PU3 MATHEMATICS

Paper 9758/02

Section A: Pure Mathematics

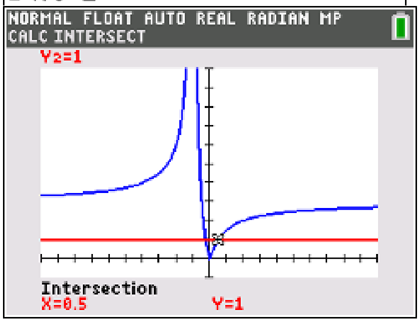
Qn	Solution
1(i) **[2]**	$y = vx^2$ ------(1) $\frac{\mathrm{d}y}{\mathrm{d}x} = v(2x) + \frac{\mathrm{d}v}{\mathrm{d}x}(x^2)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 2vx + x^2\frac{\mathrm{d}v}{\mathrm{d}x}$ ------(2) Subst (1) and (2) into $x\frac{\mathrm{d}y}{\mathrm{d}x} = x^3 + xy + 2y$, $x\left(2vx + x^2\frac{\mathrm{d}v}{\mathrm{d}x}\right) = x^3 + x(vx^2) + 2(vx^2)$ $2vx^2 + x^3\frac{\mathrm{d}v}{\mathrm{d}x} = x^3 + vx^3 + 2vx^2$ $x^3\frac{\mathrm{d}v}{\mathrm{d}x} = x^3 + vx^3$ $\frac{\mathrm{d}v}{\mathrm{d}x} = 1 + v$ (shown)
1(ii) **[3]**	**<u>Method 1</u>** $\frac{\mathrm{d}v}{\mathrm{d}x} = 1 + v$ $\int \frac{1}{1+v}\,\mathrm{d}v = \int 1\,\mathrm{d}x$ $\ln\lvert 1+v\rvert = x + c$ $\lvert 1+v\rvert = \mathrm{e}^{x+c}$ $1 + v = \pm\mathrm{e}^{x+c}$ $1 + v = A\mathrm{e}^x$ where $A = \pm\mathrm{e}^c$ $1 + \frac{y}{x^2} = A\mathrm{e}^x$ Subst $y = 1$ and $x = 1$, $1 + \frac{1}{1^2} = A\mathrm{e}$ $A = \frac{2}{\mathrm{e}}$ $1 + \frac{y}{x^2} = \left(\frac{2}{\mathrm{e}}\right)\mathrm{e}^x$ $1 + \frac{y}{x^2} = 2\mathrm{e}^{x-1}$

Qn	Solution
	<u>Method 2</u> $\frac{\mathrm{d}v}{\mathrm{d}x}=1+v$ $\int \frac{1}{1+v}\,\mathrm{d}v=\int 1\,\mathrm{d}x$ $\ln\lvert 1+v\rvert = x+c$ $\ln\left\lvert 1+\frac{y}{x^2}\right\rvert = x+c$ Subst $y=1$ and $x=1$, $\ln\left\lvert 1+\frac{1}{1^2}\right\rvert = 1+c$ $c=\ln 2-1$ $\ln\left\lvert 1+\frac{y}{x^2}\right\rvert = x+\ln 2-1$

Qn	Solution
2(a) **[5]**	$\mathrm{f}(x)=\sec^2 x$ $\mathrm{f}'(x)=2\sec x(\sec x\tan x)$ $\quad =2\sec^2 x\tan x$ $\mathrm{f}''(x)=2\left[\sec^2 x\left(\sec^2 x\right)+(\tan x)\left(2\sec x(\sec x\tan x)\right)\right]$ $\quad =2\sec^4 x+4\sec^2 x\tan^2 x$ When $x=0$, $\mathrm{f}(0)=1$ $\mathrm{f}'(x)=0$ $\mathrm{f}''(0)=2$ $\therefore\ \mathrm{f}(x)=1+0x+\frac{2}{2!}x^2+\ldots$ $\quad =1+x^2+\ldots$

