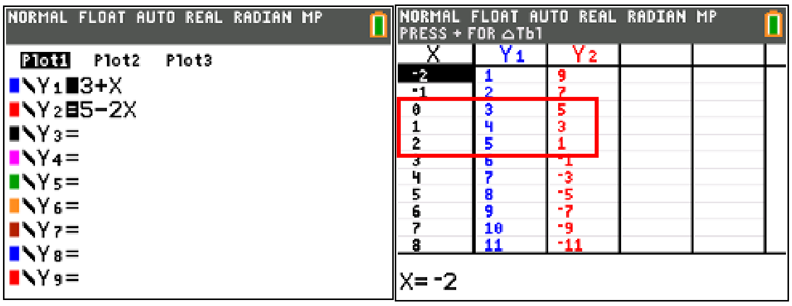
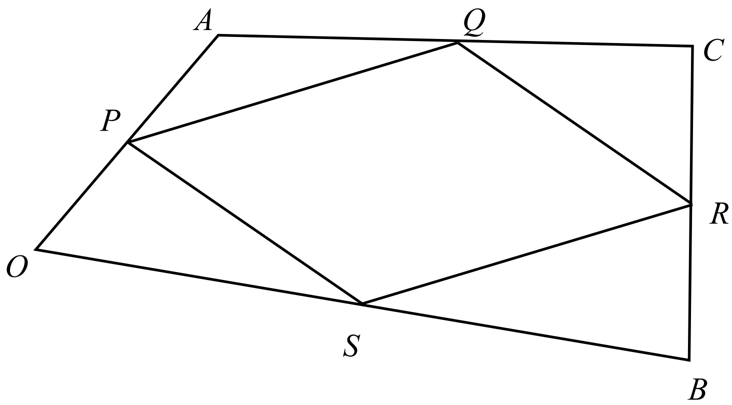
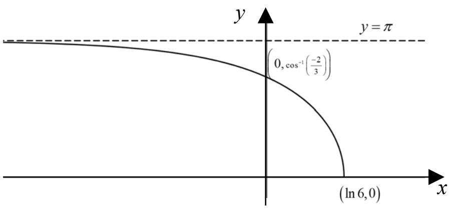
2021 ACJC H2 Math Prelim P2 Marking Scheme

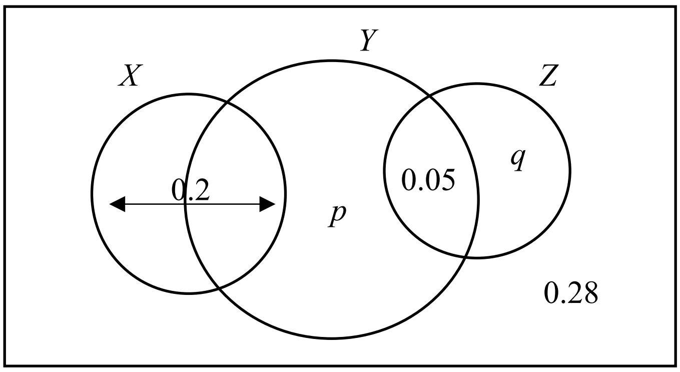
Qn	Solutions	
1	Let x, y and z be the digits on the hundreds, tens and ones position respectively. $x+y+z=8 \;-----(1)$ $(100x+10y+z)-(100z+10y+x)=297----(2)$ $\Rightarrow \begin{aligned} x+y+z&=8 \\ x-z&=3 \end{aligned}$ Using GC: $x=3+z,\ \ y=5-2z,\ \ z=z$ Since x, y and z are non-negative integer values, $3+z\geq 0 \Rightarrow z\geq -3$ $5-2z\geq 0 \Rightarrow z\leq 2.5$ $z\geq 0$ $\therefore 0\leq z\leq 2.5$ When $z=0, x=3, y=5$ When $z=1, x=4, y=3$ When $z=2, x=5, y=1$ Alternative: using GC  Hence, the possible original numbers are 350, 431 or 512.	
2(i)		

