2023 PU3 H2 MATHEMATICS PRELIMINARY EXAMINATION

Paper 9758/01

Qn	Solution
1 **[4]**	$\mathrm{f}(x)=ax^3+bx+c$ At $(1,10)$, $10=a(1)^3+b(1)+c$ $\Rightarrow \quad a+b+c=10$ At $(-2,12)$, $12=a(-2)^3+b(-2)+c$ $\Rightarrow \quad -8a-2b+c=12$ Since that $\mathrm{f}(x)$ is divisible by $x-2$, $\mathrm{f}(2)=0$. $0=a(2)^3+b(2)+c$ $\Rightarrow \quad 8a+2b+c=0$ Using GC, $a=-\frac{7}{3},\ \ b=\frac{19}{3},\ \ c=6$
2 **[4]**	$x^4\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=x+3$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{x^3}+\frac{3}{x^4}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2x^2}-\frac{1}{x^3}+C$ $y=\frac{1}{2x}+\frac{1}{2x^2}+Cx+D$ When x = 1, $\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow-\frac{1}{2}-1+C=0\Rightarrow C=\frac{3}{2}$. When x = 1, $y=\frac{23}{6}\Rightarrow\frac{1}{2}+\frac{1}{2}+\frac{3}{2}+D=\frac{23}{6}\Rightarrow D=\frac{4}{3}$. Hence, $y=\frac{1}{2x}+\frac{1}{2x^2}+\frac{3x}{2}+\frac{4}{3}$.
3(i) **[3]**	Since $x=-2+\mathrm{i}$ is a root of $x^3+ax^2-7x+b=0$, it satisfies the equation.

Qn	Solution
	$(-2+\mathrm{i})^3 + a(-2+\mathrm{i})^2 - 7(-2+\mathrm{i}) + b = 0$ $12 + 4\mathrm{i} + a(3-4\mathrm{i}) + b = 0$ $12 + 3a + b + (4-4a)\mathrm{i} = 0$ By comparing real and imaginary parts, $12 + 3a + b = 0 \quad \Rightarrow 12 = -3a - b$ $4 - 4a = 0 \quad \Rightarrow 4 = 4a$ $\Rightarrow \quad a = 1$ $\therefore b = -15$ **<u>Alternative method (remove from examiner report)</u>** Since all the coefficients of the equation are real, $x = -2+\mathrm{i}$ is a root means that $x = -2-\mathrm{i}$ is also a root. $x^3 + ax^2 - 7x + b = [x-(-2+\mathrm{i})][x-(-2-\mathrm{i})][x+C]$ $= [(x+2)-\mathrm{i}][(x+2)+\mathrm{i}][x+C]$ $= [(x+2)^2 - \mathrm{i}^2][x+C]$ $= (x^2+4x+5)(x+C)$ By comparing: Constant term: $b = 5C$ Coefficient of x: $-7 = 4C + 5 \quad \Rightarrow \quad C = -3$ Coefficient of x^2: $a = C + 4$ So, $b = -15, \quad a = 1$
3(ii) **[3]**	$x^3 + x^2 - 7x - 15 = 0$ Since all the coefficients of the equation are real, $x = -2+\mathrm{i}$ is a root means that $x = -2-\mathrm{i}$ is also a root. Quadratic factor: $[x-(-2+\mathrm{i})][x-(-2-\mathrm{i})]$ $= [(x+2)-\mathrm{i}][(x+2)+\mathrm{i}]$ $= (x+2)^2 - \mathrm{i}^2$ $= x^2 + 4x + 5$ $x^3 + x^2 - 7x - 15 = 0$ $(x^2+4x+5)(x+c) = 0$ By comparing coefficients or long-division, $5c = -15 \quad \Rightarrow \quad c = -3$ $(x^2+4x+5)(x-3) = 0$

Qn	Solution
	The last root is $x = 3$. Hence the other roots are $x = -2 - i$ and $x = 3$

