Question 1(i)

$$\left|\frac{z^3}{w^*}\right| = \frac{|z|^3}{|w|} = \frac{k^3}{3k^2} = \frac{k}{3}$$

$$\arg\left(\frac{z^3}{w^*}\right) = 3\arg(z) - \left[-\arg(w)\right] = 3\alpha + 4\alpha = 7\alpha$$

Thus, $\dfrac{z^3}{w^*} = \dfrac{k}{3}\mathrm{e}^{\mathrm{i}(7\alpha)}$

Question 1(ii)

$$\frac{z^3}{w^*} = \frac{k}{3}\mathrm{e}^{\mathrm{i}(\pi/3)} = \frac{k}{3}\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$$

$$\arg\left(\frac{z^3}{w^*}\right)^n = n\arg\left(\frac{z^3}{w^*}\right) = \frac{n\pi}{3}$$

For $\left(\dfrac{z^3}{w^*}\right)^n$ to be real, $\arg\left(\dfrac{z^3}{w^*}\right)^n = k\pi$, $k \in \mathbb{Z}$

$$\frac{n\pi}{3} = \cdots, -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \cdots$$

$$n = \cdots, -9, -6, -3, 0, 3, 6, 9, \cdots$$

Question 2(i)

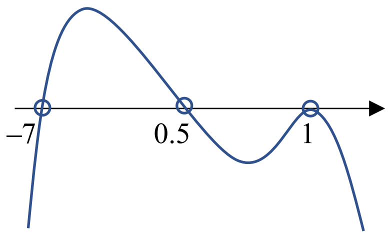
$$\frac{17x-9-2x^2-13x+7}{2x^2+13x-7}>0$$

$$\frac{-2x^2+4x-2}{(2x-1)(x+7)}>0\,;\, x\neq\frac{1}{2}\ ,-7$$

$$\frac{-2(x-1)^2}{(2x-1)(x+7)}>0$$

$$-2(x-1)^2(2x-1)(x+7)>0$$

$$\Rightarrow -7<x<\frac{1}{2}$$



Question 2(ii)

Replace x with $-x$: $\frac{17x+9}{2x^2-13x-7}<-1 \quad \rightarrow \quad \frac{17(-x)+9}{2(-x)^2-13(-x)-7}<-1$

$$\frac{-17x+9}{2x^2+13x-7}<-1$$

$$\frac{17x-9}{2x^2+13x-7}>1$$

From above, $-7<-x<\frac{1}{2}$

$$\Rightarrow -\frac{1}{2}<x<7$$

Question 3(i)

$\lambda\mathbf{a}+\mu\mathbf{b}+\gamma\mathbf{c}=\mathbf{0}$ $\Rightarrow \mathbf{a},\mathbf{b},\mathbf{c}$ lie on the same plane

$$\Rightarrow \mathbf{a}\times(\lambda\mathbf{a}+\mu\mathbf{b}+\gamma\mathbf{c})=\mathbf{a}\times\mathbf{0}$$
$$\Rightarrow \lambda(\mathbf{a}\times\mathbf{a})+\mu(\mathbf{a}\times\mathbf{b})+\gamma(\mathbf{a}\times\mathbf{c})=\mathbf{0}$$
$$\Rightarrow \mathbf{0}+\mu(\mathbf{a}\times\mathbf{b})=-\gamma(\mathbf{a}\times\mathbf{c})$$
$$\Rightarrow \mu(\mathbf{a}\times\mathbf{b})=\gamma(\mathbf{c}\times\mathbf{a}) \text{ (shown) } \ldots \text{ (A)}$$

Similarly

$$\Rightarrow \mathbf{b}\times(\lambda\mathbf{a}+\mu\mathbf{b}+\gamma\mathbf{c})=\mathbf{b}\times\mathbf{0}$$
$$\Rightarrow \lambda(\mathbf{b}\times\mathbf{a})+\mu(\mathbf{b}\times\mathbf{b})+\gamma(\mathbf{b}\times\mathbf{c})=\mathbf{0}$$
$$\Rightarrow \lambda(\mathbf{b}\times\mathbf{a})=-\gamma(\mathbf{b}\times\mathbf{c})$$
$$\Rightarrow \lambda(\mathbf{b}\times\mathbf{a})=\gamma(\mathbf{c}\times\mathbf{b}) \ \ldots \text{ (B)}$$

