta \tan 2\theta}{2\sec^2 2\theta}$$

$$= -2\sin 2\theta$$

When $\theta = 0$,

$x=-1$ and $y=-1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.

The coordinates are $(-1,-1)$ and the gradient of C at this point is 0.

As $\theta \to -\frac{\pi}{4}$,

$x \to -\infty$ and $y \to -\infty$

$\frac{\mathrm{d}y}{\mathrm{d}x} \to 2$

As $\theta \to \frac{\pi}{4}$

$x \to +\infty$ and $y \to -\infty$

$\frac{\mathrm{d}y}{\mathrm{d}x} \to -2$

Question 9(iii)

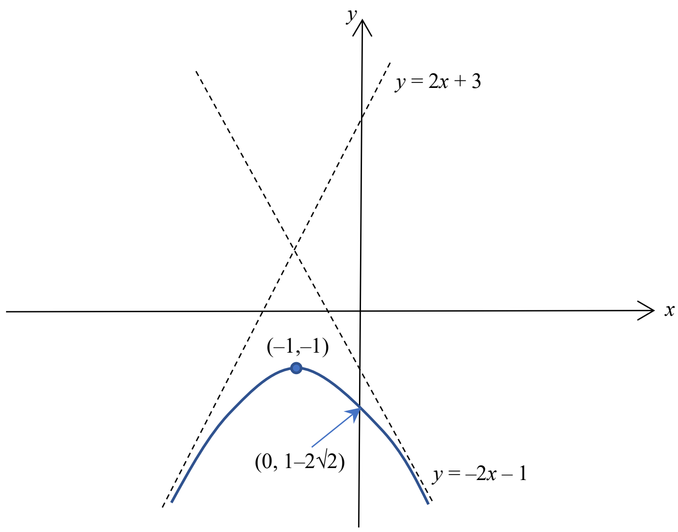
Let the equations of the asymptote be $y = 2x + a$ and $y = -2x + b$.

Substitute $x = -1$ and $y = 1$ into the equations,

$$1 = 2(-1) + a \Rightarrow a = 3$$

$$1 = -2(-1) + b \Rightarrow b = -1$$

Therefore the equations of the asymptotes are $y = 2x + 3$ and $y = -2x - 1$.



Question 9(iv)

Let $\tan 2\theta = x + 1$ and $\sec 2\theta = \dfrac{1-y}{2}$

Since $1 + \tan^2 2\theta = \sec^2 2\theta$

$$1 + (x+1)^2 = \left(\frac{1-y}{2}\right)^2$$

$$(1-y)^2 = 4 + 4(x+1)^2$$

$$y - 1 = \pm\sqrt{4 + 4(x+1)^2}$$

$$y - 1 = -\sqrt{4 + 4(x+1)^2} \qquad \text{reject } y - 1 = \sqrt{4 + 4(x+1)^2}$$

$$y = 1 - \sqrt{4 + 4(x+1)^2}$$

Question 10(i)

Let $\alpha = \angle TPB$ and $\beta = \angle CPB$

$$\tan\theta = \tan(\alpha - \beta)$$

$$= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\tan\theta = \frac{\frac{b}{x} - \frac{a}{x}}{1 + \left(\frac{b}{x}\right)\left(\frac{a}{x}\right)}$$

$$\tan\theta = \frac{(b-a)x}{x^2 + ab}$$

Question 10(ii)

$$\frac{\mathrm{d}}{\mathrm{d}x}(\tan\theta) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{(b-a)x}{x^2+ab}\right)$$

$$\sec^2\theta\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{\left(x^2+ab\right)(b-a) - (2x)(b-a)x}{\left(x^2+ab\right)^2}$$

$$\sec^2\theta\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{\left(ab - x^2\right)(b-a)}{\left(x^2+ab\right)^2}$$

OR

$$\theta = \tan^{-1}\left(\frac{(b-a)x}{x^2+ab}\right)$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{1}{1+\left(\frac{(b-a)x}{x^2+ab}\right)^2} \times \frac{\left(x^2+ab\right)(b-a) - 2x^2(b-a)}{\left(x^2+ab\right)^2}$$

$$= \vdots$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}x} = 0, \qquad \frac{\left(ab-x^2\right)(b-a)}{\left(x^2+ab\right)^2} = 0$$

$$x = \sqrt{ab} \text{ or } x = -\sqrt{ab} \text{ (rej } \because x > 0)$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{\left(ab - x^2\right)\left(b - a\right)}{\left(x^2 + ab\right)^2 \sec^2\theta}$$

Since $\left(x^2+ab\right)^2 > 0$, $\sec^2\theta > 0$ and $b > a \Rightarrow b - a > 0$, it suffices to check $\left(ab - x^2\right)$

Using first derivative test:

x	$\left(\sqrt{ab}\right)^-$	$\sqrt{ab}$	$\left(\sqrt{ab}\right)^+$
$\frac{\mathrm{d}\theta}{\mathrm{d}x}$	+ve $\because ab - x^2 > 0$	0	-ve $\because ab - x^2 < 0$

$\therefore$ θ is a maximum when $x = \sqrt{ab}$.

