ANDERSON SERANGOON JUNIOR COLLEGE

H2 MATHEMATICS **9758/02**

3 hours

Preliminary Examination Paper 2
(100 marks)

Additional Material(s): List of Formulae (MF26)

CANDIDATE NAME	
CLASS	/

READ THESE INSTRUCTIONS FIRST

Write your name and class in the boxes above.
Please write clearly and use capital letters.
Write in dark blue or black pen. HB pencil may be used for graphs and diagrams only.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions and write your answers in this booklet.
Do not tear out any part of this booklet.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

All work must be handed in at the end of the examination. If you have used any additional paper, please insert them inside this booklet.
The number of marks is given in brackets [ ] at the end of each question or part question.

Question number	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total	

Section A: Pure Mathematics [40 marks]

1 The complex numbers z and w have moduli k and $3k^2$ respectively and arguments α and 4α respectively, where k is a positive constant and $-\frac{\pi}{7} < \alpha \leq \frac{\pi}{7}$.

(i) Express $\frac{z^3}{w^*}$ in the form $re^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. [2]

(ii) It is given that $\alpha = \frac{\pi}{21}$. Find the integer values of n such that $\left(\frac{z^3}{w^*}\right)^n$ is real. [2]

2 (i) Without the use of a calculator, solve the inequality $\frac{17x-9}{2x^2+13x-7} > 1$. [3]

(ii) Hence solve the inequality $\frac{17x+9}{2x^2-13x-7} < -1$. [2]

3 Three distinct vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are each of unit length such that $\lambda\mathbf{a} + \mu\mathbf{b} + \gamma\mathbf{c} = \mathbf{0}$, where λ, μ, γ are non-zero scalars.

(i) Show that $\mu(\mathbf{a} \times \mathbf{b}) = \gamma(\mathbf{c} \times \mathbf{a})$ and $\lambda(\mathbf{b} \times \mathbf{a}) = \gamma(\mathbf{c} \times \mathbf{b})$. [3]

(ii) By considering the sines of angles between $\mathbf{b}$ and $\mathbf{c}$, $\mathbf{c}$ and $\mathbf{a}$, and $\mathbf{a}$ and $\mathbf{b}$, show that

$$\frac{|\mu|}{\sin\angle COA} = \frac{|\lambda|}{\sin\angle BOC} = \frac{|\gamma|}{\sin\angle AOB}.$$ [4]

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4 (a) The function f is defined as

$$\text{f}: x \to \ln\left|\sec 2x\right| \text{ for } \frac{\pi}{3} \le x \le \frac{3\pi}{8}.$$

Find $\text{f}^{-1}(x)$ and determine its domain. [4]

(b) The function g is one-one and is defined as

$$\text{g}: x \to 2x\sin\left(x^2\right) \text{ for } -k \le x \le k,$$

where k is positive constant.

(i) Write down the largest value of k and find the range of g, correct to 3 decimal places. [2]

(ii) Using the value of k found in part **(i)**, sketch the graphs of $y = \text{g}(x)$, $y = \text{g}^{-1}(x)$ and $y = \text{gg}^{-1}(x)$ on the same diagram. Your diagram should illustrate correctly the relationship between the graphs, showing clearly the coordinates of the end-points of all graphs. [3]

(iii) Hence find the exact values of x such that $\text{g}(x) = \text{g}^{-1}(x)$. [2]

(iv) The function h is defined as

$$\text{h}: x \to -2x\sin(x^2) \text{ , for } -1 \le x \le 1.$$

With the aid of a diagram, explain clearly why the method used in part **(iii)** to find the exact values of x such that $\text{h}(x) = \text{h}^{-1}(x)$ would not yield the complete set of solution. [2]

