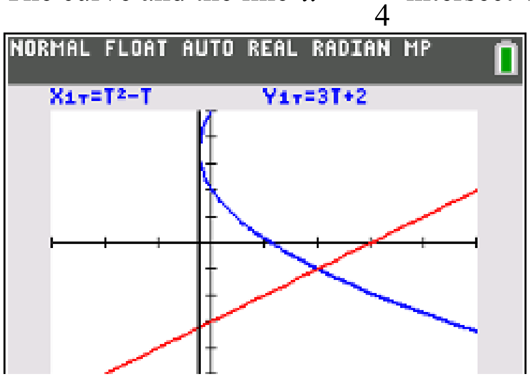
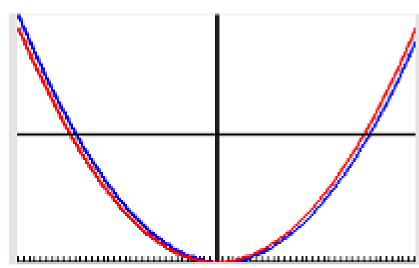
2023 Year 6 H2 Mathematics Preliminary Examination Paper 2: Solutions

1	**Solutions**
(a) **[2]**	$x=t^2-t$ $=t^2-t+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2$ $=\left(t-\frac{1}{2}\right)^2-\frac{1}{4}$ Since $\left(t-\frac{1}{2}\right)^2\geq 0$, $\left(t-\frac{1}{2}\right)^2-\frac{1}{4}\geq -\frac{1}{4}$. Hence $x\geq -\frac{1}{4}$ for all values of t.
(b) **[3]**	$\frac{\mathrm{d}y}{\mathrm{d}t}=3$, $\frac{\mathrm{d}x}{\mathrm{d}t}=2t-1$ $\therefore\ \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{3}{2t-1}$ When $t=-1$, $x=2$, $y=-1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=-1$ & gradient of normal = 1. Equation of normal at the point where $t=-1$ is $y+1=x-2 \Rightarrow y=x-3$
(c) **[4]**	The curve and the line $x=-\frac{1}{4}$ intersect when $t=\frac{1}{2}$.  Required Area $=\int_{-\frac{1}{4}}^{2} y\ \mathrm{d}x-\int_{-\frac{1}{4}}^{2}(x-3)\ \mathrm{d}x$ $=\int_{\frac{1}{2}}^{-1}(3t+2)(2t-1)\ \mathrm{d}t-\int_{-\frac{1}{4}}^{2}(x-3)\ \mathrm{d}x$ $=5.90625=5.906$ (3.d.p) **Alternative Solution 1**

	Required Area $= \int_{-1}^{\frac{7}{2}} (x+\frac{1}{4})\ \mathrm{d}y + \int_{-\frac{13}{4}}^{-1} (y+3+\frac{1}{4})\ \mathrm{d}y$ $= \int_{-1}^{\frac{1}{2}} 3(t-\frac{1}{2})^2\ \mathrm{d}t + \int_{-\frac{13}{4}}^{-1} (y+3+\frac{1}{4})\ \mathrm{d}y$ $= 5.90625 = 5.906$ (3.d.p) Note that $\int_{-\frac{13}{4}}^{-1} (y+3+\frac{1}{4})\ \mathrm{d}y$ can be seen as $\frac{1}{2}\times\frac{9}{4}\times\frac{9}{4}$. **Alternative Solution 2** Required Area $= \int_{-\frac{1}{4}}^{\frac{10}{9}} y\ \mathrm{d}x + \left[\frac{1}{2}\times\left(\frac{13}{4}+1\right)\times\frac{9}{4} - \left\lvert\int_{\frac{10}{9}}^{2} y\ \mathrm{d}x\right\rvert\right]$ $= \int_{\frac{1}{2}}^{-\frac{2}{3}} \left(6t^2+t-2\right)\mathrm{d}t + \left[\frac{1}{2}\times\left(\frac{13}{4}+1\right)\times\frac{9}{4} - \left\lvert\int_{-\frac{2}{3}}^{-1} \left(6t^2+t-2\right)\mathrm{d}t\right\rvert\right]$