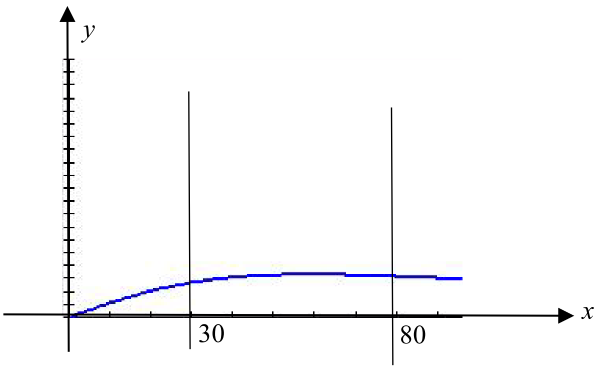
When $x = \sqrt{ab}$,

$$\tan\theta = \frac{(b-a)\sqrt{ab}}{2ab}$$

Question 10(iii)

Given $a = 50, b = 70$,

$$\tan\theta = \frac{(b-a)x}{x^2 + ab} = \frac{20x}{x^2 + 3500}$$



$$\frac{3}{22} < \tan\theta \le \frac{1}{\sqrt{35}}$$

Question 10(iv)

$$\frac{\mathrm{d}x}{\mathrm{dt}} = -3$$

When $a = 50, b = 70, x = 10$,

$$\tan\theta = \frac{(b-a)x}{x^2+ab}$$
$$= \frac{1}{18}$$

$$\therefore \sec^2\theta = \frac{325}{324}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{(x^2-ab)(a-b)}{(x^2+ab)^2\sec^2\theta} = \frac{17}{3250} \text{ or } 0.0052308$$

$$\frac{\mathrm{d}\theta}{\mathrm{dt}} = \frac{\mathrm{d}\theta}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{dt}}$$
$$= -3 \times \frac{17}{3250}$$
$$= -\frac{51}{3250}$$

The angle θ is decreasing at a rate of $\frac{51}{3250}$ rad/s.

Question 11(i)

$$p: 6x-5y+2z+5=0 \Rightarrow \mathbf{r}\cdot\begin{pmatrix}6\\-5\\2\end{pmatrix}=-5$$

$$l: \frac{x-10}{15}=\frac{y-15}{14}=\frac{z-5}{-10} \Rightarrow \mathbf{r}=\begin{pmatrix}10\\15\\5\end{pmatrix}+\lambda\begin{pmatrix}15\\14\\-10\end{pmatrix}, \lambda\in\mathbb{R}$$

Consider, $\begin{pmatrix}10\\15\\5\end{pmatrix}\cdot\begin{pmatrix}6\\-5\\2\end{pmatrix}=60-75+10=-5 \Rightarrow (10,15,5)$ lies on both l & p.

Consider, $\begin{pmatrix}15\\14\\-10\end{pmatrix}\cdot\begin{pmatrix}6\\-5\\2\end{pmatrix}=90-70-20=0 \Rightarrow l$ is parallel to p.

Therefore, the path l lies on plane p.

Question 11(ii)

When $z=0$, $\frac{x-10}{15}=\frac{1}{2} \Rightarrow x=17.5$ and $\frac{y-15}{14}=\frac{1}{2} \Rightarrow y=22$

Thus fixed point has coordinates $(17.5, 22, 0)$.

Question 11(iii)

$$m: \mathbf{r}=\begin{pmatrix}30\\-10\\0\end{pmatrix}+t\begin{pmatrix}-10\\9\\15\end{pmatrix}, t\in\mathbb{R} \text{ and } p: \mathbf{r}\cdot\begin{pmatrix}6\\-5\\2\end{pmatrix}=-5$$

Consider, $\begin{pmatrix}30-10t\\-10+9t\\15t\end{pmatrix}\cdot\begin{pmatrix}6\\-5\\2\end{pmatrix}=-5$.

$\Rightarrow 180-60t+50-45t+30t=-5$.