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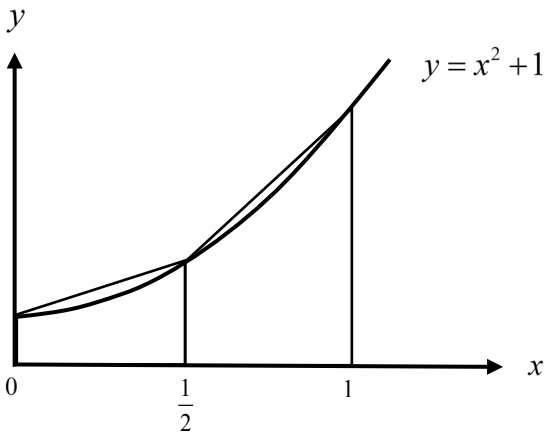
5 It is given that R is the region bounded by the x-axis, the y-axis, the line $x = 1$ and the curve $y = x^2 + 1$.

The area of R can be estimated by calculating the sum of the areas of trapeziums with equal widths. The diagram below (not drawn to scale) shows an example of two such trapeziums.

[Area of trapezium = $\frac{1}{2} \times$ sum of parallel sides $\times$ height]



(i) Find the total area of the trapeziums shown in the diagram. [2]

To better estimate the area of R, n trapeziums of equal width are drawn.

(ii) State the length of the shorter side of the k^{th} trapezium for $1 \leq k \leq n$. Hence show that its area is given by $\frac{(k-1)^2 + k^2}{2n^3} + \frac{1}{n}$ units^2. [3]

The sum of the areas of the n trapeziums is given as A units^2.

(iii) Find A, giving your answers in terms of n. [4]

[You may use $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$]

(iv) State whether A is an overestimate or an underestimate of the area of region R. [1]

(v) Use your answer in part **(iii)** to find the exact area for region R. [1]

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Section B: Probability and Statistics [60 marks]

6 A disease outbreak occurs in a particular city. It is given that p % of the residents from the city has contracted the disease. A test for the disease has recently been developed. It gives an outcome of positive or negative when administered to a person.

The test has a probability 0.9 of giving a positive result when it is administered to a person who has the disease. The test also has the same probability of giving a negative result when it is administered to a person who does not have the disease.

(i) The health authority of the city has been aggressively testing a large number of residents at random and discovered that 20% of the tests return a positive result. Find the proportion of the residents infected with the disease. [2]

(ii) As time goes by, the proportion of people infected with the disease increases to q %. Find, in terms of q, the probability that a person has the disease given that this person has been tested negative. [2]

(iii) Using your answer in part **(ii)**, discuss the effectiveness of this test in the long run. [1]

7 Four friends went for a Christmas party, each bringing a present for gift exchange. During the gift exchange, the presents were randomly distributed, such that each person received exactly one gift, which may be the same or different from the present that he brought along. It is given that the presents brought to the Christmas party were all distinct from one another.

(i) Find the total number of ways to distribute the four presents to the four people if there are no restrictions. [1]

(ii) Find the number of ways the presents can be distributed if there is exactly one person who received back their own presents. [2]

(iii) Find the number of ways the presents can be distributed given that there are exactly two persons who received back his own present. [2]

(iv) Explain why there cannot be a case where there are exactly 3 persons who received back their own gifts. [1]

(v) Hence or otherwise, find the probability that none of the friends received back their own present after the gift exchange. [2]

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8 A bag contains n blue cards and $(n+1)$ white cards which are identical in all aspects except for their colour. Two cards are drawn at random without replacement, from the bag.

(i) Find the probability that the two cards drawn are of different colour. [1]

(ii) Find the probability that the two cards drawn are of the same colour. [1]

If the cards are of different colour, two fair coins are then tossed and the number of heads is recorded. If the cards are of the same colour, the two fair coins are each tossed twice and the total number of heads is recorded. The random variable X is the total number of heads recorded when the two cards are drawn without replacement from the bag.

(iii) Show that $\mathrm{P}(X=1)=\dfrac{3n+2}{4(2n+1)}$. [2]

It is now given that there are 3 blue cards.