4(i)
[4]

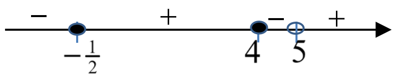
$$\frac{x^2 - 14}{x - 5} \geq 2 - x$$

$$\frac{x^2 - 14 - (2 - x)(x - 5)}{x - 5} \geq 0$$

$$\frac{x^2 - 14 - \left(2x - 10 - x^2 + 5x\right)}{x - 5} \geq 0$$

$$\frac{2x^2 - 7x - 4}{x - 5} \geq 0$$

$$\frac{(2x + 1)(x - 4)}{x - 5} \geq 0$$



$$-\frac{1}{2} \leq x \leq 4 \quad \text{or} \quad x > 5$$

4(ii)
[2]

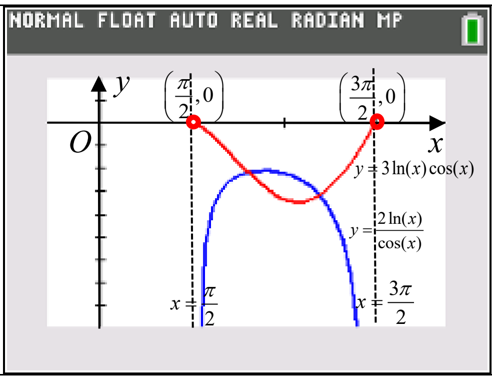
To solve $\frac{x^2 - 14}{|x| - 5} \geq 2 - |x|$, replace x with $|x|$

$$-\frac{1}{2} \leq |x| \leq 4 \quad \text{or} \quad |x| > 5$$

$$\Rightarrow 0 \leq |x| \leq 4 \quad \text{or} \quad |x| > 5$$

$$-4 \leq x \leq 4 \quad \text{or} \quad x < -5 \quad \text{or} \quad x > 5$$

5(i)
[3]



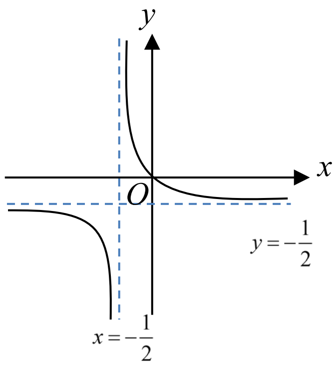
5(ii)
[3]

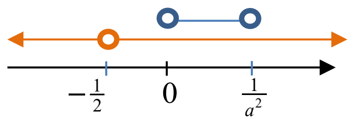
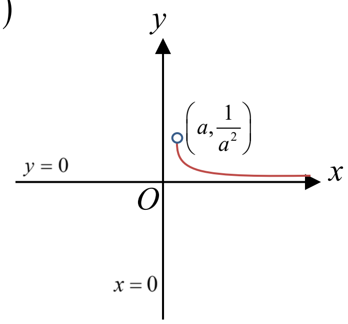
From graphing calculator, the two graphs in **(i)** intersect at (2.52611, –2.26990) (5 d.p.) and (3.75707, –3.24224) (5 d.p.)

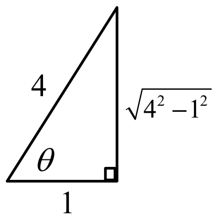
$$\text{Area of } R = \int_{2.52611}^{3.75707} \left[\frac{2\ln x}{\cos x} - 3\cos x \ln x\right] \mathrm{d}x$$

$$\approx 0.94708$$

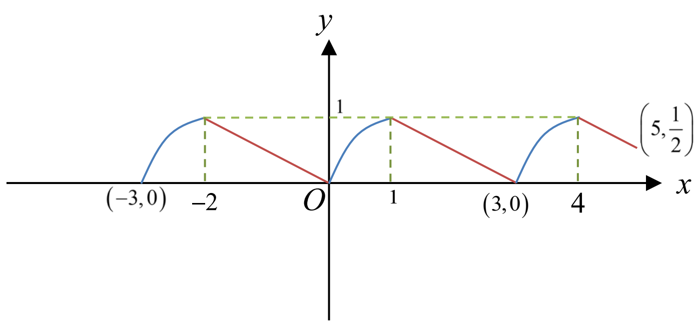
$$= 0.95 \text{ units}^2. \text{ (2 d.p.)}$$