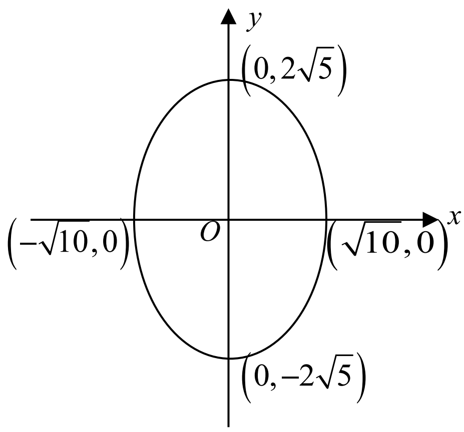
2(b) **[3]**	$(1+ax)^{-4}=1+(-4)(ax)+\frac{(-4)(-5)}{2!}(ax)^2+\ldots$ $\quad =1-4ax+10a^2x^2+\ldots$ Since the coefficients of the x and x^2 terms in the expansion are equal, $-4a=10a^2$ $10a^2+4a=0$ $2a(5a+2)=0$ $a=0$ (rejected since $a\neq 0$) or $a=-\frac{2}{5}$

Qn	Solution
3(a) **[3]**	Method 1 (most used this) $S_N=u_2+u_3+u_4\ldots+u_{N-1}+u_N$ $=\sum_{n=2}^{N}u_n$ $=\sum_{n=2}^{N}\ln\left(\frac{n-1}{n+1}\right)$ $=\sum_{n=2}^{N}\left(\ln(n-1)-\ln(n+1)\right)$ $=\ln 1-\ln 3$ $+\ln 2-\ln 4$ $+\ln 3-\ln 5$ $\vdots$ $+\ln(N-3)-\ln(N-1)$ $+\ln(N-2)-\ln N$ $+\ln(N-1)-\ln(N+1)$ $=\ln 1+\ln 2-\ln N-\ln(N+1)$ $=\ln 2-\ln\left(N(N+1)\right)$ Method 2 $S_N=u_2+u_3+u_4\ldots+u_{N-1}+u_N$ $=\ln\left(\frac{1}{3}\right)+\ln\left(\frac{2}{4}\right)+\ln\left(\frac{3}{5}\right)+\ldots+\ln\left(\frac{N-2}{N}\right)+\ln\left(\frac{N-1}{N+1}\right)$ $=\ln\left(\frac{1\times 2\times \cancel{3\times\ldots\times(N-2)\times(N-1)}}{\cancel{3\times 4\times 5\times\ldots\times(N-1)}\times(N)\times(N+1)}\right)$ $=\ln\left(\frac{2}{N(N+1)}\right)$ $=\ln 2-\ln\left(N(N+1)\right)$

Qn	Solution
3(b) **(i)** **[3]**	$\sum_{n=1}^{\infty}\left(\frac{3x}{x+1}\right)^n=\left(\frac{3x}{x+1}\right)^1+\left(\frac{3x}{x+1}\right)^2+\left(\frac{3x}{x+1}\right)^3+\ldots$ is a geometric series with $r=\frac{3x}{x+1}$. If the series has a finite sum, S_∞ exists and $\lvert r\rvert<1$. $\lvert r\rvert<1$ $\left\lvert\frac{3x}{x+1}\right\rvert<1$ <u>Method 1 (Graphical):</u>   $-0.25<x<0.5$ <u>Method 2 (Algebraic):</u> $\left\lvert\frac{3x}{x+1}\right\rvert<1$ $\lvert 3x\rvert<\lvert x+1\rvert$ $(3x)^2<(x+1)^2$ $9x^2<x^2+2x+1$ $8x^2-2x-1<0$ $(4x+1)(2x-1)<0$ $-0.25<x<0.5$

Qn	Solution
3(b) (ii) [2]	Method 1 $\sum_{n=1}^{\infty}\left(\frac{3x}{x+1}\right)^n=\sum_{n=1}^{\infty}\left(\frac{3(0.25)}{0.25+1}\right)^n$ $=\sum_{n=1}^{\infty}\left(\frac{3}{5}\right)^n$ $=\left(\frac{3}{5}\right)^1+\left(\frac{3}{5}\right)^2+\left(\frac{3}{5}\right)^3+\ldots$ $=\frac{\frac{3}{5}}{1-\frac{3}{5}}$ $=\frac{3}{2}$ Method 2 $\sum_{n=1}^{\infty}\left(\frac{3x}{x+1}\right)^n=\frac{\frac{3x}{x+1}}{1-\frac{3x}{x+1}}$ $=\frac{\frac{3x}{x+1}}{\frac{1-2x}{x+1}}$ $=\frac{3x}{1-2x}$ $=\frac{3(0.25)}{1-2(0.25)}$ since $x=0.25$ $=1.5$

Qn	Solution
4(i) [2]	$2x^2+y^2=20$ $\frac{x^2}{10}+\frac{y^2}{20}=1$ $\frac{x^2}{\left(\sqrt{10}\right)^2}+\frac{y^2}{\left(\sqrt{20}\right)^2}=1$ $\frac{x^2}{\left(\sqrt{10}\right)^2}+\frac{y^2}{\left(2\sqrt{5}\right)^2}=1$