	$\mathbf{p}=\frac{1}{2}\mathbf{a} \qquad \mathbf{q}=\frac{1}{2}(\mathbf{a}+\mathbf{c})$ $\mathbf{r}=\frac{1}{2}(\mathbf{b}+\mathbf{c}) \quad \mathbf{s}=\frac{1}{2}\mathbf{b}$ $\overrightarrow{PQ}=\mathbf{q}-\mathbf{p}=\frac{1}{2}(\mathbf{a}+\mathbf{c})-\frac{1}{2}\mathbf{a}=\frac{1}{2}\mathbf{c}$ $\overrightarrow{SR}=\mathbf{r}-\mathbf{s}=\frac{1}{2}(\mathbf{b}+\mathbf{c})-\frac{1}{2}\mathbf{b}=\frac{1}{2}\mathbf{c}=\overrightarrow{PQ}$ Therefore *PQRS* is a parallelogram.	
2(ii)	$\lvert\mathbf{p}+\mathbf{q}\rvert=\lvert\mathbf{p}\rvert+\lvert\mathbf{q}\rvert$ if and only if **p** and **q** are parallel and in the same direction.	
2(iii)	Area of *PQRS* = $\left\lvert\overrightarrow{PQ}\times\overrightarrow{PS}\right\rvert=\left\lvert\frac{1}{2}\mathbf{c}\times\frac{1}{2}(\mathbf{b}-\mathbf{a})\right\rvert=\frac{1}{4}\lvert\mathbf{c}\times\mathbf{b}-\mathbf{c}\times\mathbf{a}\rvert=\frac{1}{4}\lvert\mathbf{c}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}\rvert$ Since *O*, *A*, *B* and *C* are coplanar, then $\mathbf{c}\times\mathbf{b}$ and $\mathbf{a}\times\mathbf{c}$ are normal to the plane containing *OACB* and in the same direction. Hence Area of *PQRS* = $\frac{1}{4}\left(\lvert\mathbf{c}\times\mathbf{b}\rvert+\lvert\mathbf{a}\times\mathbf{c}\rvert\right)$ Area of *OACB* = Area of OAC + Area of OCB $=\frac{1}{2}\lvert\mathbf{a}\times\mathbf{c}\rvert+\frac{1}{2}\lvert\mathbf{b}\times\mathbf{c}\rvert=2(\text{Area of PQRS})$.	
3(i)	$\mathrm{f}(r)-\mathrm{f}(r+1)=\frac{1}{(r-1)r!}-\frac{1}{r(r+1)!}$ $=\frac{1}{(r-1)r!}-\frac{1}{r(r+1)!}$ $=\frac{r(r+1)-(r-1)}{r(r-1)(r+1)!}$ $=\frac{r^2+1}{r(r-1)(r+1)!}$	
3(ii)	$\sum_{r=2}^{n}\frac{r^2+1}{r(r-1)(r+1)!}+3^{-r}=\sum_{r=2}^{n}\mathrm{f}(r)-\mathrm{f}(r+1)+\sum_{r=2}^{n}\left(\frac{1}{3}\right)^r$ $=\begin{bmatrix}\mathrm{f}(2)-\mathrm{f}(3)\\+\mathrm{f}(3)-\mathrm{f}(4)\\+\mathrm{f}(5)-\mathrm{f}(6)\\\vdots\\+\mathrm{f}(n-1)-\mathrm{f}(n)\\+\mathrm{f}(n)-\mathrm{f}(n+1)\end{bmatrix}+\frac{\frac{1}{3}^2\left(1-\left(\frac{1}{3}\right)^{n-1}\right)}{1-\frac{1}{3}}$ $=\mathrm{f}(2)-\mathrm{f}(n+1)+\left(\frac{3}{2}\right)\left(\frac{1}{9}\right)\left(1-\frac{1}{3^{n-1}}\right)$ $=\frac{1}{(2-1)2!}-\frac{1}{(n+1-1)(n+1)!}+\frac{1}{6}\left(1-\frac{3}{3^n}\right)$ $=\frac{2}{3}-\frac{1}{n(n+1)!}-\frac{1}{2}\left(\frac{1}{3}\right)^n$	
3(iii)	As $n\to\infty$, $\frac{1}{n(n+1)!}\to 0$ and $\left(\frac{1}{3}\right)^n\to 0$.	

