EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2023

General Certificate of Education Advanced Level

Higher 2

CANDIDATE NAME			
CIVICS GROUP		INDEX NO.	

MATHEMATICS

9758/02

Paper 2 [100 marks]

21 September 2023

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **24** printed pages and **4** blank pages.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total

Section A: Pure Mathematics (40 marks)

1 The complex numbers z and w satisfy the following equations.

$$4z + 5\mathrm{i}w = 7$$
$$(1-\mathrm{i})z + 8w = 30$$

Find z and w, giving your answers in the form $a + \mathrm{i}b$ where a and b are real numbers. [4]

2 **(a)** Find the exact roots of the equation $\left|x^2 - 7x + 3\right| = 13 - x$. [3]

(b) On the same axes, sketch the curves with equations $y = \left|x^2 - 7x + 3\right|$ and $y = 13 - x$, indicating the value of the x-coordinates of any intersections. Hence solve exactly the inequality $\left|x^2 - 7x + 3\right| < 13 - x$. [5]

3 The variables x and y are related by the differential equation $\pi\dfrac{\mathrm{d}y}{\mathrm{d}x} + y\left(3 - \pi\tan x\right) = 0$.

(a) Using the substitution $y = z\sec x$, show that $\dfrac{\mathrm{d}z}{\mathrm{d}x} = \dfrac{-3z}{\pi}$. Hence, show that the particular solution for which $y = 2$ when $x = \dfrac{\pi}{3}$ can be expressed in the form $y\cos x = \mathrm{e}^{a+bx}$ where a and b are constants to be determined. [6]

(b) For the graph of the solution found in part **(a)**, find the equations of the two vertical asymptotes closest to the y-axis. [2]

4 A curve C has equation

$$x^3 + y^3 - xy = A$$

where A is a non-zero constant.

(a) Show that any stationary points on C lie on

$$y = 3x^2.$$ [2]

(b) Find, in terms of A, the x-coordinates of the stationary points, and hence determine the range of values of A for which C has two distinct stationary points. [4]

(c) Suppose that C has two stationary points. Determine the nature of each of the stationary points found in part **(b)**. [3]

5 Planes p and q are perpendicular. Plane p has equation $2x - z = 10$. Plane q contains the line l with equation $\mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$, where λ is a parameter.

(a) Show that q has equation $x + 2y + 2z = 7$. [2]

(b) Find a vector equation of the line m where p and q meet. [2]

The point A with coordinates $(2,1,-6)$ lies on p and the shortest distance from A to q is k.

(c) Find the position vector of the foot of perpendicular, F from A to q. Hence find the value of k. [4]

(d) Find vector equations of the lines in p such that the shortest distance from each line to q is k. [3]

Section B: Probability & Statistics (60 marks)

6 For the events A, B and C it is given that $\mathrm{P}(A) = \mathrm{P}(A \cap B) = \frac{1}{5}$ and $\mathrm{P}(B) = 2\mathrm{P}(A)$. It is also given that $\mathrm{P}(B \cap C) = \frac{3}{10}$.

(a) Find the greatest and least possible values of $\mathrm{P}(A \cap C)$. [4]

(b) Draw a Venn diagram showing all 3 events representing the case where the value of $\mathrm{P}(A \cap C)$ is the greatest. [1]

It is now known that $\mathrm{P}(A' \cap B' \cap C') = \frac{1}{12}$.

(c) If A and C are independent events, calculate the value of $\mathrm{P}(A \cap C)$. [2]

7 X is a normal random variable with mean 1 and variance 1, and Y is also a normal random variable with mean μ and variance 2.

(a) Given that $\mathrm{P}(0 \le X + Y < 3) > 0.44$, find the range of possible values of μ. [3]

(b) Given instead that $\mu = 10$ and the probability that the sum of n independent observations of X exceeds $2Y$ by at least 10 is less than 0.03, find the largest possible value of n. [4]

(c) State an assumption needed for the above calculations to be valid. [1]

8 A company produces ceramic vases. A fixed number of randomly chosen vases are inspected each day and the number of defective vases found in a day is denoted by X.

(a) State, in context, two assumptions needed for X to be well modelled by a binomial distribution. [2]

Assume now that X has the distribution B(30, 0.04).