2	Solutions
(a) [2]	1, 3, 5, 7, ... Number of terms in each bracket follows an AP with first term 1 and common difference 2. $\therefore$ Number of integers in the first n sets $=\frac{n}{2}\left[2(1)+(n-1)2\right]=n^2$
(b) [4]	Last integer in the nth set is the $\left(n^2\right)$th term of the AP 1, 4, 7, 10, 13, 16, ... which has first term 1 and common difference 3. Last integer of the nth set $=1+\left(n^2-1\right)3=3n^2-2$ From GC, <table><tr><td>n</td><td>$3n^2-2$</td></tr><tr><td>25</td><td>1873</td></tr><tr><td>26</td><td>2026</td></tr></table>$\therefore\ k=26$ OR: Given that 2023 occurs in the kth set, first term in the kth set $\le 2023 \le$ last term in the kth set $\left[3(k-1)^2-2\right]+3\le 2023\le 3k^2-2$ $(k-1)^2\le\frac{2022}{3}$ and $k^2\ge\frac{2025}{3}$ $-24.961\le k\le 26.961$ and $k\le -25.980$ or $k\ge 25.980$ $\therefore 25.980\le k\le 26.961$ Since $k\in\mathbb{Z}^+$, $k=26$ OR: $\left[3(k-1)^2-2\right]+3\le 2023\le 3k^2-2$  From GC, $-24.961\le k\le 26.961$ and $k\le -25.980$ or $k\ge 25.980$ $\therefore 25.980\le k\le 26.961$ Since $k\in\mathbb{Z}^+$, $k=26$
(c) [3]	Required sum

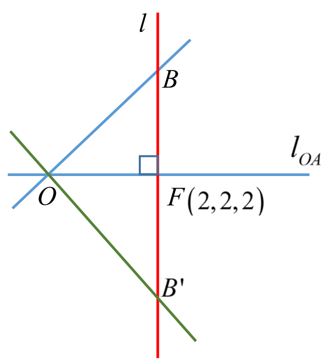
	$=$ Sum of first $\left(10^2\right)$th terms $-$ Sum of first $\left(4^2\right)$th terms $=\dfrac{10^2}{2}\left[2(1)+\left(10^2-1\right)3\right]-\dfrac{4^2}{2}\left[2(1)+\left(4^2-1\right)3\right]$ $=14\ 574$ OR: Last term in the 10th set $=3(10)^2-2=298$ Last term in the 4th set $=3(4)^2-2=46$ First term in the 5th set $=46+3=49$ To find the sum of the AP : $49,52,55,...,298$ with first term 49 and common difference 3: $298=49+(m-1)3$ $m=84$ $\therefore$ Required sum $=\dfrac{84}{2}(49+298)=14\ 574$ OR using GC: Since the 1st integer in the 5th set is the (4^2+1) th term in the AP and the last integer in the 10th set is the 10^2th term, required sum = $\displaystyle\sum_{r=17}^{100} 1+3(r-1)=14574$

3	**Solutions**
(a) **[6]**	$y = e^x \cos 3x$ $\frac{dy}{dx} = e^x \cos 3x + e^x(-3\sin 3x)$ $= y - 3e^x \sin 3x$ $\frac{d^2y}{dx^2} = \frac{dy}{dx} - 3e^x \sin 3x - 3e^x(3\cos 3x)$ $= \frac{dy}{dx} + \left(\frac{dy}{dx} - y\right) - 9y$ $= 2\frac{dy}{dx} - 10y$ (shown) $\frac{d^3y}{dx^3} = 2\frac{d^2y}{dx^2} - 10\frac{dy}{dx}$ When $x = 0$, $y = 1,\ \frac{dy}{dx} = 1,\ \frac{d^2y}{dx^2} = -8,\ \frac{d^3y}{dx^3} = -26$ By Maclaurin expansion, $y = 1 + x + \frac{(-8)}{2!}x^2 + \frac{(-26)}{3!}x^3 + \ldots$ $= 1 + x - 4x^2 - \frac{13}{3}x^3 + \ldots$
(b) **[2]**	Using standard series expansion, $e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \ldots$ $\cos 3x = 1 - \frac{(3x)^2}{2!} + \ldots = 1 - \frac{9}{2}x^2 + \ldots$

