2021 ACJC H2 Math Promo Paper Solution

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>1</td><td>

$$y=2\sin(2x+\alpha)\cos(x)\xrightarrow{\text{replace } x \text{ by } x+\alpha} y=2\sin(2x+3\alpha)\cos(x+\alpha)$$
$$\xrightarrow{\text{replace } x \text{ by } 2x} y=2\sin(4x+3\alpha)\cos(2x+\alpha)$$
$$\xrightarrow{\text{replace } y \text{ by } -y} y=-2\sin(4x+3\alpha)\cos(2x+\alpha)$$

1) Translation of the graph by α units in the negative x-direction, followed by
2) Scaling parallel to x – axis by a factor of ½ .
3) Reflection in the x – axis.
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<tr><td>OR</td><td>

$$y=2\sin(2x+\alpha)\cos(x)\xrightarrow{\text{replace } x \text{ by } 2x} y=2\sin(4x+\alpha)\cos(2x)$$
$$\xrightarrow{\text{replace } x \text{ by } x+\frac{\alpha}{2}} y=2\sin\left(4\left(x+\tfrac{\alpha}{2}\right)+\alpha\right)\cos\left(2\left(x+\tfrac{\alpha}{2}\right)\right)$$
$$=2\sin(4x+3\alpha)\cos(2x+\alpha)$$
$$\xrightarrow{\text{replace } y \text{ by } -y} y=-2\sin(4x+3\alpha)\cos(2x+\alpha)$$

1) Scaling parallel to x – axis by a factor of ½ , followed by
2) Translation of the graph by $\frac{\alpha}{2}$ units in the negative x – axis direction.
3) Reflection in the x – axis.
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<tr><td>2</td><td>

$$\frac{x+3}{x^2+x-2}>-1$$
$$\frac{x+3}{x^2+x-2}+1>0$$
$$\frac{x^2+2x+1}{x^2+x-2}>0$$
$$\frac{(x+1)^2}{(x+2)(x-1)}>0$$

Since $(x+1)^2\geq 0$ for $x\in\mathbb{R}$, $(x+2)(x-1)>0.$

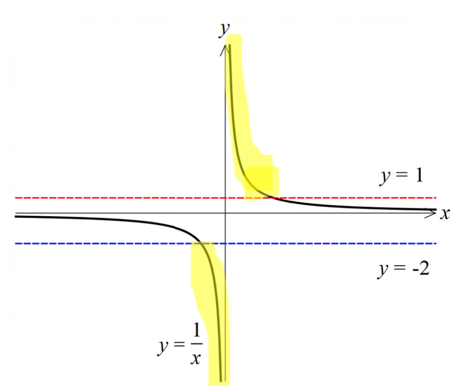
$\therefore x<-2$ or $x>1.$

$$\frac{x+3x^2}{1+x-2x^2}>-1$$
$$\frac{\frac{x+3x^2}{x^2}}{\frac{1+x-2x^2}{x^2}}>-1$$
$$\frac{\frac{1}{x}+3}{\left(\frac{1}{x}\right)^2+\left(\frac{1}{x}\right)-2}>-1$$

Replace x by $\frac{1}{x}$,

$\frac{1}{x}<-2$ or $\frac{1}{x}>1$

$\therefore -\frac{1}{2}<x<0$ or $0<x<1$



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3

$$2x^2 - 8y^2 = x^2 + xy^2 + 100$$
$$x^2 - 8y^2 = xy^2 + 100$$
$$2x - 16y\frac{\mathrm{d}y}{\mathrm{d}x} = 2xy\frac{\mathrm{d}y}{\mathrm{d}x} + y^2$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x - y^2}{2xy + 16y} \qquad \text{(shown)}$$

Suppose $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x - y^2}{2xy + 16y} = 0$

$$2x = y^2$$
$$\therefore \quad \frac{x^2 - 8x}{x^2 + 2x^2 + 100} = \frac{1}{2}$$
$$2x^2 - 16x = 3x^2 + 100$$
$$x^2 + 16x + 100 = 0$$
$$(x+8)^2 + 36 = 0$$

No solution since $(x+8)^2 + 36 > 0$ for $x \in \mathbb{R}$.