Question 3(ii)

Consider,

$$|\mathbf{b}\times\mathbf{c}|=|\mathbf{b}||\mathbf{c}|\sin\angle BOC \quad \ldots (1)$$
$$|\mathbf{c}\times\mathbf{a}|=|\mathbf{c}||\mathbf{a}|\sin\angle COA \quad \ldots (2)$$
$$|\mathbf{a}\times\mathbf{b}|=|\mathbf{a}||\mathbf{b}|\sin\angle AOB \quad \ldots (3)$$

Take $\dfrac{(2)}{(3)}:\dfrac{|\mathbf{c}\times\mathbf{a}|}{|\mathbf{a}\times\mathbf{b}|}=\dfrac{|\mathbf{c}||\mathbf{a}|\sin\angle COA}{|\mathbf{a}||\mathbf{b}|\sin\angle AOB}$

from (A): $\dfrac{\frac{|\mu|}{|\gamma|}|\mathbf{a}\times\mathbf{b}|}{|\mathbf{a}\times\mathbf{b}|}=\dfrac{\sin\angle COA}{\sin\angle AOB}\Rightarrow\dfrac{|\mu|}{\sin\angle COA}=\dfrac{|\gamma|}{\sin\angle AOB}$

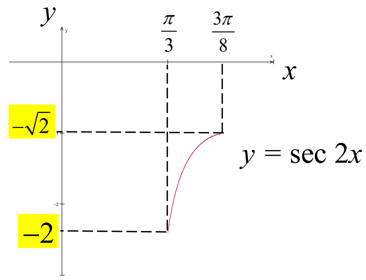
Similarly, taking $\dfrac{(1)}{(3)}:\dfrac{|\mathbf{b}\times\mathbf{c}|}{|\mathbf{a}\times\mathbf{b}|}=\dfrac{|\mathbf{b}||\mathbf{c}|\sin\angle BOC}{|\mathbf{a}||\mathbf{b}|\sin\angle AOB}$

from (B): $\dfrac{\frac{|\lambda|}{|\gamma|}|\mathbf{b}\times\mathbf{a}|}{|\mathbf{a}\times\mathbf{b}|}=\dfrac{\sin\angle BOC}{\sin\angle AOB}\Rightarrow\dfrac{|\lambda|}{\sin\angle BOC}=\dfrac{|\gamma|}{\sin\angle AOB}$

Thus, $\dfrac{|\mu|}{\sin\angle COA}=\dfrac{|\lambda|}{\sin\angle BOC}=\dfrac{|\gamma|}{\sin\angle AOB}$ (proven)

Question 4(a)

Let $y = \ln|\sec 2x| \Rightarrow \pm e^y = \sec 2x$



Since $\frac{\pi}{3} \le x \le \frac{3\pi}{8}$, then sec $2x < 0$

$\Rightarrow \quad \sec 2x = -e^y$

Then $\quad \cos 2x = -e^{-y}$

$$x = \frac{1}{2}\cos^{-1}\left(-e^{-y}\right)$$

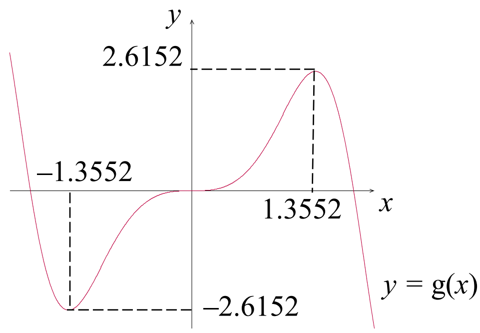
Hence $f^{-1}(x) = \frac{1}{2}\cos^{-1}\left(-e^{-x}\right)$.