Qn	Solution
5(iii) **[2]**	Volume of $S = \pi \int_{2.52611}^{3.75707} \left[\left(3\cos x \ln x\right)^2 - \left(\frac{2\ln x}{\cos x}\right)^2 \right] \mathrm{d}x$ ≈ 16.968 $= 17.0$ units^3. (3 s.f.)
6(i) **[3]**	$\mathrm{f}(x) = -\frac{x}{2x+1}$ where $\mathrm{D_f} = \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$ Let $y = \mathrm{f}(x)$ $y = -\frac{x}{2x+1}$ $2xy + y = -x$ $x(2y+1) = -y$ $x = -\frac{y}{2y+1}$ $\mathrm{f}^{-1}(y) = -\frac{y}{2y+1}$ $\Rightarrow \mathrm{f}^{-1}(x) = -\frac{x}{2x+1}$  Domain of f^{-1} = Range of f = $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$ Or Domain of $\mathrm{f}^{-1} = \mathbb{R} \setminus \left\{-\frac{1}{2}\right\}$
6(ii) **[1]**	<u>**Method 1 - Hence**</u> Since $\mathrm{f}^{-1}(x) = -\frac{x}{2x+1} = \mathrm{f}(x)$ and the domains $\mathrm{D_f} = \mathrm{D_{f^{-1}}}$ are the same, $\mathrm{f}^2(x) = \mathrm{f}\left(\mathrm{f}^{-1}(x)\right) = x$ <u>**Method 2 - Otherwise**</u> $\mathrm{f}^2(x) = \mathrm{f}\left(-\frac{x}{2x+1}\right)$ $= -\left[\frac{\frac{-x}{2x+1}}{2\left(\frac{-x}{2x+1}\right)+1}\right]$ $= \frac{x}{2x+1} \times \frac{2x+1}{-2x+(2x+1)}$ $= x$

Qn	Solution
6(iii) **[2]**	$\mathrm{f}^{2023}(x)=\mathrm{f}\left(\mathrm{f}^{2022}(x)\right)$ $\quad\quad\quad\;\;=\mathrm{f}(x)$ $\mathrm{f}^{2023}(5)=\mathrm{f}(5)=-\dfrac{5}{11}$
6(iv) **[2]**	$\mathrm{g}(x)=\dfrac{1}{ax}$ where $\mathrm{D_g}=(a,\infty)$ Range of g $=\left(0,\dfrac{1}{a^2}\right)$   Since $\mathrm{R_g}\subseteq \mathrm{D_f}$, composite function fg exists.
7(i) **[4]**	$z^2=18\mathrm{i}$ Let $z=x+y\mathrm{i}$ $(x+y\mathrm{i})^2=18\mathrm{i}$ $x^2+2xy\mathrm{i}-y^2=18\mathrm{i}$ By comparison of real and imaginary parts, $x^2-y^2=0$ $2xy=18 \;\Rightarrow\; y=\dfrac{9}{x}$ $x^2-\left(\dfrac{9}{x}\right)^2=0$ $x^4-81=0$ $x=\pm 3$ when $x=3,\;\; y=3$ when $x=-3,\;\; y=-3$ $z_1=3+3\mathrm{i},$ $z_2=-3-3\mathrm{i}$