[Or using discriminant: Since $16^2 - 4(1)(100) = -144 < 0$, there's no real roots.]

$\therefore$ There is no stationary point since $\frac{\mathrm{d}y}{\mathrm{d}x} \neq 0$ for $x \in \mathbb{R}$.

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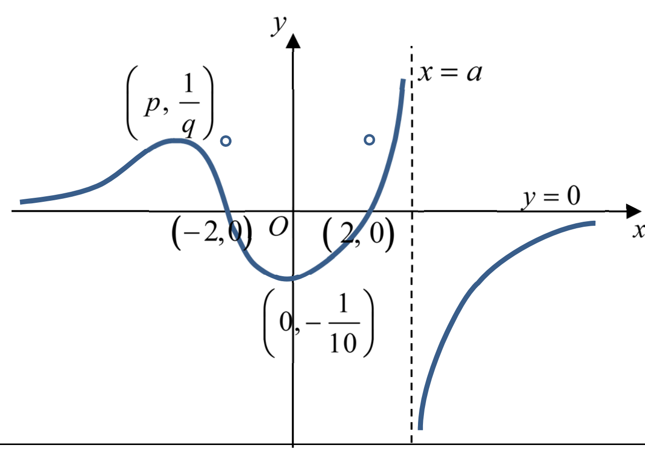
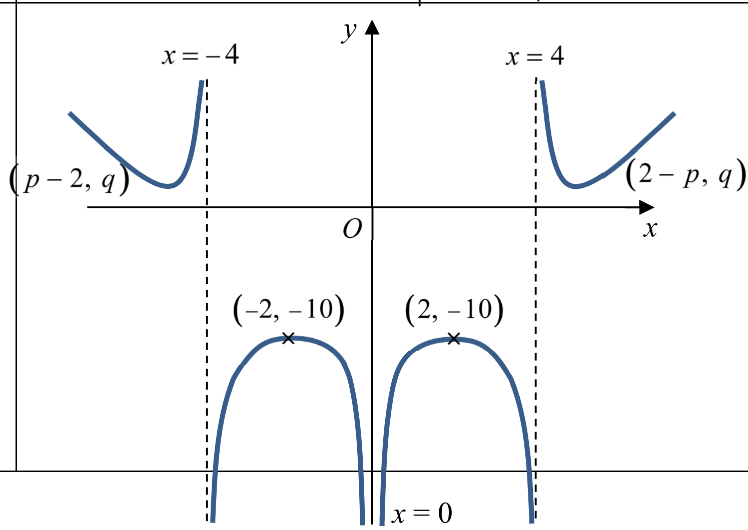
4 (i)



(ii)



$y = \mathrm{f}(x)$

$\downarrow$ replace x by $x + 2$

$y = \mathrm{f}(x + 2)$

$\downarrow$ replace x by $-x$

$y = \mathrm{f}(2 - x)$

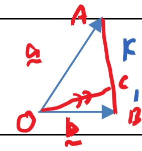
$\downarrow$ replace x by $|x|$

$y = \mathrm{f}(2 - |x|)$

5(i)

Using Ratio Theorem

$$\overrightarrow{OC} = \frac{1}{k+1}(\mathbf{a} + k\mathbf{b})$$



(ii)

Length of projection of $\overrightarrow{OC}$ onto $\overrightarrow{OA}$

$$= \frac{\left|\overrightarrow{OC} \bullet \mathbf{a}\right|}{|\mathbf{a}|} = \frac{\left|\frac{1}{k+1}(\mathbf{a}+k\mathbf{b}) \bullet \mathbf{a}\right|}{2}$$

$$= \frac{1}{2(k+1)}\left|(\mathbf{a}+k\mathbf{b}) \bullet \mathbf{a}\right| \quad \text{since } 0 < k < 1$$

$$= \frac{1}{2(k+1)}\left|(\mathbf{a} \bullet \mathbf{a} + k\mathbf{b} \bullet \mathbf{a})\right|$$

$$= \frac{1}{2(k+1)}\left||\mathbf{a}|^2 + k\mathbf{b} \bullet \mathbf{a}\right|$$

$$= \frac{1}{2(k+1)}\left|4 + k|\mathbf{a}||\mathbf{b}|\cos\left(60^o\right)\right|$$

$$= \frac{1}{2(k+1)}\left|4 + k(2)(1)\left(\frac{1}{2}\right)\right| = \frac{4+k}{2(k+1)} \ (proved)$$