	Hence $S_n \to \frac{2}{3}$ which is a constant, hence S_n is convergent. $\therefore S_\infty = \frac{2}{3}$.	
4(i)	$x = \ln(3+\theta)$, $\qquad y = \cos^{-1}\left(\frac{\theta}{3}\right)$ $\frac{dx}{d\theta} = \frac{1}{3+\theta}$ $\qquad \frac{dy}{d\theta} = -\frac{1}{\sqrt{1-\left(\frac{\theta}{3}\right)^2}} \times \frac{1}{3} = -\frac{1}{\sqrt{9-\theta^2}}$ $\frac{dy}{dx} = \frac{-\frac{1}{\sqrt{9-\theta^2}}}{\frac{1}{3+\theta}} = -\frac{3+\theta}{\sqrt{9-\theta^2}}$ $= -\frac{3+\theta}{\sqrt{(3+\theta)(3-\theta)}}$ $= -\frac{\cancel{\sqrt{(3+\theta)}}\sqrt{(3+\theta)}}{\cancel{\sqrt{(3+\theta)}}\sqrt{(3-\theta)}}$ $= -\sqrt{\frac{3+\theta}{3-\theta}}$ As $\theta \to 3$, $\frac{dy}{dx} \to \infty \Rightarrow$ The tangent is parallel to the y-axis.	
4(ii)	As $\theta \to -3,\ x = \ln(3+\theta) \to -\infty$ and $y = \cos^{-1}\left(\frac{\theta}{3}\right) \to \cos^{-1}\left(\frac{-3}{3}\right) = \pi$ $\therefore y = \pi$ is a horizontal asymptote of the curve C.  when $x = 0 \Rightarrow \ln(3+\theta) = 0 \Rightarrow \theta = -2 \Rightarrow y = \cos^{-1}\left(\frac{-2}{3}\right)$ $\Rightarrow$ y-intercept at $\left(0, \cos^{-1}\left(\frac{-2}{3}\right)\right)$ when $y = 0 \Rightarrow \cos^{-1}\left(\frac{\theta}{3}\right) = 0 \Rightarrow \theta = 3 \Rightarrow x = \ln(3+3) = \ln 6$ $\Rightarrow$ x-intercept at $(\ln 6, 0)$	

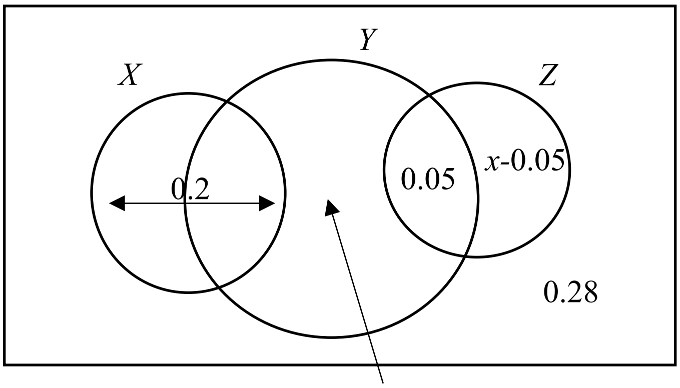
4(iii)	$y=p \Rightarrow \; p=\cos^{-1}\left(\dfrac{\theta}{3}\right) \Rightarrow \theta = 3\cos p$ $\Rightarrow \left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right\rvert_{\theta=3\cos p} = -\sqrt{\dfrac{3+3\cos p}{3-3\cos p}}$ $= -\sqrt{\dfrac{3+3\left(2\cos^2\dfrac{p}{2}-1\right)}{3-3\left(1-2\sin^2\dfrac{p}{2}\right)}}$ $= -\sqrt{\dfrac{6\cos^2\dfrac{p}{2}}{6\sin^2\dfrac{p}{2}}}$ $= -\sqrt{\cot^2\dfrac{p}{2}} \quad = -\cot\dfrac{p}{2}$ (shown) $\quad (\because 0 \le y < \pi \Rightarrow \cot\dfrac{p}{2} > 0)$ Or $x=\ln(3+\theta), y=\cos^{-1}\left(\dfrac{\theta}{3}\right) \Rightarrow \theta = 3\cos y$ $\Rightarrow x = \ln(3+3\cos y)$ $\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{-3\sin y}{3+3\cos y} = -\dfrac{\sin y}{1+\cos y} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1+\cos y}{\sin y}$ $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right\rvert_{y=p} = -\dfrac{1+\cos p}{\sin p}$ $= -\dfrac{1+\left(2\cos^2\dfrac{p}{2}-1\right)}{2\sin\dfrac{p}{2}\cos\dfrac{p}{2}}$ $= -\dfrac{2\cos^2\dfrac{p}{2}}{2\sin\dfrac{p}{2}\cos\dfrac{p}{2}}$ $= -\dfrac{\cos\dfrac{p}{2}}{\sin\dfrac{p}{2}} = -\cot\dfrac{p}{2}$ (shown)	
4(iv)	Gradient of normal = $-\dfrac{1}{\left(-\cot\dfrac{p}{2}\right)} = \tan\dfrac{p}{2}$ when $y=p \Rightarrow \; p=\cos^{-1}\left(\dfrac{\theta}{3}\right) \Rightarrow \theta = 3\cos p$ (may have been found in (iii)) $\Rightarrow x = \ln(3+3\cos p)$ $\therefore$ equation of normal: $y-p=\tan\dfrac{p}{2}\left(x-\ln(3+3\cos p)\right)$	