Qn	Solution
7(ii) **[4]**	$z^2 = 18\mathrm{i}$ $\Rightarrow \lvert z^2 \rvert = 18, \quad \arg\left(z^2\right) = \dfrac{\pi}{2}$ $w = -1 - \mathrm{i}\sqrt{3}$ $\Rightarrow \lvert w \rvert = 2, \quad \arg\left(w\right) = -\dfrac{2\pi}{3}$ $\left\lvert \left(\dfrac{w}{z^2}\right)^* \right\rvert = \dfrac{\lvert w^* \rvert}{\lvert z^2 \rvert} = \dfrac{2}{18} = \dfrac{1}{9}$ $\arg\left(\dfrac{w}{z^2}\right)^* = -\left[\arg\left(w\right) - \arg\left(z^2\right)\right]$ $= -\left(-\dfrac{2\pi}{3} - \dfrac{\pi}{2}\right)$ $= \dfrac{7\pi}{6}$ $= -\dfrac{5\pi}{6}$ $\left(\dfrac{w}{z^2}\right)^* = \dfrac{1}{9}\mathrm{e}^{\mathrm{i}\left(-\frac{5\pi}{6}\right)}$
8a(i) **[3]**	$(3\mathbf{b} - 2\mathbf{a})\cdot(3\mathbf{b} - 2\mathbf{a}) = 9\lvert \mathbf{b} \rvert^2 - 12\left(\mathbf{a}\cdot\mathbf{b}\right) + 4\lvert \mathbf{a} \rvert^2$ $\Rightarrow \lvert 3\mathbf{b} - 2\mathbf{a} \rvert^2 = 9(2)^2 - 12\left(\mathbf{a}\cdot\mathbf{b}\right) + 4(1)^2$ $\Rightarrow \left(\sqrt{34}\right)^2 = 40 - 12\left(\mathbf{a}\cdot\mathbf{b}\right)$ $\Rightarrow \mathbf{a}\cdot\mathbf{b} = \dfrac{40-34}{12} = \dfrac{1}{2}$
8a(ii) **[3]**	Now, $\mathbf{a}\cdot\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$ where θ is the angle between **a** and **b** Hence, $\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert} = \dfrac{0.5}{(1)(2)} = \dfrac{1}{4}$. $\Rightarrow \sin\theta = \dfrac{\sqrt{4^2 - 1^2}}{4} = \dfrac{\sqrt{15}}{4}$.  Hence, $\lvert\mathbf{a}\times\mathbf{b}\rvert = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\lvert\sin\theta\rvert = (1)(2)\dfrac{\sqrt{15}}{4} = \dfrac{\sqrt{15}}{2}$. Geometrical Interpretation of $\lvert\mathbf{a}\times\mathbf{b}\rvert$: (1) the perpendicular (or shortest) distance from B to the line OA OR

Qn	Solution
	(2) area of parallelogram with adjacent sides OA and OB / twice the area of triangle OAB OR (3) height of triangle OAB with OA as the base.
8(b) **[3]**	By Ratio Theorem, $\overrightarrow{OP}=\dfrac{2\mathbf{b}+\mathbf{a}}{3}$ … (1) $\overrightarrow{OM}=\dfrac{\mathbf{b}+\mathbf{c}}{2}$ … (2) $OBMN$ forms a rhombus $\overrightarrow{ON}=\overrightarrow{BM}$ $=\overrightarrow{OM}-\mathbf{b}$ $=\dfrac{\mathbf{b}+\mathbf{c}}{2}-\mathbf{b}$ $=\dfrac{\mathbf{c}-\mathbf{b}}{2}\quad ---(3)$ P is a mid-point of MN $\Rightarrow \overrightarrow{OP}=\dfrac{1}{2}\left(\overrightarrow{OM}+\overrightarrow{ON}\right)$ --- (4) **<u>Method 1</u>**: Equating two different expressions for $\overrightarrow{OP}$ Put (1), (2), (3) into (4) $\overrightarrow{OP}=\dfrac{2\mathbf{b}+\mathbf{a}}{3}=\dfrac{1}{2}\left[\left(\dfrac{\mathbf{b}+\mathbf{c}}{2}\right)+\left(\dfrac{\mathbf{c}-\mathbf{b}}{2}\right)\right]$ $\overrightarrow{OP}=\dfrac{1}{2}\mathbf{c}$ $\Rightarrow 4\mathbf{b}+2\mathbf{a}=3\mathbf{c}$ $\Rightarrow 2\mathbf{a}+4\mathbf{b}-3\mathbf{c}=\mathbf{0}$. (shown) **<u>Method 2</u>**: Equating two different expressions for $\overrightarrow{ON}$ From (4), $\overrightarrow{ON}=2\overrightarrow{OP}-\overrightarrow{OM}$ $\Rightarrow \left(\dfrac{\mathbf{c}-\mathbf{b}}{2}\right)=2\left(\dfrac{2\mathbf{b}+\mathbf{a}}{3}\right)-\left(\dfrac{\mathbf{b}+\mathbf{c}}{2}\right)$ [from (1), (2) and (3)] $\Rightarrow \dfrac{\mathbf{c}-\mathbf{b}}{2}=\dfrac{2\mathbf{a}}{3}+\dfrac{5\mathbf{b}}{6}-\dfrac{\mathbf{c}}{2}$