Domain of f^{-1} = $[\ln\sqrt{2}, \ln 2]$ or $[0.347, 0.693]$

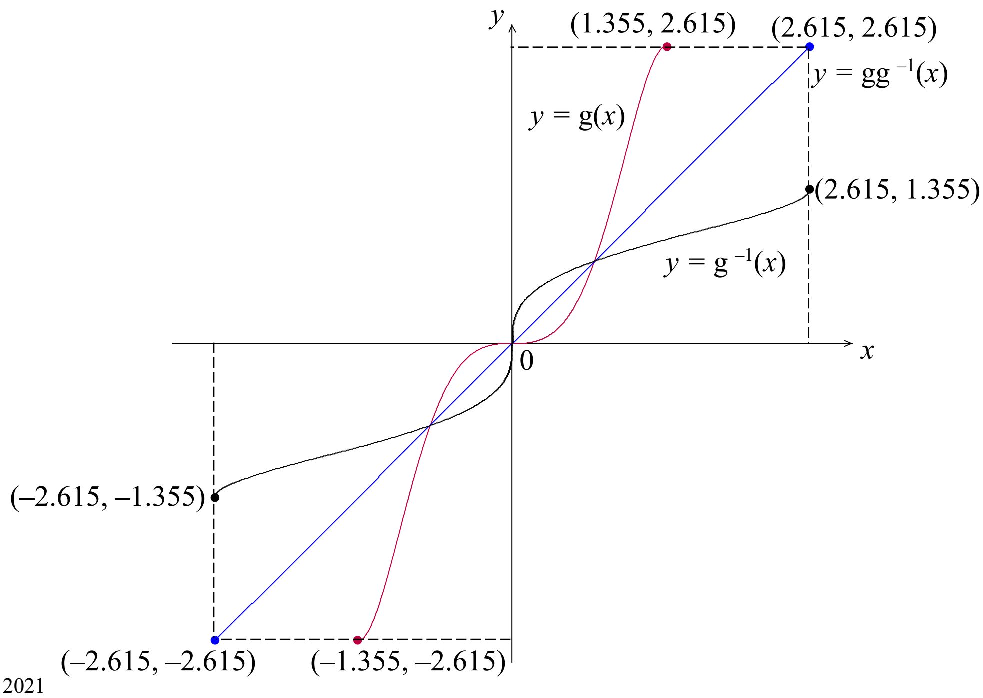
Question 4(b)(i)

From the GC, $k = 1.3552 = 1.355$ (3 dp)

$R_g = [-2.615, 2.615]$



Question 4(b)(ii)



Question 4(b)(iii)

Let $g(x) = x$.

Then $2x\sin(x^2) = x$

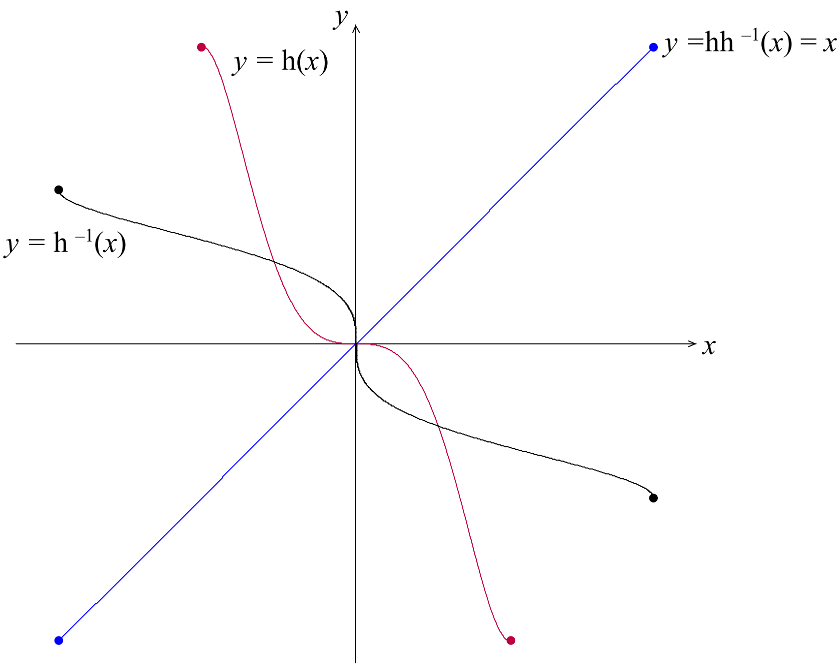
$$x\left[2\sin\left(x^2\right)-1\right]=0$$