(iii)

Area of triangle OAC

$$= \frac{1}{2}|\mathbf{a} \times \mathbf{c}| = \frac{1}{2}\left|\mathbf{a} \times \frac{1}{k+1}(\mathbf{a} + k\mathbf{b})\right|$$

$$= \frac{1}{2(k+1)}\left|(\mathbf{a} \times \mathbf{a}) + k(\mathbf{a} \times \mathbf{b})\right| \quad \text{since } 0 < k < 1$$

$$= \frac{1}{2(k+1)}\left|(k(\mathbf{a} \times \mathbf{b})\right| \quad \text{since } \mathbf{a} \times \mathbf{a} = \mathbf{0}$$

$$= \frac{k}{2(k+1)}\left||\mathbf{a}||\mathbf{b}|\sin\left(60^o\right)\right| = \frac{k}{2(k+1)}\left|2(1)\frac{\sqrt{3}}{2}\right| = \frac{\sqrt{3}k}{2(k+1)}$$

6 (i)

$$L_1: \frac{x-2}{a} = \frac{y+2}{b} = \frac{z-3}{c}$$

$$\mathbf{r} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad \text{--------(1)}$$

L_1 is perpendicular to L_2,

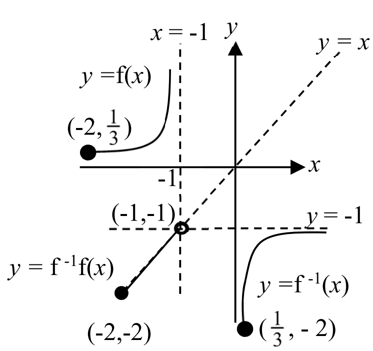
$$\begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$$

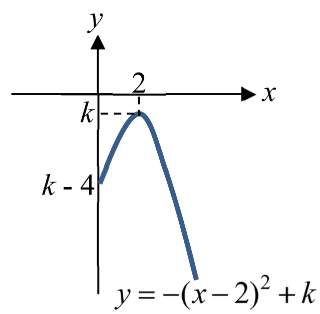
$$4a + 3b = 0$$

(ii)

Equation of line L_3 : $\mathbf{r} = \mu \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ ---------(2)

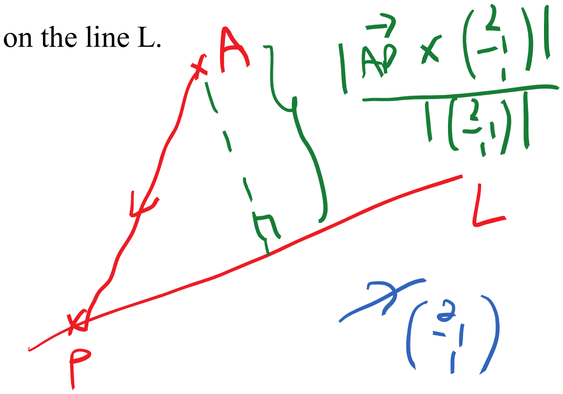
Since L_1 intersects L_3, sub (1) into (2):