	$(0,1)$ lies on the normal and thus satisfies normal equation $\Rightarrow 1-p=\tan\frac{p}{2}\left(0-\ln\left(3+3\cos p\right)\right)$ $\Rightarrow \tan\frac{p}{2}\left(\ln\left(3+3\cos p\right)\right)+1-p=0$ Using GC, since $0\le p<\pi \Rightarrow p=1.94$ (to 3sf)	
5(i)	$y=-\sqrt{hx+k}$ $(0,0): 0=-\sqrt{k} \ \Rightarrow \ k=0$ Point A has coordinates $(6,-1)$. $(6,-1): -1=-\sqrt{6h+0} \ \Rightarrow \ h=\frac{1}{6}$	
5(ii)	Volume of water required: $(10\times5\times0.8)+5\times\int_0^6\left\lvert-\sqrt{\frac{x}{6}}\right\rvert\,\mathrm{d}x=60\text{ m}^3$	
5(iii)	$y=\mathrm{e}^{0.04x^2}-3$ When $x=0,\ y=1-3 \ \Rightarrow \ y=-2$. $y+3=\mathrm{e}^{0.04x^2}$ $\ln(y+3)=0.04x^2$ $x^2=25\ln(y+3)$ Volume of water required to fill Jane's preferred pool: $\pi\int_{-2}^{0}x^2\,\mathrm{d}y=25\pi\int_{-2}^{0}\ln(y+3)\,\mathrm{d}y$ $=25\pi\left(\left[y\ln(y+3)\right]_{-2}^{0}-\int_{-2}^{0}\frac{y}{y+3}\,\mathrm{d}y\right)$ $=-25\pi\left(\int_{-2}^{0}1-\frac{3}{y+3}\,\mathrm{d}y\right)$ $=-25\pi\left(\left[y-3\ln(y+3)\right]_{-2}^{0}\right)$ $=-25\pi\left(-3\ln3-(-2)\right)$ $=(75\ln3-50)\pi\ \text{ m}^3$ $a=75,\ b=3,\ c=50.$	
5(iv)	$y=\mathrm{e}^{0.04x^2}-3$ When $y=0,\ \mathrm{e}^{0.04x^2}=3 \ \Rightarrow \ 0.04x^2=\ln3 \ \Rightarrow \ x=\pm5\sqrt{\ln3}$. (or $x=\pm5.24$) $\frac{\mathrm{d}y}{\mathrm{d}x}=0.08x\mathrm{e}^{0.04x^2}$ $\therefore S=2\pi\int_0^{5\sqrt{\ln3}}x\sqrt{1+\left(0.08x\mathrm{e}^{0.04x^2}\right)^2}\,\mathrm{d}x$ $=102.077\text{ m}^2\approx102\text{ m}^2$ (if $x=5.24$ used, $S\approx102.038$)	

