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CANDIDATE NAME

CLASS

ADMISSION NUMBER

2023 Preliminary Examination
Pre-University 3

MATHEMATICS **9758/02**

Paper 2 **19 September 2023**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your admission number, name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Qn No.	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	*	Total
Score														
Max Score	**5**	**8**	**8**	**8**	**11**	**4**	**7**	**7**	**7**	**9**	**13**	**13**		**100**

This document consists of **28** printed pages.

Section A: Pure Mathematics [40 marks]

1 **(i)** It is given that $x\frac{\mathrm{d}y}{\mathrm{d}x}=x^3+xy+2y$, $x\neq 0$. Using the substitution $y=vx^2$, show that the differential equation can be reduced to

$$\frac{\mathrm{d}v}{\mathrm{d}x}=v+1.$$ [2]

(ii) Given that $y=1$ when $x=1$, solve the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x}=x^3+xy+2y.$$ [3]

2 **(a)** Given that $\mathrm{f}(x)=\sec^2 x$, find $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$. Hence, find the Maclaurin series for $\mathrm{f}(x)$, up to and including the term in x^2. [5]

(b) Using standard series from the List of Formulae (MF26), expand $(1+ax)^{-4}$ as far as the term in x^2, where a is a non-zero constant. Hence find the value of a for which the coefficients of the x and x^2 terms in the expansion are equal. [3]

3 **(a)** A sequence $u_2, u_3, u_4,...$ is such that $u_n=\ln\left(\frac{n-1}{n+1}\right)$ where $n\geq 2$.

Find in terms of N, an expression for S_N, where $S_N=u_2+u_3+u_4...+u_{N-1}+u_N$, leaving your answer in the form $\ln a-\mathrm{f}(N)$, where a is a positive constant. [3]

(b) **(i)** Find the set of values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{3x}{x+1}\right)^n$ has a finite sum. [3]

(ii) Given that $x=0.25$, find the value of $\sum_{n=1}^{\infty}\left(\frac{3x}{x+1}\right)^n$. [2]

4 The curve C has equation $2x^2+y^2=20$.

(i) Sketch curve C, giving the exact coordinates of any points where the curve meets the axes. [2]

(ii) Show that $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{2x}{y}$. [1]

(iii) It is given that point $P(a,b)$ is on C. The normal to C at P passes through the point (1, 0). Find the possible coordinates of P. [5]

5 The plane p has equation $x+2y-5z=4$, and contains the line l with equation $\mathbf{r}=\begin{pmatrix}3\\-2\\b\end{pmatrix}+\lambda\begin{pmatrix}a\\2a\\2\end{pmatrix}$, where a and b are constants and λ is a parameter.

(i) Find the perpendicular distance from the origin to p. [2]

(ii) Show that $a=2$ and $b=-1$. [3]

Relative to O, the point Q has position vector $-4\mathbf{i}+7\mathbf{j}+3\mathbf{k}$.

(iii) The points C and D are on l such that they are a distance of $\sqrt{110}$ away from Q. Find the position vectors $\overrightarrow{OC}$ and $\overrightarrow{OD}$.

(iv) Without performing any further calculations, explain whether the shortest distance from Q to l is greater or smaller than $\sqrt{110}$. [1]

Section B: Probability and Statistics [60 marks]

6 A game is played by choosing 2 cards from 5 identical cards numbered 1 to 5, without replacement. The score, X, is the sum of the two numbers shown on the chosen cards.

(i) Determine the probability distribution of the score of this game. [2]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [2]

7 The probability that a student is present in school on a typical day is 0.96. In a particular school, there are 28 students in each class. The number of students in a class who are present in school on a typical school day is denoted by the random variable X. A student is considered absent if he is not present in school the whole day. Assume that X can be modelled by a binomial distribution.

(i) Find the probability that all students in a class are present in school on a typical school day. [1]

(ii) Find the most likely number of students who are present in school on a typical school day. [1]

An investigation by the form teacher will be carried out if there are more than 2 students in the class who are not present in school on a typical school day.

(iii) Find the probability that an investigation is carried out on a typical school day. [2]

There are 16 classes in a level. The school leaders must be alerted if an investigation has to be carried out in at least a third of the classes in the level.

(iv) Find the probability that the school leaders are alerted on a typical school day. [3]

8 For events A, B and C, it is given that $\mathrm{P}(A)=0.4$, $\mathrm{P}(C)=0.3$ and $\mathrm{P}(B\cap C)=0.1$. It is also given that events A and B are mutually exclusive, and that events A and C are independent.