$$x = 0 \quad \text{or} \quad \sin\left(x^2\right)=\frac{1}{2}$$

$$x^2 = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

$$x = \pm\sqrt{\frac{\pi}{6}}$$

Question 4(b)(iv)



From the diagram, there are 3 solutions for $h(x) = h^{-1}(x)$.

However, there is only 1 solution for $g(x) = x$.

Hence the method used in part **(iii)** would not yield the complete set of solutions.

<u>Question 5(i)</u>

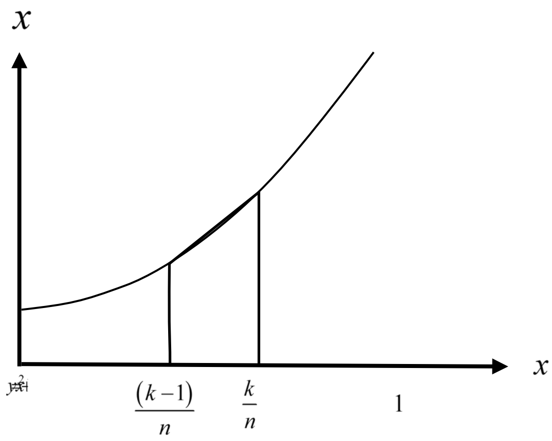
Width = $\dfrac{1}{2}$ units

Area of first trapezium = $\dfrac{1}{2}\times\left(0^2+1+\dfrac{1}{2}^2+1\right)\times\dfrac{1}{2}=\dfrac{9}{16}$ units^2

Area of second trapezium = $\dfrac{1}{2}\times\left(\dfrac{1}{2}^2+1+1^2+1\right)\times\dfrac{1}{2}=\dfrac{13}{16}$ units^2

Total area of trapeziums = $\dfrac{9}{16}+\dfrac{13}{16}=\dfrac{11}{8}$ units^2

<u>Question 5(ii)</u>



Width = $\dfrac{1}{n}$ and x-value of the shorter side of the k^{th} trapezium = $\dfrac{(k-1)}{n}$

Hence the length of the shorter side of the kth trapezium is $\left[\dfrac{(k-1)}{n}\right]^2+1$

Area of k^{th} trapezium $=\dfrac{1}{2}\times\left[\left(\dfrac{k-1}{n}\right)^2+1+\left(\dfrac{k}{n}\right)^2+1\right]\times\dfrac{1}{n}$

$$=\frac{1}{2n^3}\times\left[(k-1)^2+k^2+2n^2\right]$$

$$=\frac{(k-1)^2+k^2}{2n^3}+\frac{1}{n}\text{ units}^2.$$

Question 5(iii)

$$A=\sum_{k=1}^{n}\left[\frac{(k-1)^2+k^2}{2n^3}+\frac{1}{n}\right]$$

$$=\frac{1}{2n^3}\sum_{k=1}^{n}\left[k^2-2k+1+k^2\right]+\frac{1}{n}\sum_{k=1}^{n}1$$

$$=\frac{1}{2n^3}\left[2\sum_{k=1}^{n}k^2-2\sum_{k=1}^{n}k+\sum_{k=1}^{n}1\right]+\frac{n}{n}$$

$$=\frac{1}{2n^3}\left[\frac{n(n+1)(2n+1)}{3}-2n\left(\frac{n+1}{2}\right)+n\right]+1$$

$$=\frac{1}{6n^3}\left[2n^3+3n^2+n-3n^2-3n+3n\right]+1$$

$$=\frac{1}{3}+\frac{1}{6n^2}+1$$

$$=\frac{4}{3}+\frac{1}{6n^2}$$

Question 5(iv)

It is an overestimate.

