ANDERSON SERANGOON JUNIOR COLLEGE

H2 MATHEMATICS 9758/01

3 hours

Preliminary Exam Paper 1
(100 marks)

Additional Material(s): List of Formulae (MF26)

CANDIDATE NAME	
CLASS	/

READ THESE INSTRUCTIONS FIRST

Write your name and class in the boxes above.
Please write clearly and use capital letters.
Write in dark blue or black pen. HB pencil may be used for graphs and diagrams only.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions and write your answers in this booklet.
Do not tear out any part of this booklet.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

All work must be handed in at the end of the examination. If you have used any additional paper, please insert them inside this booklet.
The number of marks is given in brackets [ ] at the end of each question or part question.

Question number	Marks
1	
2	
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10	
11	
12	
Total	

1 **Do not use a calculator in answering this question.**

It is given that $\mathrm{f}(z) = z^4 + 2\sqrt{2}z^3 + z^2 + 8\sqrt{2}z - 12$. One of the roots of the equation $\mathrm{f}(z) = 0$ is given by $2\mathrm{i}$. By factorising $\mathrm{f}(z)$ as a product of two quadratic factors, obtain the other roots of the equation. [4]

2 Amazoon TV, BingeWatch, Cinematic are three TV streaming services which offer customers a monthly subscription package.

Promotions for monthly subscriptions are available for PhoneHub members as well as for customers who pay by credit cards. This is shown in the following table.

TV Streaming Service	Promotion for PhoneHub members	Promotion for credit card
Amazoon TV	Free for first 3 months	10% discount on all monthly subscriptions
BingeWatch	Not Applicable	
Cinematic	Free for first month	

- Aaron is a PhoneHub member but does not own a credit card. He subscribed to all three services.
- Serene is not a PhoneHub member and does not own a credit card. She decided to subscribe to BingeWatch and Cinematic services.
- Jaycee is a PhoneHub member who subscribed to all three services. She uses her credit card to pay for Amazoon TV and BingeWatch services and pays for Cinematic service by cash.

In the first year of their subscriptions, the annual expenditure of Aaron, Serene and Jaycee are \$360.00, \$300 and \$337.20 respectively. How much is the monthly subscription for each streaming service? [4]

3 **Do not use a calculator in answering this question.**

A geometric series has first term a and common ratio r, where $a > 0$ and $r \neq 0$. An arithmetic series has the same first term a and common difference d, where $d \neq 0$. It is given that the second and fourth terms of the geometric series are equal to the third and fifth terms of the arithmetic series respectively.

(i) Show that $r^3 - 2r + 1 = 0$ and explain why the common ratio cannot be 1 even though $r = 1$ is a root of the equation. [3]

(ii) Given that the geometric series is convergent, find the exact value of its common ratio. [2]

4 Curve C_1 has equation $y=\dfrac{1}{x}$ and curve C_2 has equation $y=\dfrac{1}{\sqrt{2-x^2}}$. Region Q is bounded by line $x=\dfrac{1}{\sqrt{2}}$ and both curves C_1 and C_2.

(a) Find the exact area of region Q. [3]

(b) The region bounded by the line $y=5$ and the curves C_1 and C_2 in the first quadrant is rotated about the x–axis by 2π radians. Find the volume of the solid formed. [3]

5 (i) It is given that $y=\mathrm{e}^{\sqrt{1+x}}$, $x>-1$. Show that $2\ln y\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\dfrac{2}{y}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) By further differentiation of this result, or otherwise, find the Maclaurin's series of y up to and including the term in x^3. [3]

(iii) Deduce the series expansion of $y=\dfrac{\mathrm{e}^{\sqrt{1+x}}}{\sqrt{1+x}}$ up to and including the term in x^2. [1]

6 (a) Evaluate $\int_{-a}^{a}\left|x^2+1-\mathrm{e}^x\right|\mathrm{d}x$ where $a>0$, giving your answer in terms of a. [3]