$\Rightarrow 75t=235$

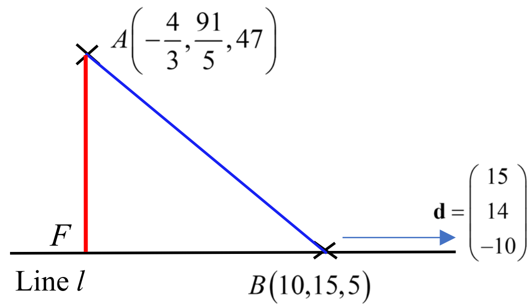
$\Rightarrow t=\frac{47}{15}=3\frac{2}{15}$

$$\mathbf{r} = \begin{pmatrix} 30-10\left(\frac{47}{15}\right) \\ -10+9\left(\frac{47}{15}\right) \\ 15\left(\frac{47}{15}\right) \end{pmatrix} = \begin{pmatrix} -\frac{4}{3} \\ \frac{91}{5} \\ 47 \end{pmatrix}$$

The unauthorised drone hits the plane p at coordinates $\left(-\frac{4}{3},\frac{91}{5},47\right)$ after taking off for 3 min 8 seconds (or 3.13 min).

<u>Question 11(iv)</u>

Let A(where unauthorised drone hits plane p) and B(point on line l) be points whose coordinates are $\left(-\frac{4}{3},\frac{91}{5},47\right)$ and $(10,15,5)$ respectively.



Let direction vector of line be $\mathbf{d} = \begin{pmatrix} 15 \\ 14 \\ -10 \end{pmatrix}$.

Thus, $\overrightarrow{BA} = \begin{pmatrix} -\frac{4}{3} \\ \frac{91}{5} \\ 47 \end{pmatrix} - \begin{pmatrix} 10 \\ 15 \\ 5 \end{pmatrix} = \begin{pmatrix} -\frac{34}{3} \\ \frac{16}{5} \\ 42 \end{pmatrix}$.

$$\begin{aligned}\text{Shortest distance } &= \left|\overrightarrow{BA} \times \frac{\mathbf{d}}{|\mathbf{d}|}\right| \\ &= \left|\begin{pmatrix} -\frac{34}{3} \\ \frac{16}{5} \\ 42 \end{pmatrix} \times \frac{1}{\sqrt{521}}\begin{pmatrix} 15 \\ 14 \\ -10 \end{pmatrix}\right| \\ &= \frac{1}{\sqrt{521}}\left|\begin{pmatrix} -620 \\ \frac{1550}{3} \\ \frac{-620}{3} \end{pmatrix}\right| = 36.49877 = 36.5 \text{ m (3 sig. fig.)}\end{aligned}$$

Question 11(v)

Let F be the foot of the perpendicular from A to line l.

$$\overrightarrow{AF}=\begin{pmatrix}10+15\lambda\\15+14\lambda\\5-10\lambda\end{pmatrix}-\begin{pmatrix}-\frac{4}{3}\\\frac{91}{5}\\47\end{pmatrix}=\begin{pmatrix}15\lambda+\frac{34}{3}\\14\lambda-\frac{16}{5}\\-10\lambda-42\end{pmatrix}$$

$$\overrightarrow{AF}\cdot\mathbf{d}=0\Rightarrow\begin{pmatrix}15\lambda+\frac{34}{3}\\14\lambda-\frac{16}{5}\\-10\lambda-42\end{pmatrix}\cdot\begin{pmatrix}15\\14\\-10\end{pmatrix}=0$$

$$\Rightarrow 521\lambda=-545.2\Rightarrow\lambda=-\frac{2726}{2605}=-1.04645$$

Thus $\overrightarrow{OF}=\begin{pmatrix}-\frac{2968}{521}\\\frac{911}{2605}\\\frac{8057}{521}\end{pmatrix}=\begin{pmatrix}-5.70\\0.350\\15.5\end{pmatrix}$.