6(a) (i)	$\mathrm{P}\left((A\cap B)'\mid A\cup B\right)=\dfrac{\mathrm{P}\left[(A\cap B)'\cap(A\cup B)\right]}{\mathrm{P}(A\cup B)}$ $=\dfrac{\mathrm{P}(A\cup B)-\mathrm{P}(A\cap B)}{\mathrm{P}(A\cup B)}$ $=\dfrac{0.37-0.03}{0.37}$ $=\dfrac{34}{37}$ or 0.919 (3sf)	
6(a) (ii)	A and B are independent events, $\mathrm{P}(A\cap B)=\mathrm{P}(A)\cdot\mathrm{P}(B)=0.03$ $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)=0.37$ $\Rightarrow \mathrm{P}(A)+\mathrm{P}(B)-0.03=0.37$ $\Rightarrow \mathrm{P}(A)+\mathrm{P}(B)=0.4$ Let $\mathrm{P}(A)=a$, $\mathrm{P}(B)=b$ So $ab=0.03$ --- (1) & $a+b=0.4$ Sub. $b=0.4-a$ into (1), $\therefore a(0.4-a)=0.03$ $a^2-0.4a+0.03=0$ Solving $a=0.3$ or 0.1 $b=0.1$ or 0.3 Since $\mathrm{P}(A)>\mathrm{P}(B)$, $P(A)=0.3$, $\mathrm{P}(B)=0.1$	
6(b)	 Method 1 From the Venn Diagram, $0.2+p+0.05+q+0.28=1\Rightarrow p=0.47-q$ $p=0.47-q$ $p\geq 0\Rightarrow q\leq 0.47$ Thus $0\leq q\leq 0.47$ Since $\mathrm{P}(Z)=0.05+q$ $\therefore$ min $\mathrm{P}(Z)=0.05$, max $\mathrm{P}(Z)=0.47+0.05=0.52$	

Alternatively

Let $\mathrm{P}(Z) = x$

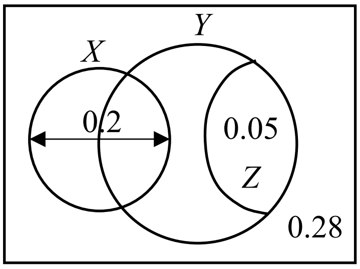


1-x-0.2-0.28 = 0.52-x

$x - 0.05 \geq 0 \Rightarrow x \geq 0.05$
$0.52 - x \geq 0 \Rightarrow x \leq 0.52$
Thus $0.05 \leq x \leq 0.52$

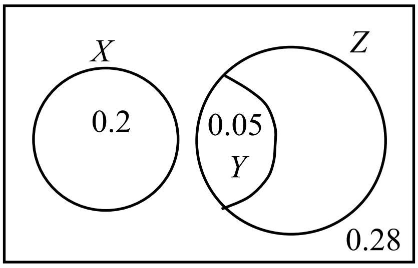
<u>Method 2</u>

For min P(Z),
Z is a subset of Y.



So min P(Z) = 0.05

For max P(Z),
Y is a subset of Z.



max P(Z) = $1 - 0.28 - 0.2 = 0.52$

7(i)

No. of ways $= {}^{12}\mathrm{C}_8 - {}^{9}\mathrm{C}_8 - {}^{8}\mathrm{C}_8$
$= 495 - 9 - 1$
$= 485$

7(ii)

$$\text{Probability} = \frac{(9-1)! \times 2 \times {}^{8}\mathrm{C}_2 \times 2!}{(12-1)!} = \frac{56}{495}$$

7(iii)

<u>Case 1: 2A, 1B, 1C</u>
No of choices of 4 students = ${}^{3}C_2 \times {}^{3}C_1 \times {}^{5}C_1$ (AX, AX, BY, CZ) $= 45$

<u>Case 2: 1A, 2B, 1C</u>
No of choices of 4 students = ${}^{3}C_1 \times {}^{3}C_2 \times {}^{5}C_1$ (AX, BY, BY, CZ)
$+ {}^{3}C_1 \times {}^{3}C_1 \times {}^{1}C_1 \times {}^{5}C_1$ (AX, BY, BZ, CZ)
$= 45 + 45 = 90$

<u>Case 3: 1A 1B 2C</u>
No of choices of 4 students = ${}^{3}C_1 \times {}^{3}C_1 \times {}^{5}C_2$ (AX, BY, CZ, CZ) $= 90$

Total no of arrangements = $(45 + 90 + 90) \times 4! \times 2 = 10800$

8(i)	$\mathrm{P}(W=0)=\frac{5}{(n+5)}\cdot\frac{4}{(n+4)}\cdot\frac{3}{(n+3)}=\frac{60}{(n+5)(n+4)(n+3)}$ $\mathrm{P}(W=1)=\frac{n}{(n+5)}\cdot\frac{5}{(n+4)}\cdot\frac{4}{(n+3)}\times 3=\frac{60n}{(n+5)(n+4)(n+3)}$ $\mathrm{P}(W=2)=\frac{n}{(n+5)}\cdot\frac{n-1}{(n+4)}\cdot\frac{5}{(n+3)}\times 3=\frac{15n(n-1)}{(n+5)(n+4)(n+3)}$ $\mathrm{P}(W=3)=\frac{n}{(n+5)}\cdot\frac{n-1}{(n+4)}\cdot\frac{n-2}{(n+3)}=\frac{n(n-1)(n-2)}{(n+5)(n+4)(n+3)}$	

w	0	1	2	3
$\mathrm{P}(W=w)$	$\frac{60}{(n+5)(n+4)(n+3)}$	$\frac{60n}{(n+5)(n+4)(n+3)}$	$\frac{15n(n-1)}{(n+5)(n+4)(n+3)}$	$\frac{n(n-1)(n-2)}{(n+5)(n+4)(n+3)}$