(b) Find the probability that, on a randomly chosen day, more than 2 defective vases are found. [2]

The number of defective vases found each day is independent of that on other days.

(c) Find the probability that, in a randomly chosen 5-day working week, more than 2 defective vases are found on at most 1 day. [2]

(d) In a particular 5-day working week, a total of 5 defective vases were found. Find the probability that all 5 of them were found on exactly 2 days that are consecutive. [4]

9 A personality test assesses 32 key traits. These 32 traits are grouped into 4 domains – Cognition, Influence, Rapport and Planning – with each domain consisting of 8 traits. The test result for a person consists of his/her top 5 traits in ascending order of strength.

(a) Find the number of different results that the personality test can produce. [2]

(b) Find the number of different results that can be produced with traits from at least 2 domains. [2]

(c) Find the number of different results that can be produced with at least one trait from each domain and neither of the top 2 traits are from the Cognition domain. [3]

(d) A result contains 2 particular traits from Rapport, 2 particular traits from Planning and 1 particular trait from Influence. Find the number of different possible results that can be produced with no consecutive traits from the same domain. [3]

10 A batch of plants is grown and then studied to examine how well they are growing. For each plant, data on two indicators are collected: the average root hair length x mm, and the amount of potassium found in the roots y mg. The data from a random sample of 8 plants are shown in the table below.

Average root hair length (x mm)	3.3	3.9	7.5	8.1	13.3	22.1	32.2	36.1
Amount of potassium (y mg)	9	t	25	27	33	37	41	44

(a) **(i)** Calculate an unbiased estimate of the population variance of the average root hair lengths. [1]

(ii) Find an expression for the unbiased estimate of the population variance of the amount of potassium in the roots, in the form

$$\frac{1}{A}\left[\left(B+t^2\right)-\frac{(C+t)^2}{D}\right],$$

where A, B, C and D are positive integers to be determined. [2]

You are now given that $t=17$.

(b) Sketch a scatter diagram of y against x. State the value of the product moment correlation coefficient between y and x. [2]

(c) With reference to your answers in part **(b)**, explain whether an equation of the form $y=mx+k$, where m and k are constants, provides an accurate model of the relationship between x and y. [1]

(d) The data is instead modelled by the regression equation $y=p\ln x+q$. Find the values of p and q, and the value of the corresponding product moment correlation coefficient. [2]

(e) Give a contextual interpretation for the value of p found in part **(d)**. [1]

(f) Hence find the estimated amount of potassium in the roots of a plant with average root hair length 30.0 mm. Explain whether your estimate is reliable. [2]

(g) It is now required to estimate the average root hair length for a plant with 28 mg of potassium in its roots. Explain whether it is appropriate to use the equation in part **(d)** to obtain this estimate. [1]

11 The amount customers spend is often correlated with the amount of time they spend in stores. According to a national survey of 500,000 shopping trips, the average time customers spend in a clothing store is 30 min. The first, second and third quartiles of the time spent, in min, are also given as follows:

Q_1	8.5
Q_2 (median)	18.5
Q_3	36.1

(a) Explain why the data suggests that shopping times do not follow a normal distribution. [1]

Jack, a clothing store manager, suspects that customers spend less time in his store than the national average.

(b) Determine whether Jack should carry out a 1-tail or a 2-tail test. State the hypotheses for the test and define any symbols you use in this context. [3]

Jack collected a random sample of 50 shopping times. The results, x (in min), are summarised as follows:

$$\sum x = 1475, \quad \sum x^2 = 45029$$

(c) Test, at the 10% significance level, whether Jack's suspicion is valid. [5]

(d) Explain what is meant by 'at the 10% significance level' in the context of the question. [1]

(e) Explain why in performing the hypothesis test, there is no need for Jack to be concerned that the shopping times of customers in his store do not follow a normal distribution. [1]

Upon closer inspection of the data, Jack realises that he has made some miscalculations. While there is no mistake in the calculation of $\sum x$, the value of $\sum x^2$ should be larger than initially calculated.

(f) Does this miscalculation affect Jack's initial conclusion? Explain your answer. [2]