	$\mathrm{e}^x \cos 3x = \left(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\ldots\right)\left(1-\frac{9}{2}x^2+\ldots\right)$ $= 1+x+\left(\frac{1}{2}-\frac{9}{2}\right)x^2+\left(\frac{1}{6}-\frac{9}{2}\right)x^3+\ldots$ $= 1+x-4x^2-\frac{13}{3}x^3+\ldots$ (verified)
(c) **[3]**	$\ln(1+\mathrm{e}^x\cos 3x)$ $\approx \ln\left[1+\left(1+x-4x^2\right)\right]$ $= \ln\left(2+x-4x^2\right)$ $= \ln 2\left(1+\frac{x}{2}-2x^2\right)$ $= \ln 2+\ln\left(1+\frac{x}{2}-2x^2\right)$ $= \ln 2+\left(\frac{x}{2}-2x^2\right)-\frac{\left(\frac{x}{2}-2x^2\right)^2}{2}+\ldots$ $= \ln 2+\frac{x}{2}-2x^2-\frac{1}{2}\left(\frac{x^2}{4}\right)+\ldots$ $= \ln 2+\frac{1}{2}x-\frac{17}{8}x^2+\ldots$ (shown)

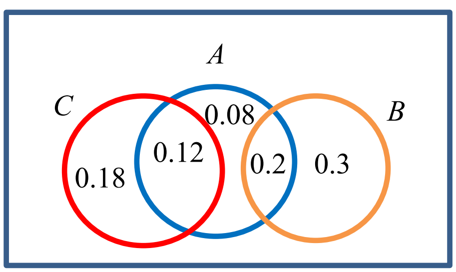
4	**Solutions**
(a) **[4]**	If $\mathbf{n}$ is perpendicular to $\mathbf{a}$, $\mathbf{n}\cdot\mathbf{a}=0$. $\left[(\mathbf{a}\cdot\mathbf{b})\mathbf{a}-(\mathbf{a}\cdot\mathbf{a})\mathbf{b}\right]\cdot\mathbf{a}$ $=(\mathbf{a}\cdot\mathbf{b})\mathbf{a}\cdot\mathbf{a}-(\mathbf{a}\cdot\mathbf{a})\mathbf{b}\cdot\mathbf{a}$ $=0$ since $\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{a}$ (shown) If $\mathbf{n}$ is parallel to plane *OAB*, it is perpendicular to $\mathbf{a}\times\mathbf{b}$ since $\mathbf{a}\times\mathbf{b}$ is perpendicular to plane *OAB*. i.e $\mathbf{n}\cdot(\mathbf{a}\times\mathbf{b})=0$.

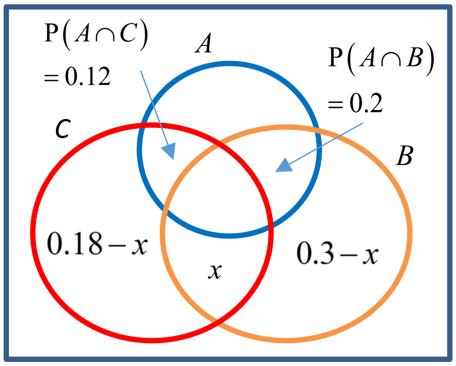
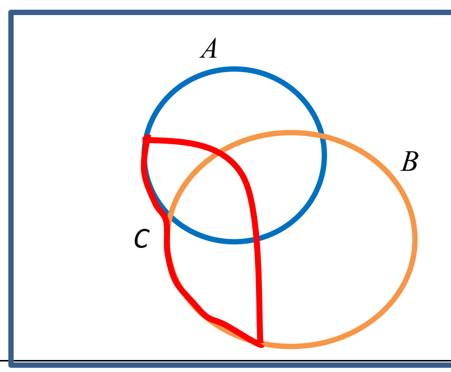
	$\left[(\mathbf{a}\cdot\mathbf{b})\mathbf{a}-(\mathbf{a}\cdot\mathbf{a})\mathbf{b}\right]\cdot(\mathbf{a}\times\mathbf{b})$ $=(\mathbf{a}\cdot\mathbf{b})\mathbf{a}\cdot(\mathbf{a}\times\mathbf{b})-(\mathbf{a}\cdot\mathbf{a})\mathbf{b}\cdot(\mathbf{a}\times\mathbf{b})$ $=0$ since $\mathbf{a}\perp\mathbf{a}\times\mathbf{b}$ and $\mathbf{b}\perp\mathbf{a}\times\mathbf{b}$ (shown) OR: Since **n** is a linear combination of non-zero and non-parallel vectors **a** and **b**, **n** lies on the same plane as **a** and **b** ie plane *OAB*. Hence **n** is parallel to the plane *OAB*.
(b) **[2]**	$\mathbf{n}=(\mathbf{a}\cdot\mathbf{b})\mathbf{a}-(\mathbf{a}\cdot\mathbf{a})\mathbf{b}$ $=\left[\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}2\\1\\3\end{pmatrix}\right]\begin{pmatrix}1\\1\\1\end{pmatrix}-\left[\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}\right]\begin{pmatrix}2\\1\\3\end{pmatrix}$ $=6\begin{pmatrix}1\\1\\1\end{pmatrix}-3\begin{pmatrix}2\\1\\3\end{pmatrix}=\begin{pmatrix}0\\3\\-3\end{pmatrix}=3\begin{pmatrix}0\\1\\-1\end{pmatrix}$ $\mathbf{m}=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$ or $\mathbf{m}=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\1\end{pmatrix}$ OR: Let vector parallel to **m** be $\begin{pmatrix}x\\y\\z\end{pmatrix}$. Then, $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix}=0\Rightarrow x+y+z=0 \quad ---(1)$ Also, $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\left[\begin{pmatrix}1\\1\\1\end{pmatrix}\times\begin{pmatrix}2\\1\\3\end{pmatrix}\right]=0$ $\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\-1\end{pmatrix}=0\Rightarrow 2x-y-z=0 \quad ---(2)$ $(1)+(2):\ x=0$ Then $y+z=0\Rightarrow y=-z$ If $y=1$, then $z=-1$ Hence $\mathbf{m}=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$.