(i) Find $\mathrm{P}(A\cap B)$. [1]

(ii) Find $\mathrm{P}(A|C)$. [1]

(iii) Given also that events B and C are independent, find $\mathrm{P}(A'\cap B'\cap C')$. [3]

(iv) Given instead that events B and C are **not** independent, find the greatest and least possible values of $\mathrm{P}(B)$. [2]

9 A class has 12 male and 8 female students. An executive committee of 4 people comprising a chairperson, a vice-chairperson, a secretary and a treasurer is to be chosen from the class.

Find the number of ways that the chairperson, the vice-chairperson, the secretary and the treasurer can be chosen from the class

(i) without any restrictions, [1]

(ii) such that there is at least a male student and at least a female student in the executive committee. [3]

At a school event, the class is allocated a table with 10 seats. It was decided that the form teacher, executive committee and 5 pre-selected students from the class will attend the event. The 10 of them sit at random around the table.

(iii) Find the probability that the executive committee members are all seated in adjacent seats and the form teacher is **not** seated next to an executive committee member. [3]

10 In an experiment to improve road safety, the following information was gathered about the braking distance d, measured in feet, for a car travelling at different velocities v, measured in miles per hour.

v	10	20	30	40	50	60	70	80
d	27	58	97	155	212	308	399	516

(i) Draw a scatter diagram for these values, labelling the axes. [1]

(ii) By calculating the relevant product moment correlation coefficients, determine whether the relationship between v and d is modelled better by $d = av + b$ or by $\ln d = av + b$. Explain how you decide which model is better, and state the equation in this case. [4]

(iii) Use your equation found in part **(ii)** to estimate the braking distance when the car is travelling at 64 miles per hour. Explain whether your estimate is reliable. [2]

(iv) Scientists from another country wish to apply the model, but the measured velocity of the car was recorded as m kilometres per hour instead. Given that $1 \text{ kilometre} = 0.621 \text{ miles}$, re-write your equation in part **(ii)** in terms of d and m so that it can be used to estimate the braking distance for a given value of m. [2]

11 A factory which produces scented candles claims that the mean mass of the scented candles is 600 grams. For quality control purposes, a quality inspector took a random sample of 30 scented candles to check if the mean mass of the candles is 600 grams.

(i) State what it means for a sample to be random in this context. [1]

The masses, x grams, of a random sample of 30 scented candles are summarised as follows.

$$\sum(x-600) = 69 \qquad \sum(x-600)^2 = 859$$

(ii) Calculate the unbiased estimates of the population mean and variance of the mass of scented candles. [2]

(iii) Carry out the test, at 1% level of significance, for the quality inspector, giving your conclusion in context. You should state your hypotheses and define any symbols you use. [5]

(iv) Explain the meaning of 'p-value' in the context of this question. [1]

The factory adopted a new manufacturing process and the mass of the scented candles produced is now known to have a normal distribution with a population variance of 25 gram^2.

(v) A random sample of n scented candles was taken, and the mean mass is 598.2 grams. Given that the quality inspector concludes at 5% level of significance that the mean mass of the scented candles has been overstated, find the range of possible values of n. [4]

12 **In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.**

A beauty company sells their best-selling perfume in two sizes. The volume of perfume, in millilitres, in a small bottle and large bottle is denoted by S and L respectively. S and L can be modelled using normal distributions with the means and standard deviations as follows.

Size	Mean (millilitres)	Standard deviation (millilitres)
Small	μ	σ
Large	75	3.5

(i) Deduce an approximate value for $\mathrm{P}(\mu \le S \le \mu+\sigma)$. [1]

10% of the small bottles contain less than 56 millilitres of perfume and 30% of the small bottles contain more than 62 millilitres of perfume.

(ii) Find the values of μ and σ. [3]

Use $\mu = 50$ and $\sigma = 2$ for the rest of the question.

(iii) Find the probability that the total volume of perfume in 2 randomly chosen small bottles and 3 randomly chosen large bottles is at least 330 millilitres. [3]

(iv) Find the probability that the total volume of perfume of 2 randomly chosen large bottles is within ± 1 millilitres of three times the volume of perfume in a randomly chosen small bottle. [4]

(v) Find the probability that the average volume of perfume of 10 randomly chosen large bottles is less than 74 millilitres. [2]

End of Paper