As the number of trapezium increase to infinity, the total of trapezium will tend to the exact area of region R.

$\therefore$ Area of region $R=\int_0^1\left(x^2+1\right)\mathrm{d}x=\lim_{n\to\infty}\left(\frac{4}{3}+\frac{1}{6n^2}\right)=\frac{4}{3}$

Question 6(i)

$$\left(p\right)\left(0.9\right) + \left(100 - p\right)\left(0.1\right) = 20$$

$(p)(0.9) + (100 - p)(0.1) = 20$

$0.9p + 10 - 0.1p = 20$

$p = 12.5$

The proportion of the residents infected is $\dfrac{12.5}{100} = \dfrac{1}{8}$ or 12.5 %

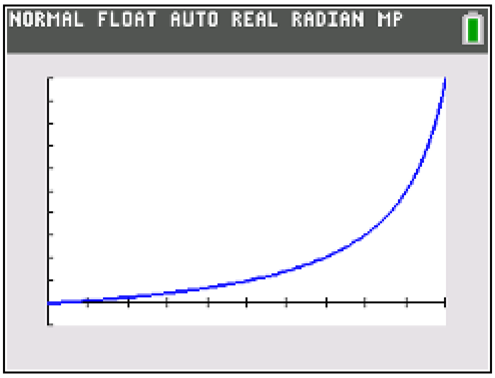
Question 6(ii)

$$\text{P(has disease | tested negative)} = \frac{\text{P(has disease \& tested negative)}}{\text{P(tested negative)}}$$

$$= \frac{0.1q}{0.1q + 0.9(100 - q)}$$

$$= \frac{q}{900 - 8q}$$

Question 6(iii)



As the proportion of people getting infected (q) increases, the probability that a person has the disease given that he has been tested negative also increases as seen in the graph. So, the test is not effective.

Question 7(i)

No of ways required = 4! = 24

Question 7(ii)

No of ways required = ${}^{4}C_1 \times {}^{2}C_1 \times {}^{1}C_1 = 8$

Alternative solution

Correct present	Listing	No of ways
A	A,C,D,B or A,D,B,C	2
B	C,B,D,A or D,B,A,C	2
C	B,D,C,A or D,A,C,B	2
D	B,C,A,D or C,A,B,D	2

Question 7(iii)

No of ways required = ${}^{4}C_2 \times {}^{1}C_1 \times {}^{1}C_1 = 6$

Alternative solution

Correct presents	Listing	No of ways
A, B	A,B,D,C	1
A, C	A,D,C,B	1
A, D	A,C,B,D	1
B, C	D,B,C,A	1
B, D	C,B,A,D	1
C, D	B,A,C,D	1

Question 7(iv)

If 3 persons have received back their own gifts, there will be one remaining gift and one person who have yet to receive a gift. But this last person is the exact person who brought this last gift to the party. If he were to receive the remaining gift, he would have received back his own gift.

Question 7(v)

No of ways required $= 24-(8+6)-1 = 9$

Probability required $= \dfrac{9}{24} = \dfrac{3}{8}$

Question 8(i)

$$\mathrm{P}(\text{different colour})=\mathrm{P}(B,W)=\frac{n}{2n+1}\cdot\frac{n+1}{2n}\cdot 2=\frac{n+1}{2n+1}.$$

Question 8(ii)

$$\mathrm{P}(\text{same colour})=\mathrm{P}(B,B)+\mathrm{P}(W,W)$$
$$=\frac{n}{2n+1}\cdot\frac{n-1}{2n}+\frac{n+1}{2n+1}\cdot\frac{n}{2n}=\frac{n-1}{2(2n+1)}+\frac{n+1}{2(2n+1)}=\frac{n}{2n+1}$$