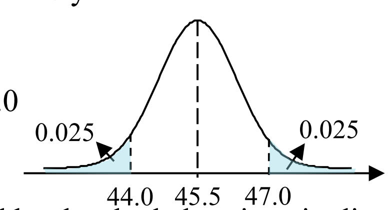
(c) **[2]**	Line OA : $\mathbf{r}=\lambda\begin{pmatrix}1\\1\\1\end{pmatrix},\ \lambda\in\mathbb{R}$ Let l be the line which passes through point B and is parallel to **m**. $l:\ \mathbf{r}=\begin{pmatrix}2\\1\\3\end{pmatrix}+\mu\begin{pmatrix}0\\1\\-1\end{pmatrix},\ \mu\in\mathbb{R}$ When line OA intersects l, $\lambda\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}2\\1\\3\end{pmatrix}+\mu\begin{pmatrix}0\\1\\-1\end{pmatrix}$ $\lambda=2$ $\lambda=1+\mu\Rightarrow\mu=1$ $\lambda=3-\mu\Rightarrow\mu=1$ $\therefore$ Coordinates of point of intersection are $(2,2,2)$. OR : Find foot of perpendicular of B to line OA
(d) **[3]**	Note that **m** // **n**, and **n** is perpendicular to **a**. Thus, line l is perpendicular to line OA. By Ratio Theorem, $\overrightarrow{OF}=\dfrac{\overrightarrow{OB}+\overrightarrow{OB'}}{2}$ $\overrightarrow{OB'}=2\overrightarrow{OF}-\overrightarrow{OB}$ $=2\begin{pmatrix}2\\2\\2\end{pmatrix}-\begin{pmatrix}2\\1\\3\end{pmatrix}=\begin{pmatrix}2\\3\\1\end{pmatrix}$  Equation of line of reflection is $\mathbf{r}=\beta\begin{pmatrix}2\\3\\1\end{pmatrix},\ \beta\in\mathbb{R}$ $\therefore$ A cartesian equation of the line of reflection is $\dfrac{x}{2}=\dfrac{y}{3}=z$.

5	**Solutions**
(a) **[2]**	Let X denote the mass of a small massage ball in grams. $X \sim \mathrm{N}\left(200, \sigma^2\right)$ $\mathrm{P}(195 < X < 205) = 0.98273$ $\mathrm{P}\left(\frac{195-200}{\sigma} < Z < \frac{205-200}{\sigma}\right) = 0.98273$ $\mathrm{P}\left(-\frac{5}{\sigma} < Z < \frac{5}{\sigma}\right) = 0.98273$ $\frac{5}{\sigma} = 2.3809$ $\sigma = 2.1000 = 2.1 \text{ (1dp)}$
(b) **[3]**	Let Y denote the mass of a medium massage ball in grams. $X \sim \mathrm{N}\left(200, 2.1^2\right)$ and $Y \sim \mathrm{N}\left(500, 1.4^2\right)$ $\mathrm{E}\left(X_1 + \cdots X_6 - 2Y\right) = 6\mathrm{E}\left(X\right) - 2\mathrm{E}\left(Y\right) = 200$ $\mathrm{Var}\left(X_1 + \cdots X_6 - 2Y\right) = 6\mathrm{Var}\left(X\right) + 4\mathrm{Var}\left(Y\right) = 34.3$ $X_1 + \cdots X_6 - 2Y \sim N(200, 34.3)$ $\mathrm{P}\left(X_1 + \cdots X_6 - 2Y > 210\right) = 0.0439 \text{ (3sf)}$
(c) **[1]**	Assume that the mass of a massage ball is independent of the mass of another massage ball.