8(ii)

$$\mathrm{E}(W)=\frac{0\times 60+1\times 60n+2\times 15n(n-1)+3\times n(n-1)(n-2)}{(n+5)(n+4)(n+3)}$$

$$=\frac{60n+30n(n-1)+3n(n-1)(n-2)}{(n+5)(n+4)(n+3)}$$

$$=\frac{3n\left[20+10(n-1)+(n-1)(n-2)\right]}{(n+5)(n+4)(n+3)}$$

$$=\frac{3n\left(n^2+7n+12\right)}{(n+5)(n+4)(n+3)}=\frac{3n}{(n+5)}$$

$$\mathrm{E}\left(W^2\right)=\frac{0\times 60+1\times 60n+4\times 15n(n-1)+9\times n(n-1)(n-2)}{(n+5)(n+4)(n+3)}$$

$$=\frac{60n+60n(n-1)+9n(n-1)(n-2)}{(n+5)(n+4)(n+3)}$$

$$=\frac{3n\left[20+20(n-1)+3(n-1)(n-2)\right]}{(n+5)(n+4)(n+3)}$$

$$=\frac{3n\left(3n^2+11n+6\right)}{(n+5)(n+4)(n+3)}$$

$$=\frac{3n(3n+2)(n+3)}{(n+5)(n+4)(n+3)}=\frac{3n(3n+2)}{(n+5)(n+4)}$$

$$\mathrm{Var}(W)=\mathrm{E}\left(W^2\right)-\left[\mathrm{E}(W)\right]^2$$

$$=\frac{3n(3n+2)}{(n+5)(n+4)}-\left[\frac{3n}{n+5}\right]^2$$

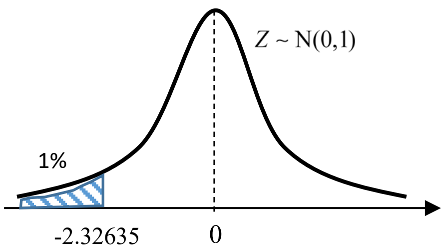
$$=\frac{3n}{n+5}\cdot\left[\frac{3n+2}{n+4}-\frac{3n}{n+5}\right]$$

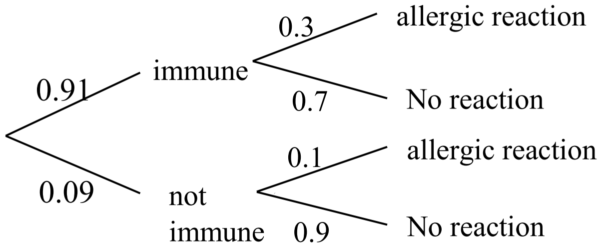
$$=\frac{3n}{n+5}\cdot\left[\frac{(3n+2)(n+5)-3n(n+4)}{(n+5)(n+4)}\right]$$

$$=\frac{3n}{n+5}\cdot\left[\frac{5n+10}{(n+5)(n+4)}\right]=\frac{15n(n+2)}{(n+5)^2(n+4)}$$