Qn	Solution
	$\Rightarrow \frac{2\mathbf{a}}{3}+\frac{5\mathbf{b}}{6}+\frac{\mathbf{b}}{2}-\frac{\mathbf{c}}{2}-\frac{\mathbf{c}}{2}=\mathbf{0}$ $\Rightarrow \frac{2\mathbf{a}}{3}+\frac{4\mathbf{b}}{3}-\mathbf{c}=\mathbf{0}$ $\Rightarrow 2\mathbf{a}+4\mathbf{b}-3\mathbf{c}=\mathbf{0}$. (shown)
9(a) i-iii [5]	**(i)** Horizontal asymptote: $y=2b$ x-intercept: $(a-1,0)$ **(ii)** Horizontal asymptote: $y=b$ x-intercept: $(-a,0)$ **(iii)** Horizontal asymptote: $y=\frac{1}{b}$ x-intercept: inconclusive (it would result in a vertical asymptote at $x=a$)
9(b) (i) [3]	
9(b) (ii) [2]	$\int_{-3}^{5} \mathrm{f}(x)\,\mathrm{d}x$ refers to the area under the curve from $x=-3$ to $x=5$. **<u>Method 1</u>** $\int_{-3}^{5} \mathrm{f}(x)\,\mathrm{d}x = 3\int_0^1 \sqrt{x}\,\mathrm{d}x + 2\int_1^3 -\frac{1}{2}x+\frac{3}{2}\,\mathrm{d}x + \int_1^2 -\frac{1}{2}x+\frac{3}{2}\,\mathrm{d}x$ $=4.75$ **<u>Method 2</u>** $\int_{-3}^{5} \mathrm{f}(x)\,\mathrm{d}x = 3\int_0^1 \sqrt{x}\,\mathrm{d}x + 2\left[\frac{1}{2}(2)(1)\right] + \frac{1}{2}\left(1+\frac{1}{2}\right)(1)$ $=4.75$