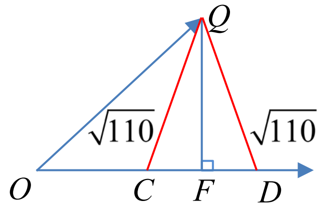
Qn	Solution
	
4(ii) **[1]**	$2x^2+y^2=20$ $4x+2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ $2y\dfrac{\mathrm{d}y}{\mathrm{d}x}=-4x$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{4x}{2y}=-\dfrac{2x}{y}$ (shown)
4(iii) **[5]**	<u>Method 1</u> Gradient of normal $=\dfrac{y}{2x}$ At (a,b), Gradient of normal $=\dfrac{b}{2a}$ $\dfrac{b}{2a}=\dfrac{b-0}{a-1}$ $ab-b=2ab$ $ab+b=0$ $b(a+1)=0$ $b=0$ or $a=-1$

Qn	Solution
	When $b=0$, $2a^2+(0)^2=20$ $a=-\sqrt{10}$ or $\sqrt{10}$ or When $a=-1$, $2(-1)^2+b^2=20$ $b^2=18$ $b=-\sqrt{18}$ or $\sqrt{18}$ $b=-3\sqrt{2}$ or $3\sqrt{2}$ The four coordinates are $(\sqrt{10},0),(-\sqrt{10},0),(-1,-3\sqrt{2})$ and $(-1,3\sqrt{2})$. <u>Method 2</u> Gradient of normal $=\dfrac{y}{2x}$ At (a,b), Gradient of normal $=\dfrac{b}{2a}$ Equation of normal at $P: y-b=\dfrac{b}{2a}(x-a)$ $y=\dfrac{b}{2a}x-\dfrac{b}{2}+b$ $y=\dfrac{b}{2a}x+\dfrac{b}{2}$ Since normal passes through (1, 0), $0=\dfrac{b}{2a}(1)+\dfrac{b}{2}$ $0=b+ab$ $b(1+a)=0$ $b=0$ or $a=-1$

Qn	Solution
	When $b=0$, $2a^2+(0)^2=20$ $a=-\sqrt{10}$ or $\sqrt{10}$ or When $a=-1$, $2(-1)^2+b^2=20$ $b^2=18$ $b=-\sqrt{18}$ or $\sqrt{18}$ $b=-3\sqrt{2}$ or $3\sqrt{2}$ The four coordinates are $\left(\sqrt{10},0\right),\left(-\sqrt{10},0\right),\left(-1,-3\sqrt{2}\right)$ and $\left(-1,3\sqrt{2}\right)$.

Qn	Solution
5(i) **[2]**	**<u>Method 1</u>** Perpendicular distance from the origin to p $=\left\lvert\dfrac{4}{\sqrt{1^2+2^2+(-5)^2}}\right\rvert$ $=\dfrac{4}{\sqrt{30}}$ units --- **<u>Method 2</u>** Note that $A(4, 0, 0)$ lies on p. Perpendicular distance from the origin to p $=\left\lvert\begin{pmatrix}4\\0\\0\end{pmatrix}\bullet\dfrac{\begin{pmatrix}1\\2\\-5\end{pmatrix}}{\sqrt{1^2+2^2+(-5)^2}}\right\rvert=\dfrac{4}{\sqrt{30}}$ units
5(ii) **[3]**	**<u>Method 1</u>** l lies on p means l is parallel to p and a point on l lies on p. l is parallel to p means direction vector of l is perpendicular to normal of p:

Qn	Solution
	$\begin{pmatrix}1\\2\\-5\end{pmatrix}\bullet\begin{pmatrix}a\\2a\\2\end{pmatrix}=0$ $a+4a-10=0$ $a=\frac{10}{5}=2$ (shown) We need $\begin{pmatrix}3\\-2\\b\end{pmatrix}$ to lie on p: $\begin{pmatrix}3\\-2\\b\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-5\end{pmatrix}=4$ $3-4-5b=4$ $5b=-5$ $b=-1$ (shown) **Method 2** Equation of plane p: $x+2y-5z=4 \Rightarrow r\bullet\begin{pmatrix}1\\2\\-5\end{pmatrix}=4$ Sub $\mathbf{r}=\begin{pmatrix}3\\-2\\b\end{pmatrix}+\lambda\begin{pmatrix}a\\2a\\2\end{pmatrix}$ into $r\bullet\begin{pmatrix}1\\2\\-5\end{pmatrix}=4$: $\begin{pmatrix}3+a\lambda\\-2+2a\lambda\\b+2\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-5\end{pmatrix}=4$ $3+a\lambda-4+4a\lambda-5b-10\lambda=4$ $5a\lambda-10\lambda-5-5b=0$ $(5a-10)\lambda+(-5-5b)=0$ Comparing coefficient of λ: $5a-10=0 \Rightarrow a=\frac{10}{5}=2$ (shown) Comparing constants: $-5-5b=0 \Rightarrow b=-1$ (shown)
5(iii) **[5]**	Let $\overrightarrow{OR}=\begin{pmatrix}3\\-2\\-1\end{pmatrix}+\lambda\begin{pmatrix}2\\4\\2\end{pmatrix}$ be the position vector of a point on l for some $\lambda\in\mathbb{R}$. $\overrightarrow{QR}=\left[\begin{pmatrix}3\\-2\\-1\end{pmatrix}+\lambda\begin{pmatrix}2\\4\\2\end{pmatrix}\right]-\begin{pmatrix}-4\\7\\3\end{pmatrix}=\begin{pmatrix}7+2\lambda\\-9+4\lambda\\-4+2\lambda\end{pmatrix}$

Qn	Solution
	$\left\|\overrightarrow{QR}\right\| = \sqrt{(7+2\lambda)^2+(-9+4\lambda)^2+(-4+2\lambda)^2} = \sqrt{110}$ $49+28\lambda+4\lambda^2+81-72\lambda+16\lambda^2+16-16\lambda+4\lambda^2=110$ $24\lambda^2-60\lambda+36=0$ $2\lambda^2-5\lambda+3=0$ $(2\lambda-3)(\lambda-1)=0$ $(2\lambda-3)=0$ or $(\lambda-1)=0$ $\lambda=\frac{3}{2}$ or $\lambda=1$ When $\lambda=\frac{3}{2}$, $\overrightarrow{OC}=\begin{pmatrix}3\\-2\\-1\end{pmatrix}+\frac{3}{2}\begin{pmatrix}2\\4\\2\end{pmatrix}=\begin{pmatrix}6\\4\\2\end{pmatrix}$ When $\lambda=1$, $\overrightarrow{OD}=\begin{pmatrix}3\\-2\\-1\end{pmatrix}+1\begin{pmatrix}2\\4\\2\end{pmatrix}=\begin{pmatrix}5\\2\\1\end{pmatrix}$
5(iv) **[1]**	Let F be the foot of perpendicular from Q to l. Then QC and QD are the hypothenuses of the right-angled triangles QFC and QFD respectively. Therefore the shortest distance from Q to l, QF, has to be smaller than QC and QD, which is $\sqrt{110}$.  OR: Assume the shortest distance from Q to l is greater than $\sqrt{110}$. But there are points C and D found on l that are a distance of $\sqrt{110}$ away from Q, which contradicts the original assumption. Therefore the shortest distance from Q to l, has to be smaller than $\sqrt{110}$. OR: There should only be one unique point on l, the foot of perpendicular, that is the shortest distance from Q. Currently there are 2 points C and D found on l that are a distance of $\sqrt{110}$ away from Q. As the distance is shortened, C and D will converge to the foot of perpendicular.

Section B: Probability and Statistics

Qn	Solution

6(i) [2]

	1	2	3	4	5
1	X	3	4	5	6
2	3	X	5	6	7
3	4	5	X	7	8
4	5	6	7	X	9
5	6	7	8	9	X

$\mathrm{P}(X=3)=2\times\frac{1}{5}\times\frac{1}{4}=\frac{1}{10}$ $\qquad$ $\mathrm{P}(X=7)=4\times\frac{1}{5}\times\frac{1}{4}=\frac{1}{5}$