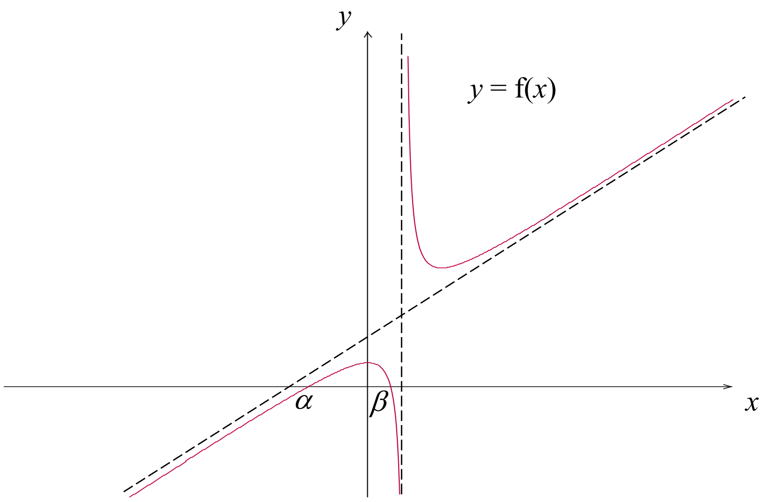
(b) By means of substituting $x=\tan\theta$, or otherwise, show that

$$\int\frac{x^2}{\left(1+x^2\right)^2}\,\mathrm{d}x=\frac{1}{2}\left(\tan^{-1}x+\mathrm{f}(x)\right)+c,$$

where c is an arbitrary unknown constant and $\mathrm{f}(x)$ is an expression to be determined in non-trigonometric form. [4]

7 (i) The curve C_1 with equation $y=\dfrac{(x+2)^2}{x+1}$ is transformed onto the curve C_2 with equation $y=\mathrm{f}(x)$. The curve C_1 has a minimum turning point (0, 4) which corresponds to the point with coordinates (a, b) on the curve C_2, where $a, b > 0$. Given that f(x) has the form $\dfrac{p^2x^2}{px-1}+q$, where p, q are positive constants, express p and q in terms of a and b. [4]

(ii) The curve of $y = \mathrm{f}(x)$ has a maximum point and a minimum point at $(0,q)$ and (a,b) respectively, and intersects the x-axis at α and β, as shown in the diagram below. The equation of the vertical asymptote is $x=\dfrac{1}{p}$.



Sketch the curve $y=\dfrac{1}{\mathrm{f}(x)}$. Your diagram should indicate clearly, in terms of a, b, α and β, the equations of any asymptote(s) as well as the coordinates of turning points and axial intercepts. [3]

8 (i) By using method of difference, show that $\sum_{r=1}^{n}\left((r+1)^3-r^3\right)=(n+1)^3-1$. [2]

(ii) By simplifying $(r+1)^3-r^3$ or otherwise, show that $\sum_{r=1}^{n}r^2=\dfrac{n}{6}(n+1)(2n+1)$. [4]

(iii) Hence find $\sum_{r=5}^{N}(r+2)^2$, leaving your answer in terms of N. [3]

9 A curve C has parametric equations

$$x = \tan 2\theta - 1, \qquad y = 1 - 2\sec 2\theta, \qquad \text{for } -\frac{\pi}{4} < \theta < \frac{\pi}{4}.$$

(i) Find the coordinates of the point where C cuts the y-axis, giving your answer in exact form. [2]

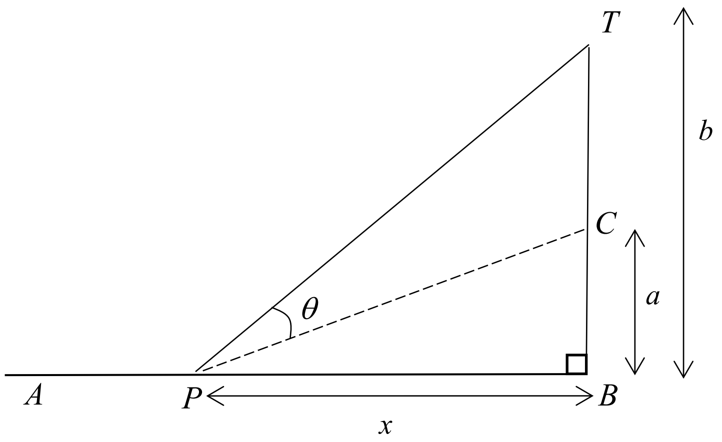
(ii) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -2\sin 2\theta$. Find the coordinates of the point where $\theta = 0$ and determine the gradient of C at this point. What can be said about the values of x and y and the gradient to C as $\theta \to -\frac{\pi}{4}$ and $\theta \to \frac{\pi}{4}$? [5]

It is given that C has two oblique asymptotes which intersect each other at the point $(-1,1)$.