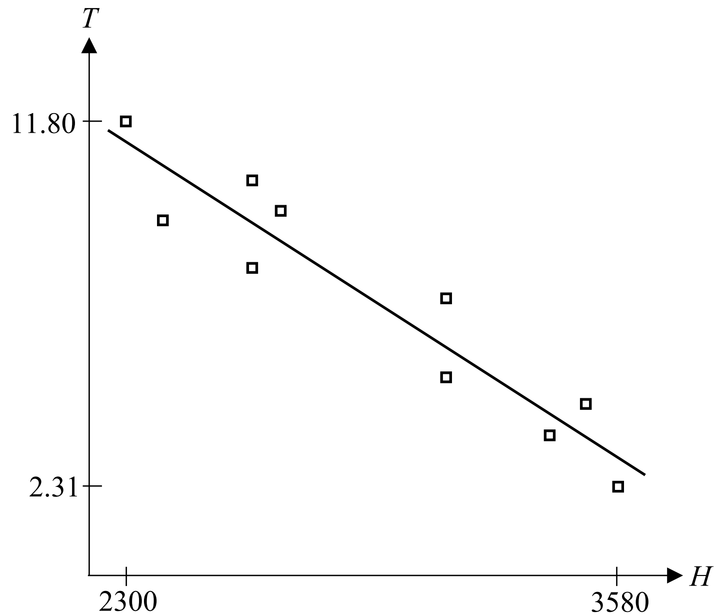
6	**Solutions**
(a) **[3]**	Arrange the 5 boys in $(5-1)!$ =24 ways. Then slot in each of the 3 girls into the 5 spaces between the boys in ${}^5\mathrm{P}_3 = 60$ ways. Total number of arrangements with no 2 girls being adjacent to each other is $24 \times 60 = 1440$
(b) **[3]**	Arrange the 3 girls within a unit in 3! = 6 ways. Then arrange the unit of 3 girls with the 5 boys in $(6-1)!$ =120 ways. Total number of arrangements with all 3 girls seated together is $6 \times 120 = 720$
(c) **[2]**	Total number of arrangements with exactly 2 of the 3 girls adjacent to each other is $((8-1)! - 1440 - 720)$ = 2880

7	**Solutions**
(a) **[2]**	Given that A and B are independent events, $\text{P}(A\cap B)=\text{P}(A)\text{P}(B)=ab$ Since $\text{P}(A'\cap B')=1-\text{P}(A\cup B)$ $=1-\left[\text{P}(A)+\text{P}(B)-\text{P}(A\cap B)\right]$ $=1-\left[a+b-ab\right]$ $=(1-a)(1-b)$ $=\text{P}(A')\text{P}(B')$ $\therefore$ A' and B' are independent events. (shown)
(b) **[2]**	$\text{P}(A\cup B)=0.7$ $\text{P}(A)+\text{P}(B)-\text{P}(A\cap B)=0.7$ $a+b-ab=0.7$ $a+0.5-0.5a=0.7$ $0.5a=0.2$ $a=0.4$ (shown)
(c) **[2]**	Given $\text{P}(A)=0.4, \text{P}(B)=0.5, \text{P}(C)=0.3$ Since A and B are independent events, $\text{P}(A\cap B)=0.4\times 0.5=0.2$ Since A and C are independent events, $\text{P}(A\cap C)=0.4\times 0.3=0.12$ Events B and C are mutually exclusive:  $\text{P}(A'\cap B'\cap C')=1-\text{P}(A)-0.18-0.3=1-0.4-0.18-0.3=0.12$
(d) **[3]**	Given $\text{P}(A)=0.4, \text{P}(B)=0.5, \text{P}(C)=0.3$ Since A and B are independent events, $\text{P}(A\cap B)=0.4\times 0.5=0.2$ Since A and C are independent events, $\text{P}(A\cap C)=0.4\times 0.3=0.12$