Qn	Solution
10 (a)(i) [2]	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\sin^{-1}\left(\mathrm{e}^{2x}\right)\right]=\frac{1}{\sqrt{1-\left(\mathrm{e}^{2x}\right)^2}}\left(\mathrm{e}^{2x}\right)(2)=\frac{2\mathrm{e}^{2x}}{\sqrt{1-\mathrm{e}^{4x}}}$.
10 (a)(ii) [3]	$\int \frac{2\mathrm{e}^{2x}+\mathrm{e}^{4x}}{\sqrt{1-\mathrm{e}^{4x}}}\,\mathrm{d}x=\int \frac{2\mathrm{e}^{2x}}{\sqrt{1-\mathrm{e}^{4x}}}+\frac{\mathrm{e}^{4x}}{\sqrt{1-\mathrm{e}^{4x}}}\,\mathrm{d}x$ $=\sin^{-1}\left(\mathrm{e}^{2x}\right)-\frac{1}{4}\int -4\mathrm{e}^{4x}\left(1-\mathrm{e}^{4x}\right)^{-\frac{1}{2}}\,\mathrm{d}x$ [From **(i)**] $=\sin^{-1}\left(\mathrm{e}^{2x}\right)-\frac{1}{4}\left[\frac{\left(1-\mathrm{e}^{4x}\right)^{\frac{1}{2}}}{\frac{1}{2}}\right]+c$ $=\sin^{-1}\left(\mathrm{e}^{2x}\right)-\frac{1}{4}\left[2\sqrt{1-\mathrm{e}^{4x}}\right]+c$ $=\sin^{-1}\left(\mathrm{e}^{2x}\right)-\frac{1}{2}\sqrt{1-\mathrm{e}^{4x}}+c$
10 (b)(i) [1]	$\sin^{n-2}\theta\cos^2\theta=\sin^{n-2}\theta\left(1-\sin^2\theta\right)$ $=\sin^{n-2}\theta-\sin^n\theta$. (shown)
10 (b)(ii) [4]	$\int_0^{\frac{\pi}{2}}\sin^n\theta\,\mathrm{d}\theta$ $=\int_0^{\frac{\pi}{2}}\sin^{n-1}\theta\sin\theta\,\mathrm{d}\theta$ $=\left[\left(\sin^{n-1}\theta\right)\left(-\cos\theta\right)\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\left(-\cos\theta\right)\left[\left(n-1\right)\sin^{n-2}\theta\cos\theta\right]\mathrm{d}\theta$ $=[0-0]+\left(n-1\right)\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta\cos^2\theta\,\mathrm{d}\theta$ $=\left(n-1\right)\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta-\sin^n\theta\,\mathrm{d}\theta$ [from **(i)**] $=\left(n-1\right)\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta\,\mathrm{d}\theta-\left(n-1\right)\int_0^{\frac{\pi}{2}}\sin^n\theta\,\mathrm{d}\theta$ $n\int_0^{\frac{\pi}{2}}\sin^n\theta\,\mathrm{d}\theta=\left(n-1\right)\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta\,\mathrm{d}\theta$ $\int_0^{\frac{\pi}{2}}\sin^n\theta\,\mathrm{d}\theta=\frac{n-1}{n}\int_0^{\frac{\pi}{2}}\sin^{n-2}\theta\,\mathrm{d}\theta$. (shown)

Qn	Solution
10 (b) (iii) [2]	$\int_0^{\frac{\pi}{2}} \sin^3\theta \, d\theta = \left(\frac{3-1}{3}\right)\int_0^{\frac{\pi}{2}} \sin\theta \, d\theta$ $= \frac{2}{3}\left[-\cos\theta\right]_0^{\frac{\pi}{2}}$ $= \frac{2}{3}[0-(-1)]$ $= \frac{2}{3}$

Qn	**Solution**

11(i) [3]

Month n	Amount in account at the start of nth month	Amount in account at the end of nth month
1 (Jan22)	500	(1.001)(500)
2	(1.001)(500)+500	$1.001[1.001(500)+500]$ $=1.001^2(500)+1.001(500)$
12	…	$1.001^{12}(500)+$ $1.001^{11}(500)+..+1.001(500)$

Total amount at the end of 12 months

$$=1.001^{12}(500)+1.001^{11}(500)+..+1.001(500)$$

$$=500\underbrace{\left(1.001+1.001^2+...+1.001^{12}\right)}_{\text{GP: }a=1.001,r=1.001,n=12}$$

$$=500\left[\frac{1.001\left(1-1.001^{12}\right)}{1-1.001}\right]$$

$$=6039.1434$$

$$=6039.14$$

total amount of money: \$6039.14

11(ii) [4]