(iv) Find the probability distribution for X. [4]

(v) Find $\mathrm{P}(X>2 \mid X\leq 3)$. [2]

9 Sleep is important for optimal cognitive functions for students. One particular study found that 85% of students from schools in Singapore obtain less than the 8 hours of nocturnal (night-time) sleep recommended by the US National Sleep Foundation, and average, only 6.5 hours a night on a school-day.

(i) Assuming that the amount of nocturnal sleep for a student per night can be modelled by a normal distribution, find its standard deviation. [2]

Three students in a school are randomly selected by a researcher to participate in a survey on sleep.

(ii) Find the probability that one of the three students has less than 5.5 hours of nocturnal sleep per night and the other two students have more than 7 hours of nocturnal sleep per night.

The researcher opines that the total amount of sleep for students should be considered as the sum of the amount of their nocturnal sleep and afternoon naps. He assumes that the total amount of sleep from afternoon naps on a school-day can be modelled using a normal distribution with mean 65 minutes and standard deviation a minutes.

(iii) Explain why the use of a normal distribution as a model may not be appropriate when $a = 60$. [1]

It is now given that $a = 10$.

(iv) Find the probability that the total amount of sleep for the first student exceeds 8 hours. [3]

(v) Find the probability that the total amount of sleep of the first two students exceeds twice the total amount of sleep of the third student by more than 1 hour. [3]

(vi) State an assumption that you have used in your calculations in part **(iv)** and explain why this assumption may be unrealistic. [2]

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10 A firm sells two types of electrical components, A and B.

Type A components are packed in boxes of 50. On average, 2% of the Type A components are faulty.

(i) State, in context, two assumptions needed for the number of faulty components of Type A in a box to be well-modelled by a binomial distribution. [2]

You are now given that the number of faulty components of type A in a box follows a binomial distribution.

(ii) Find the probability that a box of these components contains more than 1 faulty component. [2]

Five boxes of Type A components are selected at random.

(iii) Find the probability that the fifth box is the third box selected which has more than one faulty component. [2]

(iv) Find the probability that there are 3 boxes with more than 1 faulty component each. [2]

(v) Explain why the answer to part **(iv)** is greater than the answer to part **(iii).** [1]

Type B components are packed in boxes of 20 and on average 0.1% of the Type B components are faulty.

(vi) Thirty boxes of Type A and forty boxes of Type B components are randomly chosen. Find, by using suitable approximations, the probability that the total number of faulty components are no more than 15. [3]

11 An airline which cater various flights within the United States of America has a flight route that travels from New Orleans to Miami. The airline claims that the average flight time is 115 minutes. Brandon, a regular commuter of this flight route, thinks that his journey takes a shorter time on average.

(i) Explain, whether Brandon should carry out a 1-tail test or a 2-tail test. State the hypotheses for the test, defining any symbols you use. [2]

Brandon records the flight time (in minutes) of 8 randomly selected flights as shown below.

113.0 112.4 110.0 113.8 111.3 115.2 114.2 115.5

(ii) Assuming that the flight time from New Orleans to Miami follows a normal distribution that has a variance of 11.3 minutes^2, carry out the test for Brandon at 10% level of significance. Give your conclusion in the context of the question. [3]

(iii) Explain what you understand by the phrase "at 10% level of significance" in the context of the question. [1]

(iv) The same sample of 8 flights is now used to carry out a test at the 10% significance level, to test whether the mean flight times is different from what the airline claims. Using the answer in **(ii)**, state the conclusion of this test. [2]

Another flight route travels from Portland to Los Angeles. Due to bad weather conditions, the flights sometimes take a longer time to arrive at Los Angeles. The airline claims that the mean flight time is k minutes. The operation manager of the airline believes that this number is understated and wants to test this claim. Sixty flights were randomly selected, and the mean flight time of 181 minutes and standard deviation of 13.6 minutes were recorded.

(v) Find the range of values of k for which the claim by the airline is supported at 4% level of significance. Give your answer to the nearest minute. [4]

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