	Given events B and C are not mutually exclusive,  $\mathrm{P}(A'\cap B'\cap C')=1-\mathrm{P}(A)-0.18-(0.3-x)=0.12+x$ Since probabilities are all non-negative, $0\le x\le 0.18$. When x is greatest, i.e. when $x=0.18$, $\mathrm{P}(A'\cap B'\cap C')$ is greatest. $\therefore$ Greatest possible value of $\mathrm{P}(A'\cap B'\cap C')$ is $0.12+0.18=0.3$. OR: $\mathrm{P}(A'\cap B'\cap C')=1-\mathrm{P}(A\cup B\cup C)$ To maximize $\mathrm{P}(A'\cap B'\cap C')$, we need to **minimize** $\mathrm{P}(A\cup B\cup C)$. Since B and C are not mutually exclusive, we need to have $C\subseteq A\cup B$ (see diagram below). Max $\mathrm{P}(A'\cap B'\cap C')$ $=1-\mathrm{P}(A\cup B\cup C)$ $=1-\mathrm{P}(A\cup B)$ $=1-0.7$ $=0.3$ 

8	**Solutions**
(a) **[1]**	A random sample is a sample chosen in such a way that 1) every student in the school has an <u>equal</u> chance of being selected, and 2) the <u>selection</u> of one student is <u>independent</u> of the selection of any other student from being selected.
(b) **[2]**	Let $y = x - 45.5$ Then $\overline{y} = \overline{x} - 45.5$ $\overline{x} = \overline{y} + 45.5 = \dfrac{\sum y}{50} + 45.5 = 46.6$ ${s_x}^2 = {s_y}^2 = \dfrac{1}{49}\left[\sum y^2 - \dfrac{\left(\sum y\right)^2}{50}\right] = \dfrac{35856}{1225} = 29.270 = 29.3$ (3 s.f) Unbiased estimates of population mean and variance of time spent by students are 46.6 hours and 29.3 hours^2.
(c) **[4]**	Let X hours be the time spent by a student per week on social media platforms. Let μ represent the population mean time spent by students per week on social media platforms. $\mathrm{H}_0: \mu = 45.5$ $\mathrm{H}_1: \mu \neq 45.5$ We perform a 2-tail test at 5% level of significance. Under H_0, since sample size 50 is large, by Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(45.5, \dfrac{35856}{1225(50)}\right)$ approximately Reject H_0 if $\overline{x} \leq 44.0$ or $\overline{x} \geq 47.0$  So the critical region (represented by the shaded regions in diagram above) is $\left\{\overline{x} \in \mathbb{R} : \overline{x} \leq 44.0 \text{ or } \overline{x} \geq 47.0\right\}$ Or $\left\{\overline{x} \in \mathbb{R} : 0 \leq \overline{x} \leq 44.0 \text{ or } 47.0 \leq \overline{x} \leq 168\right\}$ Since $\overline{x} = 46.6 \notin \left\{\overline{x} \in \mathbb{R} : \overline{x} \leq 44.0 \text{ or } \overline{x} \geq 47.0\right\}$, we do not reject H_0 and conclude that there is insufficient evidence, at the 5% significance level, that the mean time spent by students per week on social media platforms differs from 45.5 minutes.

(d) **[1]**	It is not necessary to assume that the time spent by students per week on social media platforms follows a normal distribution in part (c). Since the sample size is large, Central Limit Theorem can be applied such that the distribution of the **sample mean time** spent by students per week on social media platforms is approximately normal.
(e) **[4]**	Null Hypothesis $\mathrm{H}_0: \ \mu = 45.5$ Alternative Hypothesis $\mathrm{H}_1: \ \mu > 45.5$ Perform a 1-tail test at α% significance level. Under H_0, $\bar{X} \sim \mathrm{N}\left(45.5, \frac{25}{50} = \frac{1}{2}\right)$. Using a z-test, p-value $= \mathrm{P}\left(\bar{X} \geq 47\right) = 0.016947$ (5 s.f) If H_0 is rejected, $p\text{-value} \leq \frac{\alpha}{100}$ $0.016947 \leq \frac{\alpha}{100}$ $\alpha \geq 1.69$ Hence the required set of values of α for which Ms Tan's belief should be accepted is [1.69, 100].