(iii) Find the equations of the asymptotes. Hence sketch C, showing clearly the features of the curve at the points where $\theta = 0$ and where $\theta \to -\frac{\pi}{4}$ and $\theta \to \frac{\pi}{4}$. [3]

(iv) Find the cartesian equation of C, expressing y in terms of x. [2]

10 In the diagram below, A and B are two fixed points on horizontal ground and an observer is standing at point P which is x metres away from the base of a building BC of height a m. A transmission tower CT is fixed at the top of the building and the total height of the building and the tower is given as b metres. You may assume that B, C and T lies on a straight line.



It is given that the angle TPC is θ radians.

(i) Show that $\tan\theta = \dfrac{(b-a)x}{x^2+ab}$. [2]

(ii) Find, in terms of a and b, the value of x such that θ is a maximum. Find also the corresponding value of $\tan\theta$. [4]

For the remaining parts of the question, take $a = 50$ and $b = 70$.

(iii) Find the range of values of $\tan\theta$ as x varies between 30 and 80. [2]

(iv) The observer at P starts to walk towards B with a steady speed of 3 ms^{-1}. Find the rate of change of θ when he is 10 m from B. [4]

11 Intercepting drones are programmed to travel in straight paths on the plane p with equation

$$6x-5y+2z+5=0\,.$$

For one particular intercepting drone, such a path is given by the line l with cartesian equation

$$\frac{x-10}{15}=\frac{y-15}{14}=\frac{z-5}{-10}.$$

(i) Show that the path l lies on plane p. [2]

This particular intercepting drone is always activated from a fixed point on the ground, which is represented by the xy plane.

(ii) Find the coordinates of the fixed point on the ground where this particular intercepting drone takes off. [2]

An unauthorised drone takes off from the ground traversing along the straight path m having an equation

$$\mathbf{r}=(30-10t)\,\mathbf{i}+(9t-10)\,\mathbf{j}+15t\ \mathbf{k}$$

where t is the time taken from take-off measured in minutes.

(iii) Find when and where the unauthorised drone hits the plane p. [3]

(iv) Determine the shortest distance between the point where the unauthorised drone hits the plane p and the path l traversed by the particular intercepting drone. [3]

(v) Find the location of the point on path l that gives this shortest distance. [3]

12 Two students are trying to do mathematical modelling for a recent disease outbreak in a particular city with a population of 10 000. There are 5 people being infected by the disease initially.

Student A proposed modelling the scenario using the following differential equation

$$\frac{\mathrm{d}I}{\mathrm{d}t} = RI \,, \qquad \text{--- (I)}$$

where I is the number of people in the city being infected at time t (days) after the initial outbreak and R is a positive constant.

(i) Show that $I = 5\mathrm{e}^{Rt}$. [3]

Another student B proposed using a geometric progression for his mathematical model. His model can be predicted by the equation

$$I_n = 5\left(2^n\right) \,, \qquad \text{--- (II)}$$

where the n^{th} term of the geometric progression, I_n, gives the number of people infected with the disease at the beginning of the n^{th} day after the initial outbreak.

It is given that the geometric progression has common ratio 2.

(ii) Explain, in the context of the infectious outbreak, the meaning of the common ratio. [1]

(iii) Show that $R = \ln 2$ [1]

(iv) Explain whether model (I) or model (II) is a better model. [1]

Both students then went to do some research and discovered another model in the form of the following differential equation

$$\frac{\mathrm{d}I}{\mathrm{d}t} = aI(10000 - I) \,, \qquad \text{--- (III)}$$

where a is a positive constant.

It is given that the greatest value of $\dfrac{\mathrm{d}I}{\mathrm{d}t}$ is 2500.

(v) Show that $a = 0.0001$. [2]

(vi) By solving the differential equation (III), show that $I = \dfrac{10000}{1 + 1999\mathrm{e}^{-t}}$. [5]

(vii) Sketch the graph showing how the number of infected people varies with time in model (III). [2]