	$2+\lambda a=0 \quad \Rightarrow \lambda=-\dfrac{2}{a}$ ------(3) $-2+\lambda b=\mu$ ------(4) $3+\lambda c=\mu$ ------(5) Sub (4) into (5): $-2+\lambda b=3+\lambda c$ --------(6) Sub (3) into (6): $-2+\left(\dfrac{-2}{a}\right)b=3+\left(\dfrac{-2}{a}\right)c$ $-2a-2b=3a-2c$ $5a+2b-2c=0$ (Shown)
(iii)	Using results in (i) & (ii), use GC to solve: $4a+3b+0c=0$ $5a+2b-2c=0$ $a=\dfrac{6}{7}c$ $b=-\dfrac{8}{7}c$
(iv)	Using result in (iii), $L_1:\quad \mathbf{r}=\begin{pmatrix}2\\-2\\3\end{pmatrix}+\lambda\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}2\\-2\\3\end{pmatrix}+\lambda\begin{pmatrix}\frac{6}{7}c\\-\frac{8}{7}c\\c\end{pmatrix}$ $L_1:\quad \mathbf{r}=\begin{pmatrix}2\\-2\\3\end{pmatrix}+\lambda\left(\dfrac{1}{7}\right)\begin{pmatrix}6c\\-8c\\7c\end{pmatrix}=\begin{pmatrix}2\\-2\\3\end{pmatrix}+\mu\begin{pmatrix}6\\-8\\7\end{pmatrix}$ Angle between L_1 and L_3: $\cos\theta=\left\|\dfrac{\begin{pmatrix}0\\1\\1\end{pmatrix}\bullet\begin{pmatrix}6\\-8\\7\end{pmatrix}}{\sqrt{2}\sqrt{6^2+8^2+7^2}}\right\|=\left\|\dfrac{-1}{\sqrt{2}\sqrt{149}}\right\|$ $\theta=86.7^0$ Acute angle between the two planes is 86.7^0
7 (i)	 Note that $D_{f^{-1}f}=D_f=[-2,-1)$

(ii)	Considering the interval $-2 \le x < -1$, $\dfrac{1}{\lvert 1-x^2\rvert} = -\dfrac{1}{1-x^2}$ $y = -\dfrac{1}{1-x^2}$ $y = \dfrac{1}{x^2-1}$ $yx^2 - y = 1$ $x^2 = \dfrac{1+y}{y}$ $x = \pm\sqrt{\dfrac{1+y}{y}}$ $x = -\sqrt{\dfrac{1+y}{y}}$ (since $-2 \le x < -1$) $\mathrm{f}^{-1}(x) = -\sqrt{\dfrac{1+x}{x}} = -\sqrt{\dfrac{1}{x}+1}$ $\mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_{\mathrm{f}} = [\frac{1}{3}, \infty)$
(iii)	Since $\mathrm{R_f} = [\frac{1}{3}, \infty) \subseteq [0, \infty) = \mathrm{D_g}$ Hence gf exists. $[-2,-1) \xrightarrow{f} [\frac{1}{3}, \infty) \xrightarrow{g} (-\infty, k]$ $\mathrm{R_{gf}} = (-\infty, k]$ 
8 **(i) (a)**	At E, $y = a(1-\cos\theta) = 0$. Hence $\cos\theta = 1$ $\therefore \theta = 2\pi$ $\therefore x = 2a\pi$ Hence $OE = 2a\pi$
(b)	When y is a maximum, $\cos\theta = -1$ OR $\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = a(\sin\theta) = 0$ $\therefore \theta = \pi$ and $y = 2a$
(ii)	$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = a(\sin\theta) = 2a\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$ and $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = a(1-\cos\theta) = a(1-1+2\sin^2\theta) = 2a\sin^2\dfrac{\theta}{2}$

	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}}=\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}=\cot\frac{\theta}{2}$
(iii)	At B, $\frac{\mathrm{d}y}{\mathrm{d}x}=\cot\frac{\beta}{2}=\frac{1}{\tan\frac{\beta}{2}}=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$ Hence $\tan\frac{\beta}{2}=\sqrt{3}$. $\frac{\beta}{2}=\frac{\pi}{3}$ $\beta=\frac{2\pi}{3}(shown)$
(iv)	Since $\frac{\mathrm{d}y}{\mathrm{d}x}=\cot\frac{\theta}{2}$ Gradient of normal at point B is $-\tan\frac{\pi}{3}=-\sqrt{3}$. Equation of normal : $y-\frac{3}{2}a=-\sqrt{3}\left(x-\left[a\left(\frac{2\pi}{3}-\sin\frac{2\pi}{3}\right)\right]\right)$ $y-\frac{3}{2}a=-\sqrt{3}\left(x-\left[a\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)\right]\right)$ $y=-\sqrt{3}\left(x-\frac{2\pi a}{3}+\frac{a\sqrt{3}}{2}\right)+\frac{3}{2}a$ $y=-\sqrt{3}x+\frac{2\pi a}{\sqrt{3}}$ $\sqrt{3}y=-3x+2\pi a$ $\sqrt{3}y=-\left(\sqrt{3}\right)^{2}x+2\pi a$
9(a)	$\sum_{r=7}^{n+1}\left(2^{r}+r^{2}-r\right)$ $=\sum_{r=7}^{n+1}\left(2^{r}\right)+\sum_{r=7}^{n+1}\left(r^{2}\right)-\sum_{r=7}^{n+1}(r)$ $=\frac{2^{7}\left(2^{n-5}-1\right)}{2-1}+\sum_{r=1}^{n+1}\left(r^{2}\right)-\sum_{r=1}^{6}\left(r^{2}\right)-\left(\frac{n-5}{2}\right)(7+n+1)$ $=2^{7}\left(2^{n-5}-1\right)+\frac{(n+1)}{6}(n+2)(2n+3)-\left(\frac{6}{6}\right)(7)(13)-\left(\frac{n-5}{2}\right)(8+n)$ $=2^{7}\left(2^{n-5}-1\right)+\frac{(n+1)}{6}(n+2)(2n+3)-91-\frac{(n-5)(8+n)}{2}$ Alternative Method:

	$\sum_{r=7}^{n+1}\left(2^r+r^2-r\right)$ $=\sum_{r=1}^{n+1}\left(2^r+r^2-r\right)-\sum_{r=1}^{6}\left(2^r+r^2-r\right)$ $=\frac{2\left(2^{n+1}-1\right)}{2-1}+\frac{1}{6}(n+1)(n+2)(2n+3)-\frac{n+1}{2}(1+n+1)$ $-\frac{2\left(2^6-1\right)}{2-1}-91+\frac{6}{2}(1+6)$ $=2^{n+2}+\frac{1}{6}(n+1)(n+2)(2n+3)-\frac{(n+1)(n+2)}{2}-198$ $=2^{n+2}+\left[\frac{(n+1)(n+2)}{6}\right](2n+3-3)-198$ $=2^{n+2}+\frac{1}{3}n(n+1)(n+2)-198$
9(b)(i)	Using partial fractions, $\frac{1}{r^2-1}=\frac{1}{2}\left(\frac{1}{r-1}-\frac{1}{r+1}\right)$ $\sum_{r=2}^{n}\frac{1}{r^2-1}=\frac{1}{2}\sum_{r=2}^{n}\left(\frac{1}{r-1}-\frac{1}{r+1}\right)$ $=\frac{1}{2}\left[\begin{array}{l}\frac{1}{1}-\cancel{\frac{1}{3}}\\+\frac{1}{2}-\cancel{\frac{1}{4}}\\+\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}\\+\cancel{\frac{1}{4}}-\cancel{\frac{1}{6}}\\\vdots\\+\cancel{\frac{1}{n-3}}-\cancel{\frac{1}{n-1}}\\+\cancel{\frac{1}{n-2}}-\frac{1}{n}\\+\cancel{\frac{1}{n-1}}-\frac{1}{n+1}\end{array}\right]$ $=\frac{1}{2}\left(\frac{3}{2}-\frac{1}{n}-\frac{1}{n+1}\right)$ $=\frac{3}{4}+\frac{-\frac{1}{2}}{n}+\frac{-\frac{1}{2}}{n+1}$ $\therefore A=-\frac{1}{2}.$