Question 8(iii)

$$\mathrm{P}(X=1)=\left(\frac{n+1}{2n+1}\right)\left(\frac{1}{2}\right)^2(2)+\left(\frac{n}{2n+1}\right)\left(\frac{1}{2}\right)^4\frac{4!}{3!}=\frac{3n+2}{4(2n+1)}$$

Alternatively,

$$\mathrm{P}(X=1)=\mathrm{P}(B,W,H,T)+\mathrm{P}(B,B,H,T,T,T)+\mathrm{P}(W,W,H,T,T,T)$$
$$=\left[\frac{n+1}{2n+1}\right]\left(\frac{1}{2}\right)^2(2)+\left[\frac{n-1}{2(2n+1)}\right]\left(\frac{1}{2}\right)^4\frac{4!}{3!}+\left[\frac{n+1}{2(2n+1)}\right]\left(\frac{1}{2}\right)^4\frac{4!}{3!}=\frac{3n+2}{4(2n+1)}$$

Question 8(iv)

If there are 3 blue cards $\Rightarrow n=3$

$$\mathrm{P}(B,W)=\frac{n+1}{2n+1}=\frac{4}{7};\ \mathrm{P}(B,B)=\frac{n-1}{2(2n+1)}=\frac{1}{7};\mathrm{P}(W,W)=\frac{n+1}{2(2n+1)}=\frac{2}{7}$$
$$\mathrm{P}(X=1)=\frac{3n+2}{4(2n+1)}=\frac{11}{28}$$
$$\mathrm{P}(X=4)=\mathrm{P}(B,B,H,H,H,H)+\mathrm{P}(W,W,H,H,H,H)$$
$$=\frac{1}{7}\left(\frac{1}{2}\right)^4+\frac{2}{7}\left(\frac{1}{2}\right)^4=\frac{3}{112}$$
$$\mathrm{P}(X=3)=\mathrm{P}(B,B,H,H,H,T)+\mathrm{P}(W,W,H,H,H,T)$$
$$=\frac{1}{7}\left(\frac{1}{2}\right)^4\left(\frac{4!}{3!}\right)+\frac{2}{7}\left(\frac{1}{2}\right)^4\left(\frac{4!}{3!}\right)=\frac{3}{28}$$

$$\mathrm{P}(X=0)=\mathrm{P}(B,W,T,T)+\mathrm{P}(B,B,T,T,T,T)+\mathrm{P}(W,W,T,T,T,T)$$
$$=\frac{4}{7}\left(\frac{1}{2}\right)^2+\frac{1}{7}\left(\frac{1}{2}\right)^4+\frac{2}{7}\left(\frac{1}{2}\right)^4=\frac{19}{112}$$

$$\mathrm{P}(X=2)=1-\mathrm{P}(X=0)-\mathrm{P}(X=1)-\mathrm{P}(X=3)-\mathrm{P}(X=4)$$
$$=1-\left(\frac{19}{112}+\frac{11}{28}+\frac{3}{28}+\frac{3}{112}\right)$$
$$=1-\frac{39}{56}=\frac{17}{56}$$

Alternatively,

x	0	1	2	3	4
$\mathrm{P}(X=x)$	$\frac{4}{7}\left(\frac{1}{2}\right)^2+\frac{3}{7}\left(\frac{1}{2}\right)^4$ $=\frac{19}{112}$	$\frac{3(3)+2}{4(2(3)+1)}$ $=\frac{11}{28}$	$\frac{4}{7}\left(\frac{1}{2}\right)^2+{}^4C_2\frac{3}{7}\left(\frac{1}{2}\right)^4$ $=\frac{17}{56}$	${}^4C_1\frac{3}{7}\left(\frac{1}{2}\right)^4$ $=\frac{3}{28}$	$\frac{3}{7}\left(\frac{1}{2}\right)^4$ $=\frac{3}{112}$

Question 8(v)

When $n=3$, $\mathrm{P}(X>2\mid X\le 3)=\dfrac{\mathrm{P}(2<X\le 3)}{\mathrm{P}(X\le 3)}$

$$=\frac{\mathrm{P}(X=3)}{1-\mathrm{P}(X=4)}$$
$$=\frac{\frac{3}{28}}{1-\frac{3}{112}}$$
$$=\frac{12}{109}$$

Question 9(i)

Let X denote the random variable representing the amount of nocturnal sleep (hours) for a randomly selected student on a school-day night. Then, $X \sim \mathrm{N}(6.5, \sigma^2)$.