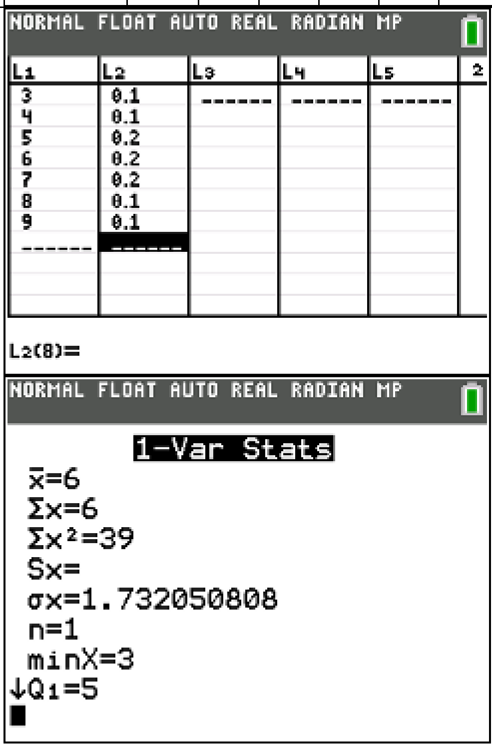
$\mathrm{P}(X=4)=2\times\frac{1}{5}\times\frac{1}{4}=\frac{1}{10}$ $\qquad$ $\mathrm{P}(X=8)=2\times\frac{1}{5}\times\frac{1}{4}=\frac{1}{10}$

$\mathrm{P}(X=5)=4\times\frac{1}{5}\times\frac{1}{4}=\frac{1}{5}$ $\qquad$ $\mathrm{P}(X=9)=2\times\frac{1}{5}\times\frac{1}{4}=\frac{1}{10}$

$\mathrm{P}(X=6)=4\times\frac{1}{5}\times\frac{1}{4}=\frac{1}{5}$

x	3	4	5	6	7	8	9
$\mathrm{P}(X=x)$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$

6(ii) [2]



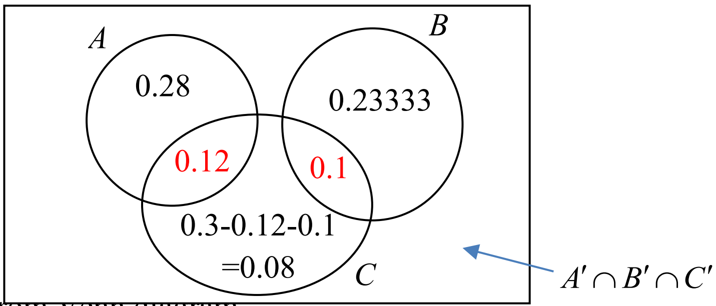
$\mathrm{E}(X)=6$

$\mathrm{Var}(X)=1.7320^2$

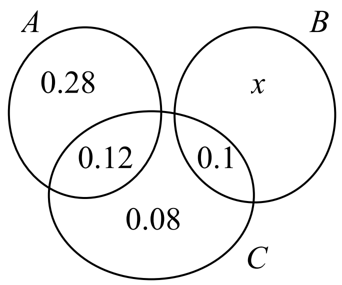
$\qquad = 2.9998 \approx 3.00$ (3 s.f.)

Qn	Solution
7(i) **[1]**	$X \sim \mathrm{B}(28, 0.96)$ $\mathrm{P}(X = 28) = 0.31886$ $= 0.319$ (3 s.f.)
7(ii) **[1]**	x | $\mathrm{P}(X = x)$ 26 | 0.20925 27 | 0.37200 28 | 0.31886 The most likely number of students who are present in school on a typical school day is 27.
7(iii) **[2]**	**Method 1** If there are more than 2 students not present, it means that there are less than 26 students present. $\mathrm{P}(X < 26) = \mathrm{P}(X \le 25)$ $= 0.099896$ (5 s.f.) $= 0.0999$ (3 s.f.) **Method 2** Let Y be the number of students who are not present in school, out of 28. Then $Y \sim \mathrm{B}(28, 0.04)$. $\mathrm{P}(Y > 2) = 1 - \mathrm{P}(Y \le 2)$ $= 0.099896$ (5 s.f.) $= 0.0999$ (3 s.f.)
7(iv) **[3]**	Let W be the number of classes where there was an investigation, out of 16. Then $W \sim \mathrm{B}(16, 0.099896)$. $\mathrm{P}\left(W \ge \frac{16}{3}\right) = \mathrm{P}(W \ge 6)$ $= 1 - \mathrm{P}(W \le 5)$ $= 0.0032794$ (5 s.f.) $= 0.00328$ (3 s.f.)