$\mathrm{g}(n)=15n(n+2)$ OR $15n^2+30n$

8(iii)	$R+W=3 \Rightarrow R=3-W$ $\text{Var}(R)=\text{Var}(3-W)=\text{Var}(W)=\dfrac{15n(n+2)}{(n+5)^2(n+4)}$	
9(i)	A sample is random if <u>every mask has an equal chance of being selected</u> and that the selections are independent of each other. Or A sample is random if <u>every subset of *n* masks has an equal chance of being selected to be part of the sample</u>.	
9(ii)	There is no need for any assumptions to be made about the population distribution of PFE of masks since $n = 50$ is large, by central limit theorem, the sample mean PFE will follow a normal distribution approximately.	
9(iii)	Let X be random variable for the PFE of Brand BEY masks. Unbiased estimate of population mean, $\bar{x}=\dfrac{289}{50}+90=95.78$ Unbiased estimate of population variance, $s^2=\dfrac{1}{49}\left[1670.56-\dfrac{(289)^2}{50}\right]=0.0028571$ Let μ be the population mean PFE of Brand BEY masks. To test $\text{H}_0:\mu=95.8$ against $\text{H}_1:\mu<95.8$ at 1% level of significance. Under H_0, since $n = 50$ is large, $\bar{X}\sim\text{N}\left(95.8,\ \dfrac{0.0028571}{50}\right)$ approximately by central limit theorem $Z=\dfrac{\bar{X}-95.8}{\sqrt{0.0028571/50}}\sim\text{N}(0,1)$ Value of test statistic $z=-2.646$ p-value = 0.00408 Since $z=-2.646<-2.326$ or p-value = 0.00408 < 0.01, reject H_0 at 1% level of significance There is sufficient evidence at 1% level of significance that the manager's suspicion is justified, that is, mean PFE of the Brand BEY surgical masks is compromised.	
9(iv)	$\text{H}_0:\mu=95.8$ $\text{H}_1:\mu<95.8$ at 1% level of significance. Now $\bar{x}=95.5$, $s^2=\dfrac{n}{n-1}$(sample variance)$=\dfrac{40}{39}(k^2)$	

	Under H_0, $\overline{X} \sim N\left(95.8, \dfrac{40k^2/39}{40}\right)$ by CLT since n is large i.e. $\overline{X} \sim N\left(95.8, \dfrac{k^2}{39}\right)$ Value of Test Statistic, $z = \dfrac{\bar{x} - 95.8}{\sqrt{k^2/39}}$  H_0 is not rejected if $z > -2.32635$ $\dfrac{95.5 - 95.8}{\sqrt{\dfrac{k^2}{39}}} > -2.32635$ $-0.3 > -2.32635\sqrt{\dfrac{k^2}{39}}$ $\dfrac{0.3}{2.32635} < \dfrac{k}{\sqrt{39}}$ $k > 0.80534$ $k > 0.805$ (to 3 s.f.)	
10(i)	The probability that a randomly selected person who will develop immunity after taking the vaccine is constant at 0.91. Whether a randomly selected person will develop immunity after taking the vaccine is independent of whether other selected people will develop immunity.	
10(ii)	A is the random variable for the number of people, out of n, who develop immunity after taking the vaccine. $A \sim B(n, 0.91)$ Mode of A = 19 $P(A=19) > P(A=18)$ & $P(A=19) > P(A=20)$	

	$\mathrm{P}(A=19)>\mathrm{P}(A=18)$ ${}^{n}\mathrm{C}_{19}(0.91)^{19}(0.09)^{n-19}>{}^{n}\mathrm{C}_{18}(0.91)^{18}(0.09)^{n-18}$ $\frac{n!}{(n-19)!19!}\frac{(0.09)^{n-19}}{(0.09)^{n-18}}>\frac{n!}{(n-18)!18!}\frac{(0.91)^{18}}{(0.91)^{19}}$ $(n-18)(0.91)>19(0.09)$ $n-18>\frac{19(0.09)}{0.91}$ $n>19.879$ $\mathrm{P}(A=19)>\mathrm{P}(A=20)$ ${}^{n}\mathrm{C}_{19}(0.91)^{19}(0.09)^{n-19}>{}^{n}\mathrm{C}_{20}(0.91)^{20}(0.09)^{n-20}$ $\frac{n!}{(n-19)!19!}\frac{(0.09)^{n-19}}{(0.09)^{n-20}}>\frac{n!}{(n-20)!20!}\frac{(0.91)^{20}}{(0.91)^{19}}$ $20(0.09)>(n-19)(0.91)$ $n-19<\frac{20(0.09)}{0.91}$ $n<20.978$ Hence $19.879<n<20.978$ $\therefore n=20$.	
10(iii)	$A'\sim\mathrm{B}(25,0.09)$ $\mathrm{P}(A'>3)=1-\mathrm{P}(A'\le 3)=0.18315=0.183$	
10(iv)	Let X_k be the random variable for the number of undesirable samples, out of k. $X_k\sim\mathrm{B}(k,0.18315)$ $\text{Probability}=\mathrm{P}(X_7=0)\times 0.18315\times\mathrm{P}(X_{22}\le 1)$ $=0.24267\times 0.18315\times 0.069250$ $=0.00308$	
10(v)	 $\mathrm{P}(\text{immune}\mid\text{allergic reaction})=\frac{\mathrm{P}(\text{immune}\cap\text{allergic reaction})}{\mathrm{P}(\text{allergic reaction})}$ $=\frac{0.91\times 0.3}{0.91\times 0.3+0.09\times 0.1}$ $=0.968$	