$\mathrm{P}(X < 8) = 0.85 \quad \Rightarrow \quad \mathrm{P}\left(Z < \dfrac{8-6.5}{\sigma}\right) = 0.85$

From the GC, $\dfrac{1.5}{\sigma} = 1.03643338$

$\therefore\ \sigma = 1.4473 = 1.45$ (3 sf)

Hence, the required standard deviation is 1.45 hrs.

Question 9(ii)

Probability required $= 3\mathrm{P}(X_1 < 5.5) \times \mathrm{P}(X_2 > 7) \times \mathrm{P}(X_3 > 7)$

$= 3(0.24480)(0.36487)^2$

$= 0.097771$

$= 0.0978$ (3 sf)

Question 9(iii)

Let Y denote the random variable representing the amount of sleep (in minutes) from daily afternoon naps on a school-day for a student. Then $Y \sim \mathrm{N}(65, a^2)$.

When $a = 60$, $\mathrm{P}(Y < 0) = 0.139$. Since 0.139 is not negligible, the amount of sleep from daily afternoon naps for a student on a school-day may not be appropriately modelled by a normal distribution.

Question 9(iv)

Let T and W denote the random variables representing the total amount of sleep (in hours) on a school-day for a student and the amount of sleep from daily afternoon naps on a school-day for a student. $\therefore\ T = X + W$ where $X \sim \mathrm{N}(6.5, 1.4473^2)$ and $W \sim \mathrm{N}\left(\dfrac{65}{60}, \left(\dfrac{10}{60}\right)^2\right)$

$$\mathrm{E}(T) = \mathrm{E}(X+W) = \mathrm{E}(X) + \mathrm{E}(W) = 6.5 + \frac{65}{60} = \frac{91}{12} = 7.5833$$

$$\mathrm{Var}(T) = \mathrm{Var}(X+W) = \mathrm{Var}(X) + \mathrm{Var}(W) = 1.44727^2 + \left(\frac{1}{6}\right)^2 = 2.1224$$

$\therefore\ T \sim \mathrm{N}(7.5833, 2.1224)$

$\Rightarrow \mathrm{P}(T > 8) = 0.38743 = 0.387$ (3 sf)

Question 9(iv)

Probability required = $P(T_1 + T_2 - 2T_3 > 1)$

$E(T_1 + T_2 - 2T_3) = E(T_1) + E(T_2) - 2E(T_3) = 0$

$\mathrm{Var}(T_1 + T_2 - 2T_3) = \mathrm{Var}(T_1) + \mathrm{Var}(T_2) + 4\mathrm{Var}(T_3) = 2.1224 + 2.1224 + 4(2.1224) = 12.735$

$\therefore\ T_1 + T_2 - 2T_3 \sim N(0,\ 12.735)$

$P(T_1 + T_2 - 2T_3 > 1) = 0.38965 = 0.390$ (3 sf)

Question 9(v)

Assumptions: The amount of nocturnal sleep on a school-day night (X) is independent of the amount of sleep from afternoon naps (W) on the same day.

The assumption may be unrealistic because a student who has slept longer in the afternoon may require less nocturnal sleep. Hence X and W may not be independent.

Question 10(i)

The event that a Type A component is faulty is independent of the event of any other Type A component that is faulty.

The probability that a Type A component is faulty is constant at 0.02.