Qn	Solution
8(i) **[1]**	Since A and B are mutually exclusive, $\mathrm{P}(A \cap B) = 0$
8(ii) **[1]**	Since A and C are independent,

Qn	Solution
	$\mathrm{P}(A \mid C)=\mathrm{P}(A)=0.4$
8(iii) **[3]**	Since A and C are independent, $\mathrm{P}(A\cap C)=\mathrm{P}(A)\times\mathrm{P}(C)$ $=0.4\times0.3$ $=0.12$ Since B and C are independent, $\mathrm{P}(B)\times\mathrm{P}(C)=\mathrm{P}(B\cap C)$ $\mathrm{P}(B)\times0.3=0.1$ $\mathrm{P}(B)=\frac{1}{3}$  From Venn diagram, $\mathrm{P}(A'\cap B'\cap C')=1-0.4-\frac{1}{3}-0.08$ $=0.18667$ $=0.187$ (3 s.f.)

8(iv)
[2]



$$y = 1 - 0.28 - 0.12 - 0.08 - 0.1 - x$$
$$= 0.42 - x$$

<u>Method 1</u>

Let $\mathrm{P}(\mathrm{B}) = 0.1 + x$

For least $\mathrm{P}(B)$, let $x = 0$. So least $\mathrm{P}(B) = 0.1$

Let $\mathrm{P}(A' \cap B' \cap C') = y$.

$$y = 1 - 0.28 - 0.12 - 0.08 - 0.1 - x$$
$$= 0.42 - x$$

For greatest $\mathrm{P}(B)$, let $y = 0$.

Then $0.42 - x = 0 \Rightarrow x = 0.42$

So greatest $\mathrm{P}(B) = 0.1 + 0.42 = 0.52$

<u>Method 2</u>

$\mathrm{P}(A' \cap B' \cap C') = y$.

$$y = 1 - 0.28 - 0.12 - 0.08 - 0.1 - x$$
$$= 0.42 - x$$

$x \geq 0$ and $y \geq 0$

$x \geq 0$ and $0.42 - x \geq 0$

$x \geq 0$ and $x \leq 0.42$

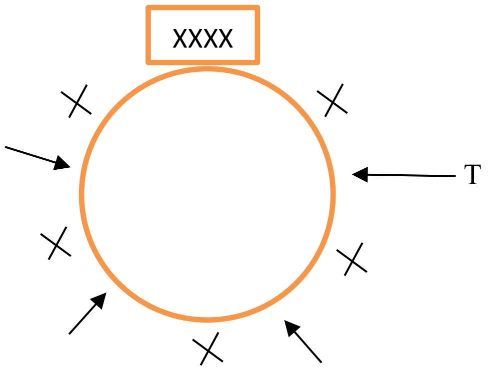
$0 \leq x \leq 0.42$

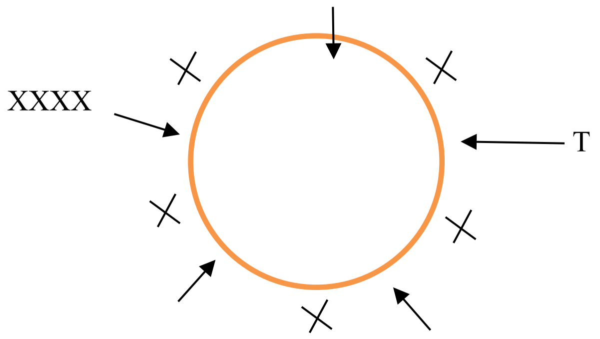
Since $\mathrm{P}(\mathrm{B}) = 0.1 + x$

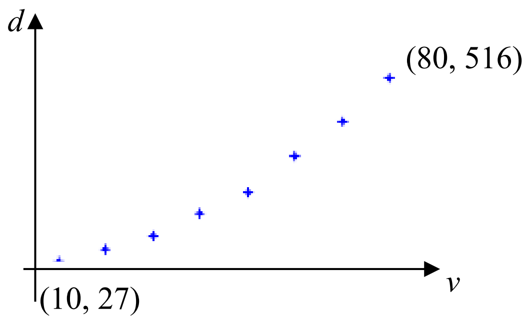
$0 + 0.1 \leq \mathrm{P}(\mathrm{B}) \leq 0.42 + 0.1$