Alternatively,

$$\overrightarrow{BA}=\begin{pmatrix}-\frac{4}{3}\\\frac{91}{5}\\47\end{pmatrix}-\begin{pmatrix}10\\15\\5\end{pmatrix}=\begin{pmatrix}-\frac{34}{3}\\\frac{16}{5}\\42\end{pmatrix}.$$

$$\overrightarrow{BF}=\left(\overrightarrow{BA}\cdot\frac{\mathbf{d}}{|\mathbf{d}|}\right)\frac{\mathbf{d}}{|\mathbf{d}|}$$

$$=\left[\begin{pmatrix}-\frac{34}{3}\\\frac{16}{5}\\42\end{pmatrix}\cdot\frac{1}{\sqrt{521}}\begin{pmatrix}15\\14\\-10\end{pmatrix}\right]\frac{1}{\sqrt{521}}\begin{pmatrix}15\\14\\-10\end{pmatrix}\qquad=-\frac{2726}{2605}\begin{pmatrix}15\\14\\-10\end{pmatrix}$$

$$\overrightarrow{OF}=-\frac{2726}{2605}\begin{pmatrix}15\\14\\-10\end{pmatrix}+\begin{pmatrix}10\\15\\5\end{pmatrix}==\begin{pmatrix}-\frac{2968}{521}\\\frac{911}{2605}\\\frac{8057}{521}\end{pmatrix}=\begin{pmatrix}-5.70\\0.350\\15.5\end{pmatrix}$$

Question 12(i)

$\dfrac{dI}{dt} = RI$

$\int \dfrac{1}{I} dI = \int R dt$

$\ln I = Rt + c_1$, since $I > 0$

$I = A_1 e^{Rt}$, where $A_1 = e^{c_1}$

When $t = 0$, $I = 5$,

$5 = A_1 e^0 \Rightarrow A_1 = 5$

$\therefore I = 5e^{Rt}$

Question 12(ii)

It means that the number of people being infected will double after every day.

Question 12(iii)

When $t = 1$, model (I) gives $I = 5e^R$

model (II) gives $I_1 = 5\left(2^1\right)$

Equating $5e^R = 5\left(2^1\right)$,

$e^R = 2$

$R = \ln 2$

Question 12(iv)

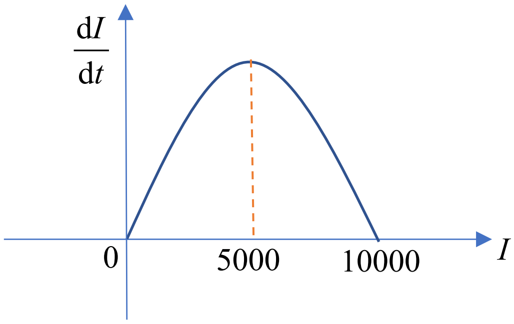
Model (I) is a better model because it can be used to predict the number of infected people at any one time whereas model (II) can only predict so at the start of every new day.

Alternative explanation

The number of infected people must be of integer values. Hence Model (II) is a better model than model (I).

to be awarded for any plausible explanation.

Question 12(v)



From the graph of $\frac{\mathrm{d}I}{\mathrm{d}t}$ against t, there are 5000 people infected at the instance when the rate of infection is the greatest.

Sub $I = 5000$ and $\frac{\mathrm{d}I}{\mathrm{d}t} = 2500$ into $\frac{\mathrm{d}I}{\mathrm{d}t} = aI(10000 - I)$,

$$2500 = 5000(10000 - 5000)a \Rightarrow a = 0.0001$$

Question 12(vi)

$$\frac{\mathrm{d}I}{\mathrm{d}t} = 0.0001I(10000 - I)$$

$$\int \frac{1}{I(10000 - I)}\,\mathrm{d}I = \int 0.00001\,\mathrm{d}t$$

$$\int \frac{1}{I^2 - 10000I}\,\mathrm{d}I = \int -0.00001\,\mathrm{d}t$$

$$\int \frac{1}{(I - 5000)^2 - 5000^2}\,\mathrm{d}I = \int -0.00001\,\mathrm{d}t$$

$$\frac{1}{2(5000)}\ln\left|\frac{(I - 5000) - 5000}{(I - 5000) + 5000}\right| = -0.00001t + c_2$$

$$\ln\left|\frac{I - 10000}{I}\right| = -t + 10000c_2$$

$$\frac{I - 10000}{I} = A_2\mathrm{e}^{-t}, \text{ where } A_2 = \pm\,\mathrm{e}^{10000c_2}$$

$$I - 10000 = A_2 I\mathrm{e}^{-t}$$

$$I\left(1 - A_2\mathrm{e}^{-t}\right) = 10000$$

$$I = \frac{10000}{1 - A_2\mathrm{e}^{-t}}$$

When $t = 0$, $I = 5$,

$$5 = \frac{10000}{1 - A_2\mathrm{e}^{0}} \Rightarrow A_2 = -1999$$

Sub $A_2 = -1999$ in $I = \frac{10000}{1 - A_2\mathrm{e}^{-t}}$, we get $I = \frac{10000}{1 + 1999\mathrm{e}^{-t}}$

Question 12(vii)