Total amount at the end of n months

$$=1.001^n(500)+1.001^{n-1}(500)+..+1.001(500)$$

$$=500\underbrace{\left(1.001+1.001^2+...+1.001^n\right)}_{\text{GP: }a=1.001,r=1.001,n\text{ terms}}$$

$$=500\left[\frac{1.001\left(1-1.001^n\right)}{1-1.001}\right]$$

$$=-500500\left(1-1.001^n\right)$$

$$=500500\left(1.001^n-1\right)$$

For $500500\left(1.001^n-1\right)>20\ 000$:

Method 1

n	$500500\left(1.003^n-1\right)$
39	19895
40	20415
41	20936

Minimum number of months is 40.

	Method 2 $500500\left(1.001^n-1\right)>20\ 000$ $1.001^n>\frac{1041}{1001}$ $n\ln 1.001>\ln\left(\frac{1041}{1001}\right)$ $n>39.2$ Minimum number of months is 40.
11(iii) **[3]**	(table below)

Month m	Amount in account at the start of mth month	Amount in account at the end of mth month
1 (Jan22)	50 000	$1.001(50000)-k$
2	$1.001(50000)-k$	$1.001\left(1.001(50000)-k\right)-k$ $=1.001^2(50000)-1.001k-k$
3	$1.001^2(50000)-1.001k-k$	$1.001\left(1.001^2(50000)-1.001k-k\right)-k$ $=1.001^3(50000)-(1.001)^2k-1.001k-k$

Amount of money in account at the end of mth month

$$=1.001^m(50000)-(1.001)^{m-1}k-\ldots-1.001k-k$$

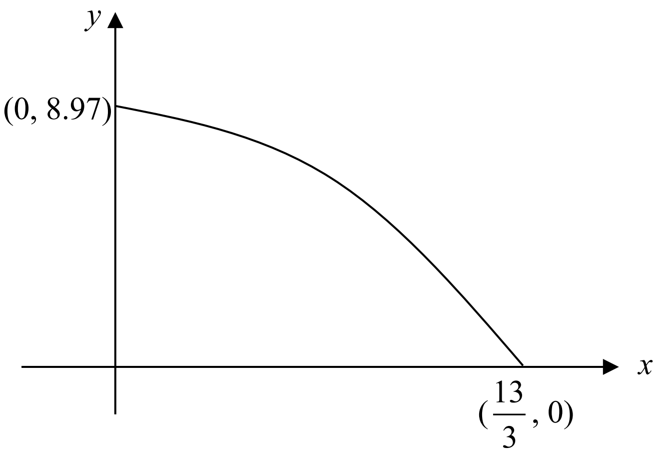
$$=1.001^m(50000)-k\underbrace{\left(1+1.001+\ldots+1.001^{m-1}\right)}_{\text{GP: }a=1,r=1.001,m\text{ terms}}$$

$$=1.001^m(50000)-k\left(\frac{1\left(1-1.001^m\right)}{1-1.001}\right)$$

$$=1.001^m(50000)+1000k\left(1-1.001^m\right)$$

$$=1.001^m\left[50000-1000k\right]+1000k$$

11(iv) **[3]**	At the end of 2024 refers to a duration of 3 years. let m = 36, $1.001^{36}\left[50000-1000k\right]+1000k=0$ $k\left[1.001^{36}(1000)-1000\right]=1.001^{36}(50000)$ Using GC, $k=1414.73$ Maximum amount of money is \$1414.