$0.1 \leq \mathrm{P}(\mathrm{B}) \leq 0.52$

So least $\mathrm{P}(B) = 0.1$, greatest $\mathrm{P}(B) = 0.52$

9(i) **[1]**	Required number of ways $= {}^{20}C_4 \times 4!$ $= 116280$
9(ii) **[3]**	**Method 1: Complement** Number of ways for all males $= {}^{12}C_4 \times 4! = 11880$ Number of ways for all females $= {}^{8}C_4 \times 4! = 1680$ Required number of ways $= 116280 - 11880 - 1680$ $= 102720$ **Method 2: Direct cases** Number of ways for 1M3F $= {}^{12}C_1 \times {}^{8}C_3 \times 4! = 16128$ Number of ways for 2M2F $= {}^{12}C_2 \times {}^{8}C_2 \times 4! = 44352$ Number of ways for 3M1F $= {}^{12}C_3 \times {}^{8}C_1 \times 4! = 42240$ Total number of ways $= 16128 + 44352 + 42240$ $= 102720$
9(iii) **[3]**	**Method 1: Insert form teacher**  Required Probability $= \dfrac{(6-1)! \times {}^{4}C_1 \times 4!}{(10-1)!}$ $= 0.031746$ $= 0.0317$ (3 s.f.)

	<u>Method 2: Insert teacher and 2 executive committee members</u>  Required Probability $= \dfrac{(5-1)! \times {}^5C_2 \times 2! \times 4!}{(10-1)!}$ $= 0.031746$ $= 0.0317$ (3 s.f.)

Qn	Solution
10(i) **[1]**	
10(ii) **[4]**	$d = av + b$: r-value = 0.97883 $\ln d = av + b$: r-value = 0.98315 Since the r-value for the model $\ln d = av + b$ is <u>closer to 1</u>, $\ln d = av + b$ is the better model. Using GC, $\ln d = 0.040564v + 3.2116$ $\ln d = 0.0406v + 3.21$ (3 s.f.)
10(iii) **[2]**	$\ln d = 0.040564(64) + 3.2116$ $\ln d = 5.807696$ $d = 332.85$ (5 s.f.) The estimated braking distance is 333 feet. Since <u>r is close to 1</u> and <u>$v = 64$ is within the given data range of v</u> $(10 \le v \le 80)$, the estimate is reliable.

Qn	Solution
10(iv) **[2]**	1 kilometre = 0.621 miles Then v (in miles/h) = $0.621m$ (in km/h). $\ln d = 0.040564(0.621m)+3.2116$ $\ln d = 0.025190m+3.2116$ (5 s.f.) $\ln d = 0.0252m+3.21$(3 s.f.)

Qn	Solution
11(i) **[1]**	Each scented candle has an **equal chance** of being **selected**. The **selection** of the scented candles is **independent** of each other.
11(ii) **[2]**	Unbiased estimates of the population mean, $\bar{x}$ $=\dfrac{69}{30}+600$ $=602.3$ Unbiased estimates of the population variance, s^2 $=\dfrac{1}{29}\left(859-\dfrac{69^2}{30}\right)$ $=24.148$ $\approx 24.1(3\text{ s.f.})$
11(iii) **[5]**	Let X be the mass, in grams, of a scented candle. Let μ be the **population mean mass** of the scented candles. $\text{H}_0 : \mu = 600$ $\text{H}_1 : \mu \neq 600$ Under H_0, since $n = 30$ is large, by Central Limit Theorem, $\bar{X} \sim \text{N}\left(600, \dfrac{24.148}{30}\right)$ approximately. Use z test at $\alpha = 0.01$. Using GC, p-value = 0.010359 > 0.01. Do not reject H_0. There is insufficient evidence at 1% level of significance to conclude that the population mean mass of scented candles is not 600 grams.
11(iv) **[1]**	The p-value of 0.0104 is the probability of observing from a sample, a **sample mean value at least as extreme as 602.3g**, under the **assumption that the population mean mass is 600g**.

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>11(v)
[4]</td><td>

$H_0 : \mu = 600$

$H_1 : \mu < 600$

Under H_0, $\bar{X} \sim N\left(600, \frac{25}{n}\right)$.

Test statistics $Z = \frac{\bar{X} - 600}{\sqrt{\frac{25}{n}}}, Z \sim N(0,1)$

Use z test at $\alpha = 0.05$.

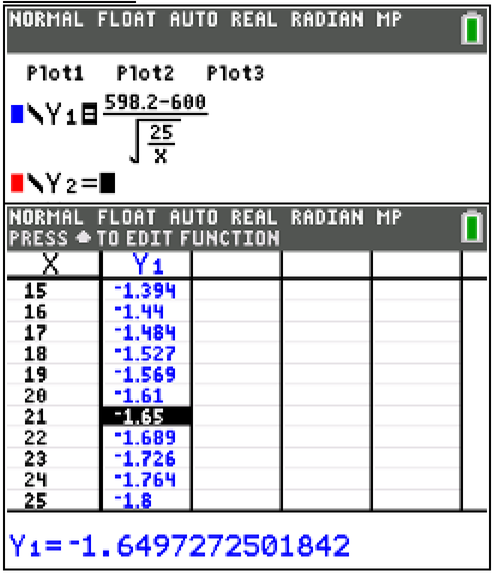
Test statistic value $z = \frac{598.2 - 600}{\sqrt{\frac{25}{n}}}$

Critical value: -1.6448

Critical region: $z \leq -1.6448$

Given conclusion: "The quality inspector concludes that he has overstated the mean mass of the scented candles" implies H_0 is rejected.