9	Solutions
(a) **[2]**	
(b) **[2]**	The product moment correlation coefficient between T and H is $r = -0.948$ (3 s.f.). Since r is close to -1, there is a strong negative linear correlation between T and H. The temperature decreases as the altitude increases.
(c) **[2]**	The readings taken during descending are H: 2300, 2626, 2700, 3131, 3450 T: 11.80, 10.29, 9.52, 7.26, 4.43 From GC, $a = 26.7$ (3 s.f.), $b = -0.00633$ (3 s.f.).
(d) **[1]**	None of the regression lines is suitable. This is because 26 degree Celsius is outside the data range for T.
(e) **[2]**	$F = \frac{9}{5}\left(23.1 - 0.00574H\right) + 32$ $= 73.58 - 0.010332H$ $\therefore c = 73.58$ and $d = -0.010332$.
(f) **[3]**	When $H = 2800$, $F = 44.6504$

	The estimate is reliable because the value $H = 2800$ is within the data range of $2390 \leq H \leq 3580$ and the product moment correlation coefficient $r = -0.999$ is close to -1 indicating a strong negative linear correlation between F and H.

10	Solutions
(a) [3]	$\mathrm{P}(R_2=1)=\mathrm{P}$ (both scores are odd numbers) $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$ $\mathrm{P}(R_2=0)=1-\frac{1}{4}=\frac{3}{4}$

OR:

R_2	**1**	**2**	**3**	**4**	**5**	**6**
1	1	0	1	0	1	0
2	0	0	0	0	0	0
3	1	0	1	0	1	0
4	0	0	0	0	0	0
5	1	0	1	0	1	0
6	0	0	0	0	0	0

OR:

$D_1\times D_2$	**1**	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

$\mathrm{P}(R_2=1)=\frac{9}{36}=\frac{1}{4}$, $\mathrm{P}(R_2=0)=\frac{27}{36}=\frac{3}{4}$

Probability distribution table for R_2:

r_2	0	1
$\mathrm{P}(R_2=r_2)$	$\frac{3}{4}$	$\frac{1}{4}$

$\mathrm{E}(R_2)=0\times\frac{3}{4}+1\times\frac{1}{4}=\frac{1}{4}$

(b) [1]

$R_6=0$ when the product is divisible by 6.

R_6	**1**	**2**	**3**	**4**	**5**	**6**
1						0
2			0			0
3		0		0		0
4			0			0
5						0
6	0	0	0	0	0	0

Alternatively,

$D_1\times D_2$	**1**	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

$\mathrm{P}(R_6=0)=\mathrm{P}(\text{products divisible by 6})=\frac{15}{36}=\frac{5}{12}$ (shown)

OR

$$\mathrm{P}(R_6 = 0)$$
$$= \mathrm{P}(\text{one die shows 6}) + \mathrm{P}(\text{one die shows 3, another shows 2 or 4})$$
$$= 1 - \mathrm{P}(\text{both no 6}) + 2\left(\frac{1}{6}\times\frac{2}{6}\right)$$
$$= 1 - \frac{5}{6}\times\frac{5}{6} + 2\left(\frac{1}{6}\times\frac{2}{6}\right) = \frac{5}{12}$$

(c)
[4]

$$X \sim \mathrm{B}\left(m, \frac{5}{12}\right)$$

In MF26

Distribution of X	$\mathrm{P}(X = x)$
Binomial $\mathrm{B}(n,p)$	$\binom{n}{x} p^x (1-p)^{n-x}$

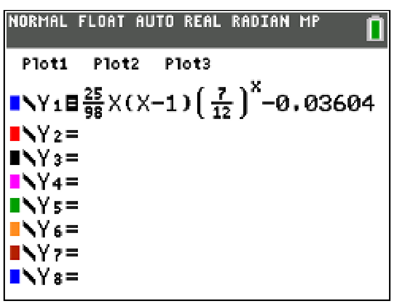
$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

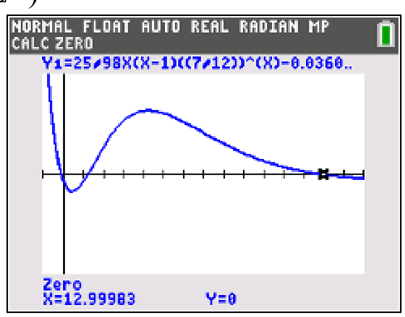
$$\mathrm{P}(X = 2) = \frac{m!}{2!(m-2)!}\left(\frac{5}{12}\right)^2\left(\frac{7}{12}\right)^{m-2}$$
$$= \frac{m(m-1)}{2}\left(\frac{5}{12}\right)^2\left(\frac{7}{12}\right)^m\left(\frac{12}{7}\right)^2$$
$$= \frac{25}{98}m(m-1)\left(\frac{7}{12}\right)^m \quad \text{where } A = \frac{25}{98} \ (\text{shown})$$