Question 10(ii)

Let X be the random variable denoting the number of faulty Type A components

in a box of 50.

$$X \sim \mathrm{B}(50, 0.02)$$

$$\begin{aligned}\mathrm{P}(X>1) &= 1-\mathrm{P}(X\le 1)\\ &= 0.26423 \text{ (to 5 s.f.)}\\ &= 0.264 \text{ (to 3 s.f.)}\end{aligned}$$

Question 10(iii)

$$\text{Required probability} = (0.26423)^2(1-0.26423)^2(0.26423)\times\frac{4!}{2!2!}$$

$$= 0.0599 \text{ (to 3 s.f.)}$$

OR

Let Q be the random variable denoting the number of boxes with more than 1 faulty component out of 4.

$$Q \sim \mathrm{B}(4, 0.26423)$$

$$\mathrm{P}(Q=2)\times(0.26423) = 0.0599$$

Question 10(iv)

Let Y be the random variable denoting the number of boxes with more than 1 faulty component out of 5.

$$Y \sim \mathrm{B}(5, 0.26423)$$

$$\mathrm{P}(Y=3) = 0.0999$$

Question 10(v)

Event in part (iii) is a subset of event in part (iv).

Question 10(vi)

Let W be the random variable denoting the number of faulty Type B components in a box of 20.

$$W \sim \mathrm{B}(20,\ 0.001)$$

Since sample size is large for both types of components, by Central Limit Theorem,

$W_1 + W_2 + \ldots + W_{40} \sim \mathrm{N}\left(0.8,\ 0.7992\right)$ approx

$X_1 + X_2 + \ldots + X_{30} \sim \mathrm{N}(30, 29.4)$ approx.

Let $T = X_1 + X_2 + \ldots + X_{30} + W_1 + W_2 + \ldots + W_{40}$

$T \sim \mathrm{N}\left(30.8, 30.1992\right)$ approximately

$\mathrm{P}\left(T \le 15\right) = 0.00202$ (to 3 s.f.)

Question 11(i)

Since Brandon suspects that the average flight time from New Orleans to Miami is shorter than 115 minutes, he should carry out a 1-tail test.

Let X be the random variable denoting the flight time from New Orleans to Miami in minutes and μ be the mean flight time.

$H_0 : \mu = 115$

$H_1 : \mu < 115$

Question 11(ii)

Using one-tailed test at 10% significance level.

Under H_0, $\overline{X} \sim \mathrm{N}\left(115, \frac{11.3}{8}\right)$.

Sample readings : $\overline{x} = 113.175$

Using Z-test, p- value = 0.062322.

Since the p-value < 0.1 (the significance level), we reject H_0 and conclude that there is sufficient evidence at the 10% level of significance to conclude that the mean flight time from New Orleans to Miami is less than 115 minutes.

Question 11(iii)

10% level of significance refers to a probability of 0.1 that we conclude that the average flight time from New Orleans to Miami is less than 115 minutes when in fact it is 115 minutes.

Question 11(iv)

p-value = $2 \times 0.062322 = 0.124644$

Since the p-value > 0.1 (the significance level), we do not reject H_0 and conclude that there is insufficient evidence at the 10% level of significance to conclude that the mean flight time from New Orleans to Miami is not 115 minutes.

Question 11(v)

Let Y be the random variable denoting the flight time from Portland to Los Angeles in minutes and μ be the mean time.

$$H_0 : \mu = k$$

$$H_1 : \mu > k$$

Using one-tailed test at 4% significance level.

Under H_0, since $n = 60$ is large, by central limit theorem,

$$\overline{Y} \sim \mathrm{N}\left(k, \frac{\frac{60}{59}(13.6)^2}{60}\right) \text{ approximately.}$$

$$\Rightarrow \frac{\overline{Y} - k}{\frac{13.6}{\sqrt{59}}} = Z \sim \mathrm{N}(0,1)$$

Since $\overline{y} = 181$, test statistics $= \dfrac{181 - k}{\frac{13.6}{\sqrt{59}}}$

Critical value = 1.7507 (by InvNorm)

Since H_0 is not rejected,

$$\frac{181 - k}{\frac{13.6}{\sqrt{59}}} < 1.7507$$

$\Rightarrow k > 177.90$

$\Rightarrow k > 178$ (3 s.f.) or $k \geq 178$