<u>Method 1:</u>



$n \geq 21$

<u>Method 2:</u>

$\frac{598.2 - 600}{\sqrt{\frac{25}{n}}} \leq -1.6448$

$-1.8 \leq -1.6448\sqrt{\frac{25}{n}}$

$\sqrt{n} \geq 4.5688$

$n \geq 20.9$ (3 s.f.), where n is a positive integer.

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Qn	Solution
12(i) **[1]**	<u>Method 1</u> $\mathrm{P}(\mu-\sigma \le S \le \mu+\sigma) \approx 0.68$ $\mathrm{P}(\mu \le S \le \mu+\sigma) \approx \frac{0.68}{2} = 0.34$ <u>Method 2</u> $\mathrm{P}(\mu \le S \le \mu+\sigma) = \mathrm{P}\left(\frac{\mu-\mu}{\sigma} \le Z \le \frac{\mu+\sigma-\mu}{\sigma}\right)$ $= \mathrm{P}(0 \le Z \le 1)$ $= 0.341$
12(ii) **[3]**	$\mathrm{P}(S < 56) = 0.10$ $\mathrm{P}\left(Z < \frac{56-\mu}{\sigma}\right) = 0.10$ $\frac{56-\mu}{\sigma} = -1.2815$ $\mu - 1.2815\sigma = 56 ------(1)$ $\mathrm{P}(S > 62) = 0.30$ $\mathrm{P}\left(Z > \frac{62-\mu}{\sigma}\right) = 0.30$ $\frac{62-\mu}{\sigma} = 0.52440$ $\mu + 0.52440\sigma = 62 ------(2)$ Using GC, $\mu = 60.2577 \approx 60.3$ (3 s.f.) $\sigma = 3.3224 \approx 3.32$ (3 s.f.)
12(iii) **[3]**	Required Probability $= \mathrm{P}(S_1 + S_2 + L_1 + L_2 + L_3 \ge 330)$ Let $T = S_1 + S_2 + L_1 + L_2 + L_3$. $\mathrm{E}(T) = 2(50) + 3(75) = 325$ $\mathrm{Var}(T) = 2(2^2) + 3(3.5^2) = 44.75$ $T \sim \mathrm{N}(325, 44.75)$ $P(T \ge 330) = 0.22740 \approx 0.227$ (3 s.f.)
12(iv) **[4]**	Required probability $= \mathrm{P}(3S - 1 \le L_1 + L_2 \le 3S + 1)$ $= \mathrm{P}(-1 \le L_1 + L_2 - 3S \le 1)$ Let $X = L_1 + L_2 - 3S$

	$\mathrm{E}(X)=2(75)-3(50)=0$ $\mathrm{Var}(X)=2(3.5^2)+3^2(2^2)=60.5$ $X\sim \mathrm{N}(0,60.5)$ $P(-1\le X\le 1)=0.10229\approx 0.102\,(3\text{ s.f.})$
12(v) **[2]**	<u>Method 1 (Using sample mean)</u> Let $\bar{L}=\dfrac{L_1+\ldots+L_{10}}{10}$. $\mathrm{E}(\bar{L})=75$ $\mathrm{Var}(\bar{L})=\dfrac{3.5^2}{10}=1.225$ $\bar{L}\sim \mathrm{N}(75,1.225)$ $P(\bar{L}<74)=0.18312\approx 0.183\,(3\text{ s.f.})$ <u>Method 2 (Using sample sum)</u> Required probability $=\mathrm{P}\left(\dfrac{L_1+\ldots+L_{10}}{10}<74\right)$ $=\mathrm{P}(L_1+\ldots+L_{10}<740)$ Let $Y=L_1+\ldots+L_{10}$. $\mathrm{E}(Y)=10(75)=750$ $\mathrm{Var}(Y)=10(3.5^2)=122.5$ $Y\sim \mathrm{N}(750,122.5)$ $P(Y<740)=0.18312\approx 0.183\,(3\text{ s.f.})$