(ii)	As $n \to \infty, \frac{1}{n} \to 0, \frac{1}{n+1} \to 0$, therefore $\sum_{r=2}^{\infty} \frac{1}{r^2-1}$ converges. $\sum_{r=2}^{\infty} \frac{1}{r^2-1} = \frac{3}{4}$.
(iii)	$\frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \ldots = \sum_{r=2}^{\infty} \frac{2}{r^2}$ Since $r^2 - 1 < r^2$, $\therefore \frac{2}{r^2-1} > \frac{2}{r^2}$. $\sum_{r=2}^{\infty} \frac{2}{r^2-1} > \sum_{r=2}^{\infty} \frac{2}{r^2}$ $\sum_{r=2}^{\infty} \frac{2}{r^2} < 2\left(\frac{3}{4}\right)$ $\therefore \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \ldots < \frac{3}{2}$ (shown)
10(i)	$\overrightarrow{OA} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} \quad \overrightarrow{OB} = \begin{pmatrix} \alpha \\ -1 \\ 2 \end{pmatrix} \quad \overrightarrow{OC} = \begin{pmatrix} -1 \\ -7 \\ \beta \end{pmatrix}$ Since A, B and C are collinear, $\overrightarrow{AB} = k\overrightarrow{AC}$ $\begin{pmatrix} \alpha - 4 \\ 1 \\ 2 \end{pmatrix} = k \begin{pmatrix} -5 \\ -5 \\ \beta \end{pmatrix} \Rightarrow \begin{cases} \alpha - 4 = -5k \\ 1 = -5k \\ 2 = k\beta \end{cases} \Rightarrow \begin{cases} k = -\frac{1}{5} \\ \alpha = 5 \\ \beta = -10 \end{cases}$
(ii)	$\overrightarrow{OP} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$ is position vector of a point on the line L. $\overrightarrow{AP} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \\ 0 \end{pmatrix}$ Distance from A to L $= \left\| \overrightarrow{AP} \times \frac{1}{\sqrt{4+1+1}} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \right\|$ $= \frac{1}{\sqrt{6}} \left\| \begin{pmatrix} -2 \\ 5 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \right\|$ $= \frac{1}{\sqrt{6}} \left\| \begin{pmatrix} 5 \\ 2 \\ -8 \end{pmatrix} \right\| = \frac{1}{\sqrt{6}} \sqrt{25+4+64} = \sqrt{\frac{93}{6}} = \sqrt{\frac{31}{2}}$ 

(iii)	Normal of plane $\pi = \overrightarrow{AB}\times\begin{pmatrix}2\\-1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\2\end{pmatrix}\times\begin{pmatrix}2\\-1\\1\end{pmatrix} = \begin{pmatrix}3\\3\\-3\end{pmatrix} = 3\begin{pmatrix}1\\1\\-1\end{pmatrix}$ Equation of plane π: $\mathbf{r}\cdot\begin{pmatrix}1\\1\\-1\end{pmatrix} = \begin{pmatrix}2\\3\\0\end{pmatrix}\bullet\begin{pmatrix}1\\1\\-1\end{pmatrix} = 2+3=5$ Cartesian equation is $x+y-z=5$
(iv)	Let F be the foot of perpendicular from $A(4,-2,0)$ to the plane π Equation of line AF : $\mathbf{r} = \begin{pmatrix}4\\-2\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\-1\end{pmatrix} = \begin{pmatrix}4+\lambda\\-2+\lambda\\-\lambda\end{pmatrix}$ To find the point of intersection of line AF and plane π, substitute equation of line into equation of plane $x+y-z=5$, $4+\lambda-2+\lambda+\lambda=5 \quad\Rightarrow\quad 3\lambda+2=5 \Rightarrow \lambda=1$ $\therefore \overrightarrow{OF} = \begin{pmatrix}4\\-2\\0\end{pmatrix} + 1\begin{pmatrix}1\\1\\-1\end{pmatrix} = \begin{pmatrix}5\\-1\\-1\end{pmatrix}$
	Let the reflection of A about plane π be A’*(x,y,z)* $\overrightarrow{AF} = \overrightarrow{FA'}$ $\overrightarrow{OA'} = 2\overrightarrow{OF} - \overrightarrow{OA}$ $\overrightarrow{OA'} = 2\begin{pmatrix}5\\-1\\-1\end{pmatrix} - \begin{pmatrix}4\\-2\\0\end{pmatrix} = \begin{pmatrix}6\\0\\-2\end{pmatrix}$ Alternatively: $\dfrac{4+x}{2} = 5 \Rightarrow x = 6$ $\dfrac{-2+y}{2} = -1 \Rightarrow y = 0$ $\dfrac{0+z}{2} = -1 \Rightarrow z = -2$ Since the line AB is parallel to π, then the reflected line about π will also be parallel to π, i.e. also parallel to the line AB. Equation of reflected line is: $\mathbf{r} = \begin{pmatrix}6\\0\\-2\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\2\end{pmatrix}, \lambda\in\mathbb{R}$
11(i) (a)	$A = \frac{1}{2}(10\sin\theta)(10+10+2(10\cos\theta)) = \frac{1}{2}(10\sin\theta)(20+20\cos\theta) = (100\sin\theta)(1+\cos\theta)$ $V = (50)(100\sin\theta)(1+\cos\theta) = (5000\sin\theta)(1+\cos\theta)$