Given $\frac{25}{98}m(m-1)\left(\frac{7}{12}\right)^m = 0.03604$

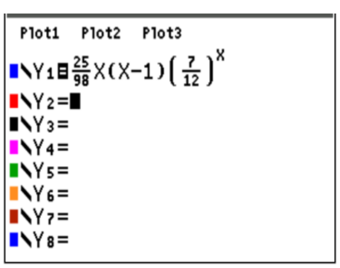
From GC, $m = -0.11850,\ 1.2233$ or 13.000

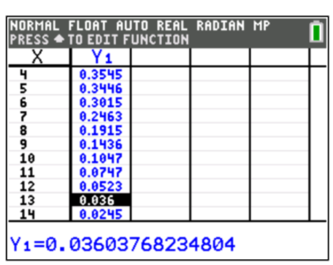
Since $m \geq 2$, $m = 13$ (Note: $m \in \mathbb{Z}^+$)





<u>OR</u> Since m is an integer, we can choose to use table to find the integer m that best satisfies the equation.





m	$\mathrm{P}(X=2)$
12	0.05227
13	0.03604, which is closest to 0.03604
14	0.02453

Thus m = 13 since m is an integer.

(d)
[5]

Let C and D denote the number of products that are divisible by 6 obtained in 10 throws of 2 fair dice by a student from Class C and a student from class D respectively.

$C, D \sim \mathrm{B}\left(10, \frac{5}{12}\right)$

From MF26

Distribution of X	$\mathrm{P}(X=x)$	Mean	Variance
Binomial $\mathrm{B}(n,p)$	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$

$\mathrm{E}(C) = \mathrm{E}(D) = 10 \times \frac{5}{12} = \frac{25}{6}$ or 4.1667 (5sf)

$\mathrm{Var}(C) = \mathrm{Var}(D) = 10 \times \frac{5}{12} \times \frac{7}{12} = \frac{175}{72}$ or 2.4306 (5sf)

Since sample sizes, 30 and 35 are large, by Central Limit Theorem,

$\overline{C} \sim \mathrm{N}\left(\frac{25}{6}, \frac{175}{72(30)} = \frac{35}{432}\right)$ approximately and

$\overline{D} \sim \mathrm{N}\left(\frac{25}{6}, \frac{175}{72(35)} = \frac{5}{72}\right)$ approximately.

$\therefore \overline{C} - \overline{D} \sim \mathrm{N}\left(0, \frac{35}{432} + \frac{5}{72} = \frac{65}{432}\right)$ approximately.

$\mathrm{P}\left(\overline{C} - \overline{D} > 0.2\right) = 0.303$ (3sf)

OR (Less preferred)

Let T_c and T_D be the total number of products divisible by 6 for class C and class D respectively.

$T_c \sim \mathrm{B}\left(350, \frac{5}{12}\right)$ and $T_D \sim \mathrm{B}\left(300, \frac{5}{12}\right)$.

Now since sample sizes of 350 and 300 are both large, by Central Limit Theorem,

$T_c \sim \mathrm{N}\left(350\left(\tfrac{5}{12}\right), 350\left(\tfrac{5}{12}\right)\left(\tfrac{7}{12}\right)\right)$ approximately, and

$T_D \sim \mathrm{N}\left(300\left(\tfrac{5}{12}\right), 300\left(\tfrac{5}{12}\right)\left(\tfrac{7}{12}\right)\right)$ approximately

$\mathrm{E}\left(\frac{T_C}{35} - \frac{T_D}{30}\right) = 0$ and $\mathrm{Var}\left(\frac{T_C}{35} - \frac{T_D}{30}\right) = \frac{1}{35^2}\mathrm{Var}(T_C) + \frac{1}{30^2}\mathrm{Var}(T_D)$

	$\frac{T_C}{35}-\frac{T_D}{30} \sim \mathrm{N}\left(0, \frac{65}{432}\right)$ approximately $\mathrm{P}\left(\frac{T_C}{35}-\frac{T_D}{30}>0.2\right)=0.303$ (3sf)