Qn	Solution
12(i) **[2]**	$x = t - \frac{10}{t}, \quad y = 6t - t^2, \quad \sqrt{10} \le t \le 6$  Note: When $t = \sqrt{10}$, $x = 0$ When $t = 6$, $x = \frac{13}{3}$
12(ii) **[3]**	$x = t - \frac{10}{t}, \quad y = 6t - t^2, \quad \sqrt{10} \le t \le 6$ When $x = 0$, $t = \sqrt{10}$ When $x = \frac{13}{3}$, $t = 6$ $\frac{\mathrm{d}x}{\mathrm{d}t} = 1 + \frac{10}{t^2}$ Required area $= \int_0^{\frac{13}{3}} y \, \mathrm{d}x$ $= \int_{\sqrt{10}}^{6} y \frac{\mathrm{d}x}{\mathrm{d}t} \, \mathrm{d}t$ $= \int_{\sqrt{10}}^{6} \left(6t - t^2\right)\left(1 + \frac{10}{t^2}\right) \mathrm{d}t$ $= 26.5917$ $= 26.6 \text{ km}^2$ (3 s.f)

12(iii) **[3]**	Let A be the Area of $OPRQ$. $A = xy = \left(t - \frac{10}{t}\right)\left(6t - t^2\right)$ $A = 6t^2 - t^3 - 60 + 10t$ $\frac{\mathrm{d}A}{\mathrm{d}t} = 12t - 3t^2 + 10$ At maximum area, Let $\frac{\mathrm{d}A}{\mathrm{d}t} = 0$ $12t - 3t^2 + 10 = 0$ Using GC, $t = -0.70801$ (rejected since $\sqrt{2} \le t \le 5$) or $t = 4.7080$ $t = 4.71$ (3 s.f.)
12(iv) **[4]**	$x = t - \frac{10}{t}, \quad y = 6t - t^2, \quad \sqrt{10} \le t \le 6$ Let the fence intersects the curve at B with parameter b. Then B has coordinates $\left(b - \frac{10}{b},\ 6b - b^2\right)$. --- **Method 1**: Use area to the left of the fence $\frac{1}{2}(\text{Area of land}) = \int_0^{b-\frac{10}{b}} y \,\mathrm{d}x$ $\frac{1}{2}(26.5917) = \int_{\sqrt{10}}^{b} \left(6t - t^2\right)(1 + \frac{10}{t^2}) \,\mathrm{d}t$ $\frac{1}{2}(26.5917) = \int_{\sqrt{10}}^{b} 6t + \frac{60}{t} - t^2 - 10 \,\mathrm{d}t$ $\frac{1}{2}(26.5917) = \left[3t^2 + 60\ln t - \frac{t^3}{3} - 10t\right]_{\sqrt{10}}^{b}$ $13.29585 = \left[\left(3b^2 + 60\ln b - \frac{b^3}{3} - 10b\right) - \left(3\left(\sqrt{10}\right)^2 + 60\ln\left(\sqrt{10}\right) - \frac{\left(\sqrt{10}\right)^3}{3} - 10\left(\sqrt{10}\right)\right)\right]$ Using GC (Graph), $b = 4.0282$ Hence equation of fence (vertical line) is $x = 4.0282 - \frac{10}{4.0282}$ $x = 1.55$ (3 s.f.) ---

	Method 2: Use area to the right of the fence $\frac{1}{2}(\text{Area of land}) = \int_{25b-\frac{50}{b}}^{\frac{14}{3}} y \, \mathrm{d}x$ $\frac{1}{2}(26.5917) = \int_{b}^{6} \left(6t-t^2\right)(1+\frac{10}{t^2}) \, \mathrm{d}t$ $13.29585 = \left[3t^2 + 60\ln t - \frac{t^3}{3} - 10t\right]_b^6$ $13.29585 = \left[\left(3(6)^2 + 60\ln(6) - \frac{(6)^3}{3} - 10(6)\right) - \left(3b^2 + 60\ln b - \frac{b^3}{3} - 10b\right)\right]$ Using GC (Graph), $b = 4.0282$ Hence equation of fence (vertical line) is $x = 4.0282 - \frac{10}{4.0282}$ $x = 1.55$ (3 s.f.)