(b)	$\frac{dV}{d\theta}=(5000\cos\theta)(1+\cos\theta)+(5000\sin\theta)(-\sin\theta)$ $=5000(\cos\theta+\cos^2\theta-\sin^2\theta)$ $=5000(2\cos^2\theta+\cos\theta-1)$ $\frac{\mathrm{d}V}{\mathrm{d}\theta}=0$ $(2\cos^2\theta+\cos\theta-1)=0$ $(2\cos\theta-1)(\cos\theta+1)=0$ Since θ is acute $\cos\theta\neq-1$ $\therefore\theta=\frac{\pi}{3}$ $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}=5000(-4\cos\theta\sin\theta-\sin\theta)=5000(-\frac{3\sqrt{3}}{2})\approx-12990<0$ when $\theta=\frac{\pi}{3}$ V is a maximum when $\theta=\frac{\pi}{3}$ Max $V=$ $(5000\frac{\sqrt{3}}{2})(1+\frac{1}{2})=$ $\frac{15000\sqrt{3}}{4}$ Maximum volume is $\frac{15000\sqrt{3}}{4}$ cm^3 = $3750\sqrt{3}$ cm^3.
(ii) (a) **(b)**	Volume of water = $V=[\frac{1}{2}h(20+2h\tan\frac{\pi}{4})]50$ $V=[h(10+h)]50=500h+50h^2$ $\frac{dV}{dh}=500+100h$. When $h=3$ cm , $\frac{dV}{dh}=800$ $\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}=\frac{100}{800}$ cm s^{-1} = $\frac{1}{8}$ cm s^{-1} = 0.125 cm s^{-1} When the depth of the water is h cm, area of water surface = $y=(10+2h\tan\frac{\pi}{4})(50)=$ $500+100h$ $\frac{dy}{dt}=100\frac{dh}{dt}=$ $\frac{100}{8}$ cm^2 s^{-1} = 12.5 cm^{2}s^{-1}
12(i)	$U_n>6550$ $3000+(n-1)100>6550$ $n>36.5$ $\therefore 37^{\text{th}}$ month
12(ii)	$S_n=\frac{n}{2}\left[6000+(n-1)100\right]$ $\frac{n}{2}\left[6000+(n-1)100\right]\geq 200{,}000$ <u>Method 1</u> By GC,

When $n = 40, y = 198,000 < 200,000$

When $n = 41, y = 205,000 > 200,000$

<u>Method 2</u>

$$\frac{n}{2}\left[6000+(n-1)100\right] \geq 200,000$$

$$100n^2 + 5900n - 400,000 \geq 0$$

$$(n-40.287)(n+99.287) \geq 0$$

$n \leq -99.3$ (rej) or $n \geq 40.3$

$\therefore$ 41 months

$$S_{40} = \frac{40}{2}\left[6000+(40-1)100\right] = \$198,000$$

$\$200,000 - \$198,000 = \$2000$

(iii)

n	**End of the month**
1	$1.003(500000-x)$
2	$1.003^2(500000-x)-(1.003)x$
3	$1.003^3(500000-x)-(1.003)^2x-(1.003)x$
$\vdots$	$\vdots$
n	

At the end of the *n*th month, the outstanding amount would be

$$1.003^n(500000-x)-(1.003)^{n-1}x-\ldots-(1.003)x$$

$$=1.003^n(500000)-(1.003^n)x-\ldots-(1.003)x$$

$$=1.003^n(500000)-x\left[1.003+1.003^2+\ldots+1.003^n\right]$$

$$=1.003^n(500000)-x\left[\frac{1.003(1.003^n-1)}{1.003-1}\right]$$

$$=1.003^n(500000)-\frac{1003}{3}x(1.003^n-1)$$

$$\therefore A = 500000, B = \frac{1003}{3}$$

(iv)

$$1.003^{96}(500000)-\frac{1003}{3}x(1.003^{96}-1)=0$$

Using GC, $x = \$5984.09$.

(v)

Total paid: $\$5984.09 \times 12 \times 8 = \$574,472.64$

Interest: $\$574,472.64 - \$500,000 = \$74,472.64$