11(1st part)

X denotes the random variable for the time in minutes past 6pm at which Billy leaves his workplace
Y denotes the random variable for the time taken for the journey from Billy's workplace to the childcare centre $X \sim \mathrm{N}(12,3^2), Y \sim \mathrm{N}(m,10^2)$

$\therefore X+Y \sim \mathrm{N}(12+m,\ 3^2+10^2)$

Given: $\mathrm{P}(X+Y>60)=0.22176$

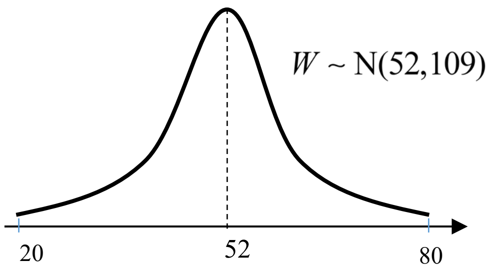
$\Rightarrow \mathrm{P}\left(Z>\dfrac{60-12-m}{\sqrt{109}}\right)=0.16$

$\Rightarrow \dfrac{60-12-m}{\sqrt{109}}=0.76626$

$\therefore m=40.0$ (to 3 s.f.)

11(i)

$W=X+Y \sim \mathrm{N}(52,\ 109)$



Note: $\mathrm{P}(20<W<80)=0.995$

11(ii)

$W=X+Y \sim \mathrm{N}(52,\ 109)$
$\mathrm{P}(W<k)\geq 0.98$
From GC, $k \geq 73.44177$
Smallest k = 73.4 mins = 1 h 13.4min
Latest time to leave workplace is 5.46pm.

11(iii)

$\overline{Y}=\dfrac{Y_1+Y_2+\ldots+Y_n}{n} \sim \mathrm{N}(40.0,\dfrac{10^2}{n})$

Given: $\mathrm{P}(\overline{Y}>42)\leq 0.2$
By GC,

n	$\mathrm{P}(\overline{Y}>42)$
17	$0.20479>0.2$
18	$0.019807<0.2$
19	$0.19166<0.2$

Since $n \leq 22$, $\therefore n=18,19,20,21,22$

Alternatively
Given: $\mathrm{P}(\overline{Y}>42)\leq 0.2$

$\Rightarrow \mathrm{P}\left(Z>\dfrac{42-40}{\sqrt{\dfrac{10^2}{n}}}\right)\leq 0.2$

$\Rightarrow \dfrac{42-40}{\sqrt{\dfrac{10^2}{n}}}\geq 0.84162$

	$\Rightarrow \dfrac{2\sqrt{n}}{10} \geq 0.84162$ $\Rightarrow n \geq 17.7$ Since $n \leq 22$, $\therefore n = 18, 19, 20, 21, 22$	
11(iv)	Let T be random variable for the time in minutes after 7pm at which Billy arrives at the childcare centre. Given: $S \sim \mathrm{N}(65,\ 6^2)$ $T = X + S - 60$ $\mathrm{E}(T) = 12 + 65 - 60, \quad \mathrm{Var}(T) = 6^2 + 3^2 = 45$ $T \sim \mathrm{N}(17,\ 45)$ $T_1 + T_2 + \ldots + T_{10} \sim \mathrm{N}(170,\ 450)$ $\mathrm{P}(\text{Total fine} > 180) = \mathrm{P}(T_1 + T_2 + \ldots + T_{10} > \dfrac{180}{1.5})$ $= \mathrm{P}(T_1 + T_2 + \ldots + T_{10} > 120)$ $=0.991$ Alternatively, F be random variable for total penalty fines for 10 randomly chosen days. $F = 1.5(T_1 + T_2 + \ldots + T_{10})$ $\mathrm{E}(F) = 1.5(10)(17) = 255, \quad \mathrm{Var}(F) = 1.5^2(10)(45) = 1012.5$ $F \sim \mathrm{N}(255, 1012.5)$ $\mathrm{P}(F > 180) = 0.991$	